P
10

10. PRAKTIKA: EREMU BEKTORIALAK

```
Clear["Global`*"]
```

▼ Proposatutako Ariketa P-10.1

Ondorengo eremu bektoriala emanik:

$$\vec{F}(x,y)=x*y\,\vec{i}+\left(x^2-y^2\right)\vec{j},$$

kalkulatu korronte lerroak eta irudikatu eremu bektorialarekin batera.

▼ Soluzioa P-10.1

★ Korronte lerroen E.D. lortuko dugu

Puntu bakoitzeko kurbarekiko bektore ukitzailea (m(x,y),n(x,y)) da eta hauxe izango da eremu bektoriala, bere malda n(x,y)/m(x,y) izanik

```
m[x_, y_] = x * y;
n[x_, y_] = x^2 - y^2;
```

Korronte lerroen E.D. definituko dugu eta ebatzi egingo dugu

```
edlc = y'[x] == n[x, y[x]] / m[x, y[x]]
```

$$y'[x] == \frac{x^2 - y[x]^2}{x\,y[x]}$$

```
S = DSolve[edlc, y[x], x]
```

$$\left\{\left\{y[x] \to -\frac{\sqrt{x^4 + 2\,C[1]}}{\sqrt{2}\;x}\right\}, \left\{y[x] \to \frac{\sqrt{x^4 + 2\,C[1]}}{\sqrt{2}\;x}\right\}\right\}$$

Lortutako emaitzetik E.D.A.-ren emaitza diren bi funtzio definituko ditugu

```
s1[x_, c_] = S[[1, 1, 2]] /. C[1] → c
s2[x_, c_] = S[[2, 1, 2]] /. C[1] → c
```

$$-\frac{\sqrt{2\,c+x^4}}{\sqrt{2}\;x}$$

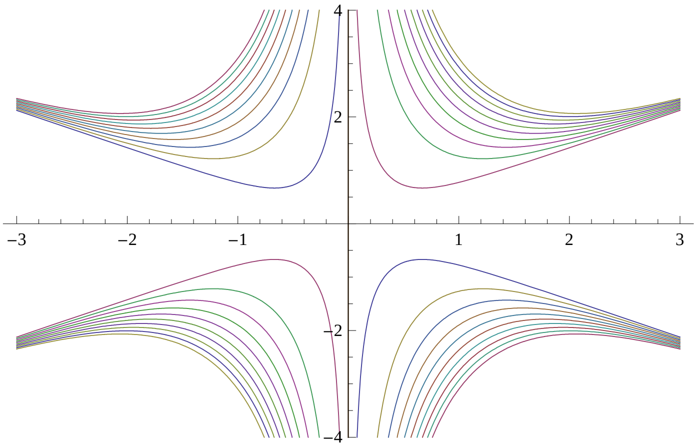
$$\frac{\sqrt{2\,c+x^4}}{\sqrt{2}\;x}$$

Ondorengo zerrenda erabilita soluzio familia bat lortuko dugu eta irudikatuko dugu

```
sol = Flatten[Table[{s1[x, c], s2[x, c]}, {c, 0.1, 10, 1}], 2]
```

$$\left\{-\frac{\sqrt{0.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{0.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{2.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{2.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{4.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{4.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{6.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{6.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{8.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{8.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{10.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{10.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{12.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{12.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{14.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{14.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{16.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{16.2+x^4}}{\sqrt{2}\;x}, -\frac{\sqrt{18.2+x^4}}{\sqrt{2}\;x}, \frac{\sqrt{18.2+x^4}}{\sqrt{2}\;x}\right\}$$

```
korrontelerroak = Plot[Evaluate[sol], {x, -3, 3}, PlotRange → {-4, 4}]
```

