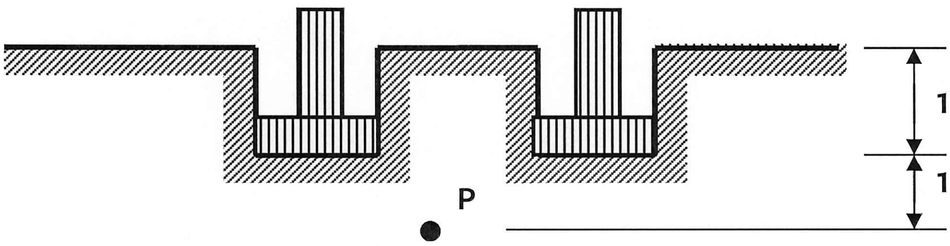
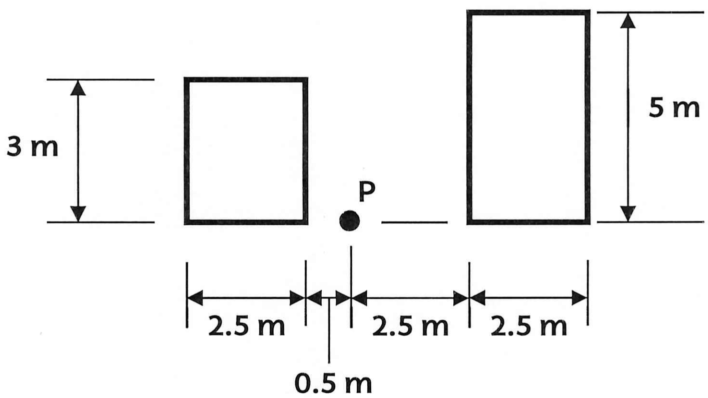
EXERCISE 3

Two footings located 1.5 m below the ground surface (see figure) carry the same uniform load of 110 kN/m². Calculate the vertical stress increment at point P, taking into account both footings.

Additional data: unit weight of soil = 20 kN/m³.





Answer: $\Delta\sigma_v = 8.96$ **kN/m²**.

This load case corresponds to a uniform load on a rectangular area (slides 14 to 17 in the lecture material). According to theory of elasticity, the vertical stress increment at point P will be, taking into account both footings,

$$\Delta\sigma_{v_P} = \Delta\sigma_{v_P}\,(\text{left footing}) + \Delta\sigma_{v_P}\,(\text{right footing})$$

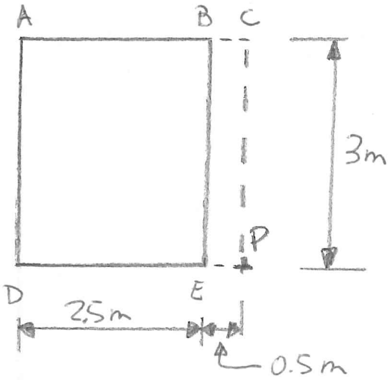
$\Delta\sigma_{v_P}$ (left footing) $= q_{NET_L} \cdot I_{LEFT}$

where q_{NET_L} is the net load applied by the left footing, and
I_{LEFT} is the influence coefficient (slide 15 of the lecture material) of the left footing

q_{NET_L} is calculated as follows

$$q_{NET_L} = q_L - \gamma \cdot H_{exc} = 110 - 20 \cdot 1.5 = 80 \text{ kN/m}^2$$

$\boxed{I_{LEFT}}$ This coefficient is determined using the chart in slide 15 or the table in slide 16, but taking into account that chart and table can only be used when point P is beneath a corner of the load area. As, in our case, point P is not beneath a corner, it is necessary to build the real rectangular load area as an addition/substraction of rectangular areas, as follows:



$I_{LEFT} = I_{ACPD} - I_{BCPE}$

where both rectangles keep to the condition.

$\underline{I_{ACPD}}$: $a = 3\text{ m}, b = 3\text{ m}, z = 1\text{m}$

$$m = \frac{a}{z} = \frac{3}{1} = 3 \;;\; n = \frac{b}{z} = \frac{3}{1} = 3$$

$$\Rightarrow \text{(in Table)} \rightarrow I_{ACPD} = 0.244$$

$\underline{I_{BCPE}}$: $a = 0.5\text{ m}, b = 3\text{ m}, z = 1\text{ m}$

$$m = \frac{a}{z} = \frac{0.5}{1} = 0.5 \;;\; n = \frac{b}{z} = \frac{3}{1} = 3 \Rightarrow I_{BCPE} = 0.137 \text{ (in Table)}$$

Then, $I_{LEFT} = 0.244 - 0.137 = 0.107$

Substituting q_{NET_L} and I_{LEFT}

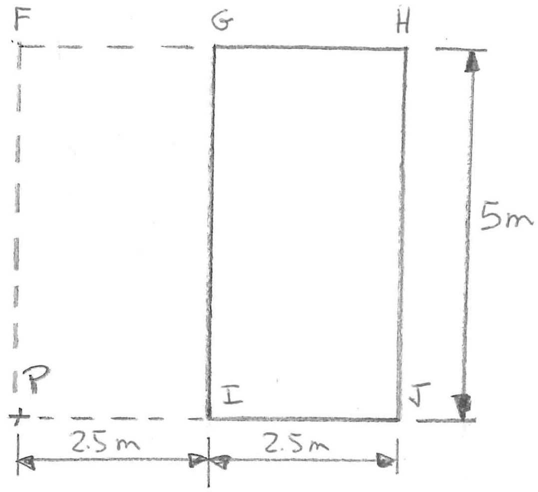
$$\Delta\sigma_{v_P} \text{(left footing)} = 80 \cdot 0.107 = 8.56 \text{ kN/m}^2$$

A similar procedure is taken for the right footing.

$$\Delta\sigma_{vP}\,(\text{right footing}) = q_{NET_R} \cdot I_{RIGHT}$$

$$q_{NET_R} = q_R - \gamma \cdot H_{exc} = 110 - 20 \cdot 1.5 = 80 \text{ kN/m}^2$$

$\boxed{I_{RIGHT}}$



$$I_{RIGHT} = I_{FHJP} - I_{FGIP}$$

$\underline{I_{FHJP}}$: $a = 5\text{m},\ b = 5\text{m},\ z = 1\text{m}$

$$m = \frac{a}{z} = \frac{5}{1} = 5;\quad n = \frac{b}{z} = \frac{5}{1} = 5$$

$$\Rightarrow I_{FHJP} = 0.249 \text{ (in Table)}$$

$\underline{I_{FGIP}}$: $a = 2.5\text{m},\ b = 5\text{m},\ z = 1\text{m}$

$$m = \frac{a}{z} = \frac{2.5}{1} = 2.5;\quad n = \frac{b}{z} = \frac{5}{1} = 5 \Rightarrow I_{FGIP} = 0.244 \text{ (in Table)}$$

Then, $I_{RIGHT} = 0.249 - 0.244 = 0.005$

Substituting q_{NET_R} and I_{RIGHT}

$$\Delta\sigma_{vP}\,(\text{right footing}) = 80 \cdot 0.005 = 0.4 \text{ kN/m}^2$$

And finally, the vertical stress increment at point P, due to both footings, will be

$$\underline{\underline{\Delta\sigma_{vP}}} = 8.56 + 0.4 = \underline{\underline{8.96 \text{ kN/m}^2}}$$