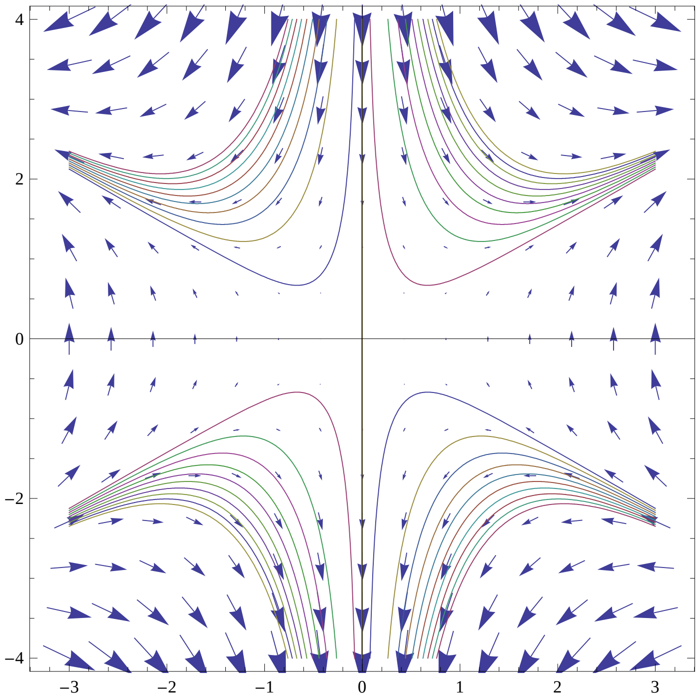


Puntu bakoitzeko kurbarekiko bektore ukitzailea (m(x,y),n(x,y)) da eta hauxe izango da eremu bektoriala

```
eremubek = VectorPlot[{m[x, y], n[x, y]}, {x, -3, 3}, {y, -4, 4}, Axes → True];
```

★ Eremu bektoriala eta korronte lerroak batera irudikatuko ditugu

```
Show[{eremubek, korrontelerroak}, PlotRange → {-4, 4}]
```



▼ Proposatutako Ariketa P-10.2

Ondorengo eremu bektoriala emanik:

$$\vec{F}(x,y)=y\,\vec{i}-x\,\vec{j}$$

a) Kalkulatu eta irudikatu lotutako eremu bektoriala.

b) Kalkulatu korronte lerroen ekuazio diferentziala eta kalkulatu emaitza orokorra.

c) Kalkulatu eta irudikatu soluzioen familia bat.

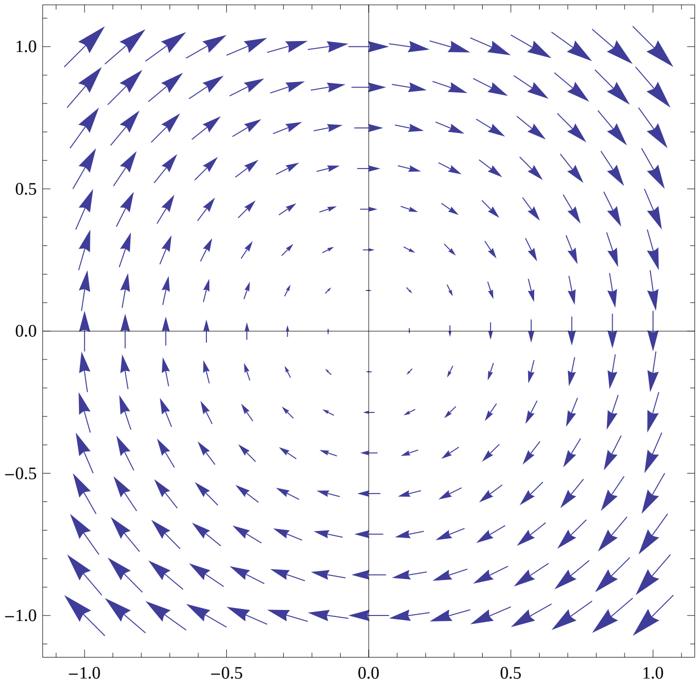
d) Irudikatu kurba familia eta eremu bektoriala biak batera.

▼ Soluzioa P-10.2

★ a) Atala

```
m[x_, y_] = y;
n[x_, y_] = -x;
```

```
eremubek = VectorPlot[{m[x, y], n[x, y]}, {x, -1, 1}, {y, -1, 1}, Axes → True]
```



★ b) Atala

```
edlc = n[x, y[x]] / m[x, y[x]] == y'[x]
```

$$-\frac{x}{y[x]} == y'[x]$$

```
S = DSolve[edlc, y[x], x]
```

$$\left\{\left\{y[x] \to -\sqrt{-x^2 + 2\,C[1]}\right\}, \left\{y[x] \to \sqrt{-x^2 + 2\,C[1]}\right\}\right\}$$

```
s1[x_, c_] = S[[1, 1, 2]] /. C[1] → c / 2
s2[x_, c_] = S[[2, 1, 2]] /. C[1] → c / 2
```

$$-\sqrt{c - x^2}$$

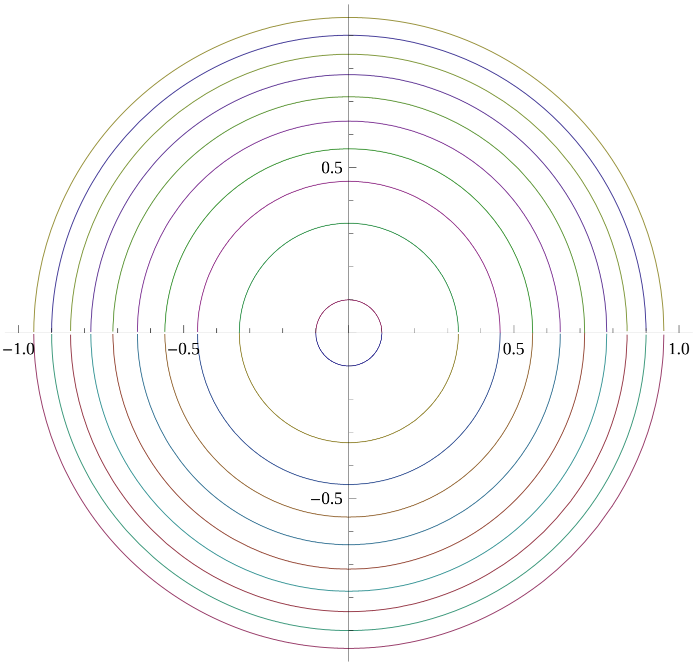
$$\sqrt{c - x^2}$$

★ c) Atala

```
sol = Flatten[Table[{s1[x, c], s2[x, c]}, {c, 0.01, 1, .1}], 2]
```

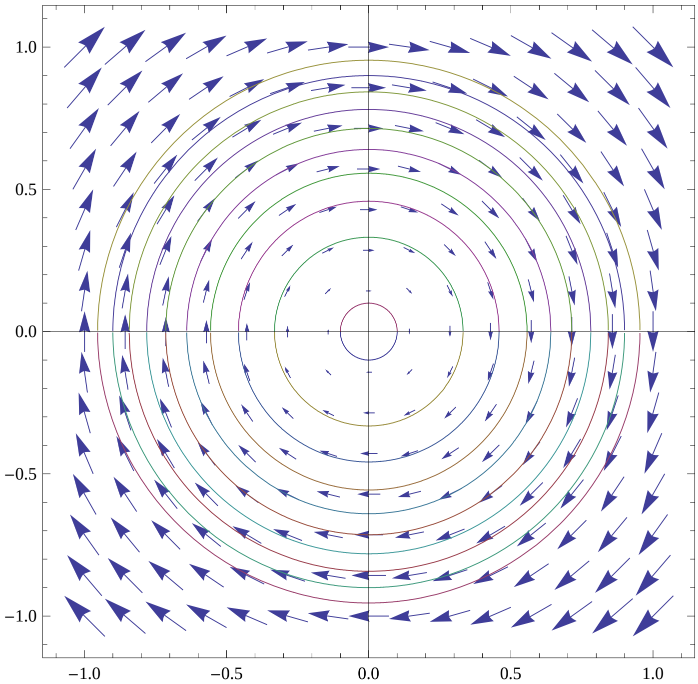
$$\left\{-\sqrt{0.01 - x^2}, \sqrt{0.01 - x^2}, -\sqrt{0.11 - x^2}, \sqrt{0.11 - x^2}, -\sqrt{0.21 - x^2}, \sqrt{0.21 - x^2}, -\sqrt{0.31 - x^2}, \sqrt{0.31 - x^2}, -\sqrt{0.41 - x^2}, \sqrt{0.41 - x^2}, -\sqrt{0.51 - x^2}, \sqrt{0.51 - x^2}, -\sqrt{0.61 - x^2}, \sqrt{0.61 - x^2}, -\sqrt{0.71 - x^2}, \sqrt{0.71 - x^2}, -\sqrt{0.81 - x^2}, \sqrt{0.81 - x^2}, -\sqrt{0.91 - x^2}, \sqrt{0.91 - x^2}\right\}$$

```
familiasol = Plot[Evaluate[sol], {x, -1, 1}, AspectRatio → Automatic]
```



★ d) Atala

```
Show[{eremubek, familiasol}]
```



▼ Proposatutako Ariketa P-10.3

Ondorengo eremu bektoriala emanik $\vec{F}(x,y)=x\,\vec{i}+2\,y\,\vec{j}$

a) Kalkulatu eta irudikatu lotutako eremu bektoriala.

b) Kalkulatu korronte lerroen ekuazio diferentziala eta kalkulatu emaitza orokorra.

c) Irudikatu kurba familia eta eremu bektoriala biak batera.

d) Kalkulatu ibilbide ortogonalen E.D.

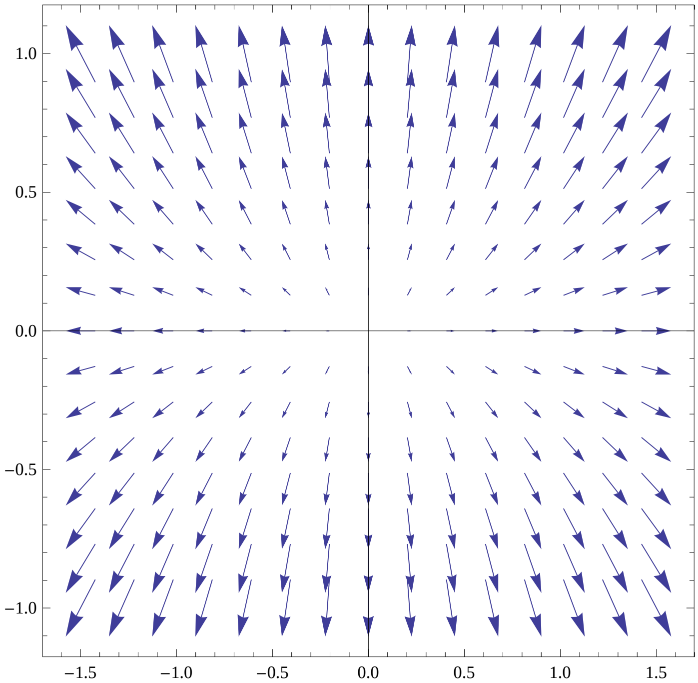
e) **Irudikatu grafiko berean kurba familia eta ibilbide ortogonalei lotutako eremu bektoriala.**

f) **Irudikatu grafiko berean kurba familia biak eta beraiei lotutako eremu bektorialak.**

▼ Soluzioa P-10.3

★ a) Atala

```
m[x_, y_] = x;
n[x_, y_] = 2 y;
eremubeki = VectorPlot[{m[x, y], n[x, y]}, {x, -1.5, 1.5}, {y, -1, 1}, Axes → True]
```



★ b) Atala

```
edlc = n[x, y[x]] / m[x, y[x]] == y'[x]
```

$$\frac{2\, y[x]}{x} == y'[x]$$

```
Si = DSolve[edlc, y[x], x]
```

$\{\{y[x] \to x^2\, C[1]\}\}$

```
si[x_, c_] = Si[[1, 1, 2]] /. C[1] → c
```

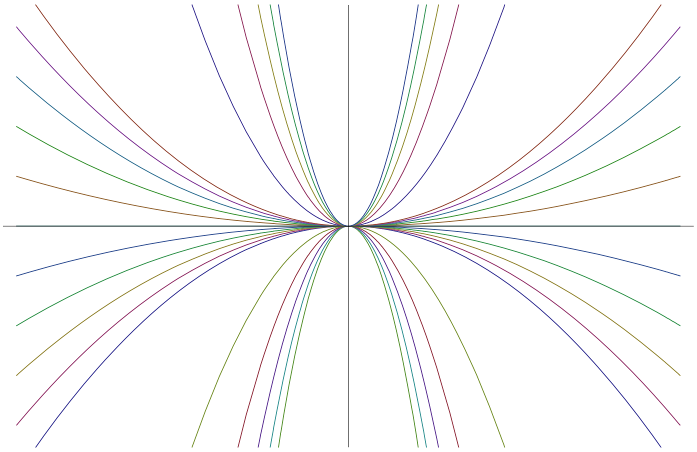
$c\, x^2$

★ c) Atala

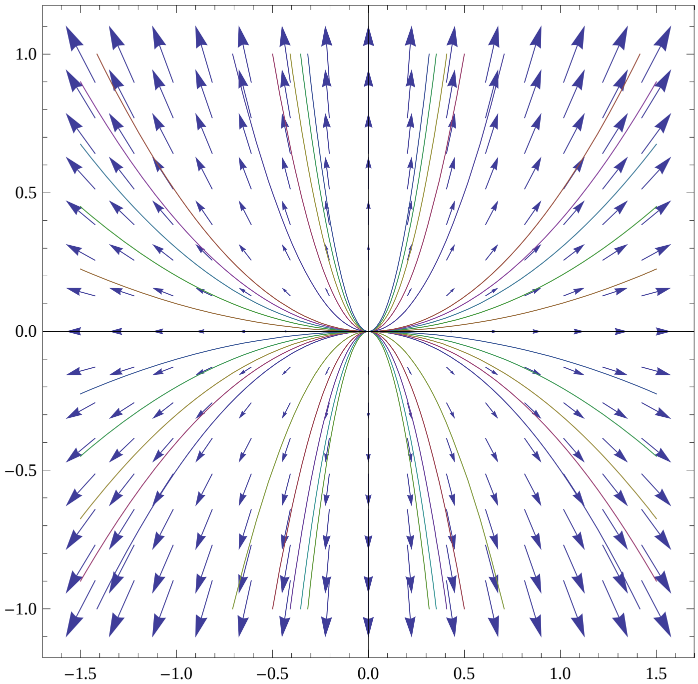
```
solti = Flatten[{Table[si[x, c], {c, -0.5, 0.5, .1}], Table[si[x, c], {c, -10, 10, 2}]}, 2]
```

$\{-0.5\, x^2, -0.4\, x^2, -0.3\, x^2, -0.2\, x^2, -0.1\, x^2, 0., 0.1\, x^2, 0.2\, x^2, 0.3\, x^2,$
$0.4\, x^2, 0.5\, x^2, -10\, x^2, -8\, x^2, -6\, x^2, -4\, x^2, -2\, x^2, 0, 2\, x^2, 4\, x^2, 6\, x^2, 8\, x^2, 10\, x^2\}$

```
famsolti = Plot[Evaluate[solti], {x, -1.5, 1.5},
  PlotRange → {-1, 1}, AspectRatio → Automatic, Ticks → None]
```



```
Show[{eremubeki, famsolti}]
```



★ d) Atala

```
edto = y'[x] == -m[x, y[x]] / n[x, y[x]]
```

$$y'[x] == -\frac{x}{2\, y[x]}$$

```
Sto = DSolve[edto, y[x], x]
```

$$\left\{\left\{y[x] \to -\frac{\sqrt{-x^2 + 4\, C[1]}}{\sqrt{2}}\right\}, \left\{y[x] \to \frac{\sqrt{-x^2 + 4\, C[1]}}{\sqrt{2}}\right\}\right\}$$

```
sto1[x_, c_] = Sto[[1, 1, 2]] /. C[1] → c / 4
sto2[x_, c_] = Sto[[2, 1, 2]] /. C[1] → c / 4
```

$$-\frac{\sqrt{c - x^2}}{\sqrt{2}}$$

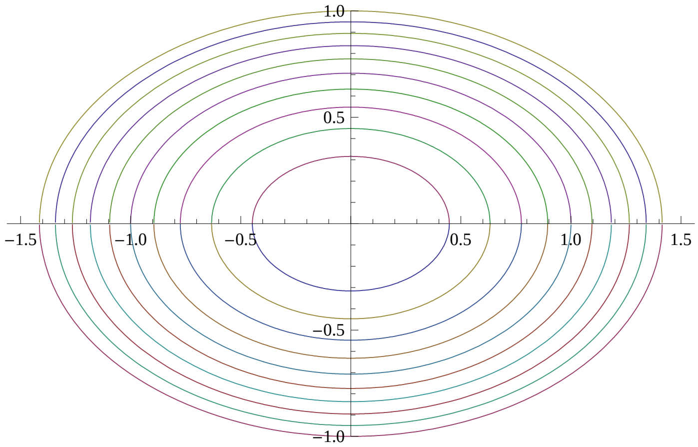
$$\frac{\sqrt{c - x^2}}{\sqrt{2}}$$

★ e) Atala

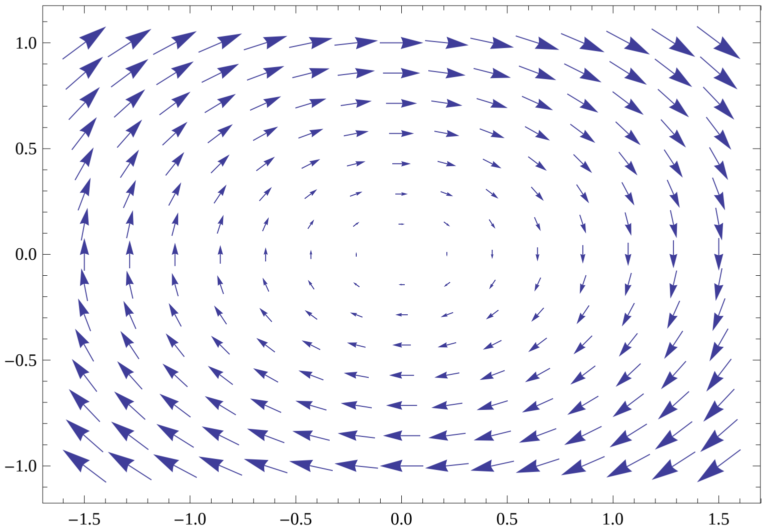
```
solto = Flatten[Table[{sto1[x, c], sto2[x, c]}, {c, 0.2, 2, .2}], 2]
```

$$\left\{-\frac{\sqrt{0.2 - x^2}}{\sqrt{2}}, \frac{\sqrt{0.2 - x^2}}{\sqrt{2}}, -\frac{\sqrt{0.4 - x^2}}{\sqrt{2}}, \frac{\sqrt{0.4 - x^2}}{\sqrt{2}}, -\frac{\sqrt{0.6 - x^2}}{\sqrt{2}}, \frac{\sqrt{0.6 - x^2}}{\sqrt{2}}, -\frac{\sqrt{0.8 - x^2}}{\sqrt{2}}, \frac{\sqrt{0.8 - x^2}}{\sqrt{2}}, -\frac{\sqrt{1. - x^2}}{\sqrt{2}}, \frac{\sqrt{1. - x^2}}{\sqrt{2}}, -\frac{\sqrt{1.2 - x^2}}{\sqrt{2}}, \frac{\sqrt{1.2 - x^2}}{\sqrt{2}}, -\frac{\sqrt{1.4 - x^2}}{\sqrt{2}}, \frac{\sqrt{1.4 - x^2}}{\sqrt{2}}, -\frac{\sqrt{1.6 - x^2}}{\sqrt{2}}, \frac{\sqrt{1.6 - x^2}}{\sqrt{2}}, -\frac{\sqrt{1.8 - x^2}}{\sqrt{2}}, \frac{\sqrt{1.8 - x^2}}{\sqrt{2}}, -\frac{\sqrt{2. - x^2}}{\sqrt{2}}, \frac{\sqrt{2. - x^2}}{\sqrt{2}}\right\}$$

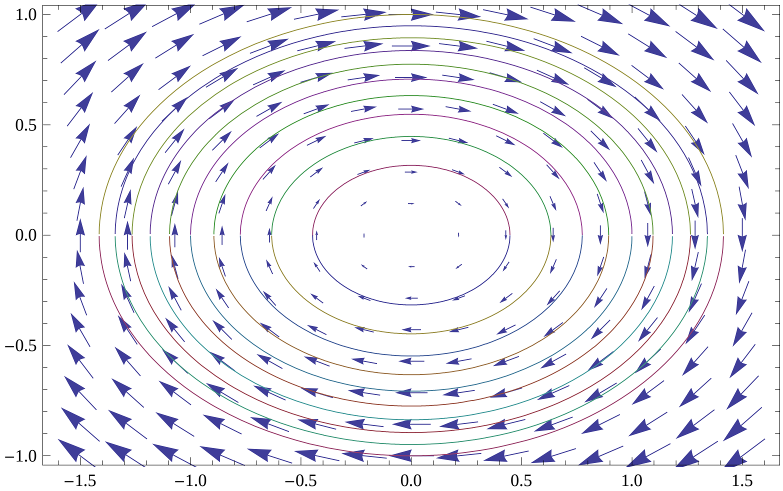
```
famsolto = Plot[Evaluate[solto], {x, -1.5, 1.5}, PlotRange → {-1, 1}]
```




```
eremubekto = VectorPlot[{2 y, -x}, {x, -1.5, 1.5}, {y, -1, 1}, AspectRatio → Automatic]
```

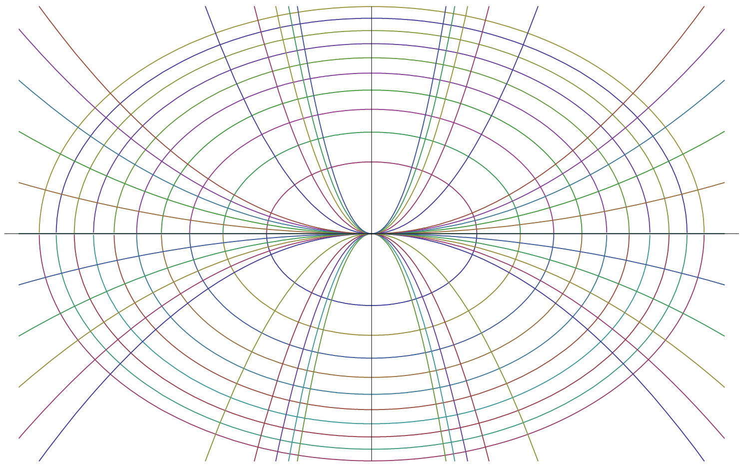


```
Show[{eremubekto, famsolto}, PlotRange → {-1, 1}, Ticks → None, AspectRatio → Automatic]
```

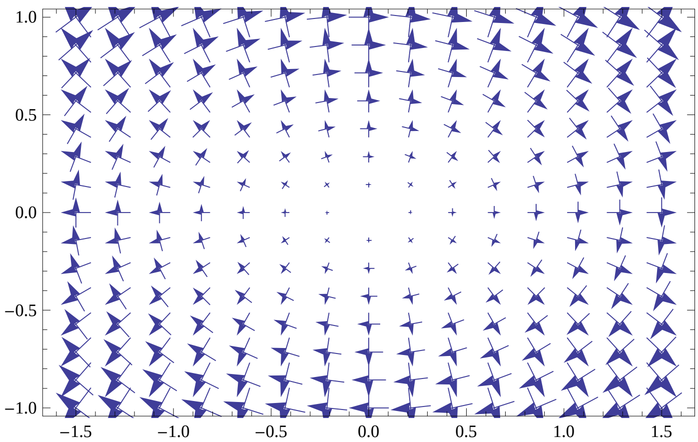


★ f) Atala

```
Show[{famsolto, famsolti}, PlotRange → {-1, 1}, Ticks → None]
```



```
Show[{eremubekto, eremubeki}, PlotRange → {-1, 1}, AspectRatio → Automatic]
```



```
Show[{eremubekto, eremubeki, famsolto, famsolti},
 PlotRange → {-1, 1}, AspectRatio → Automatic]
```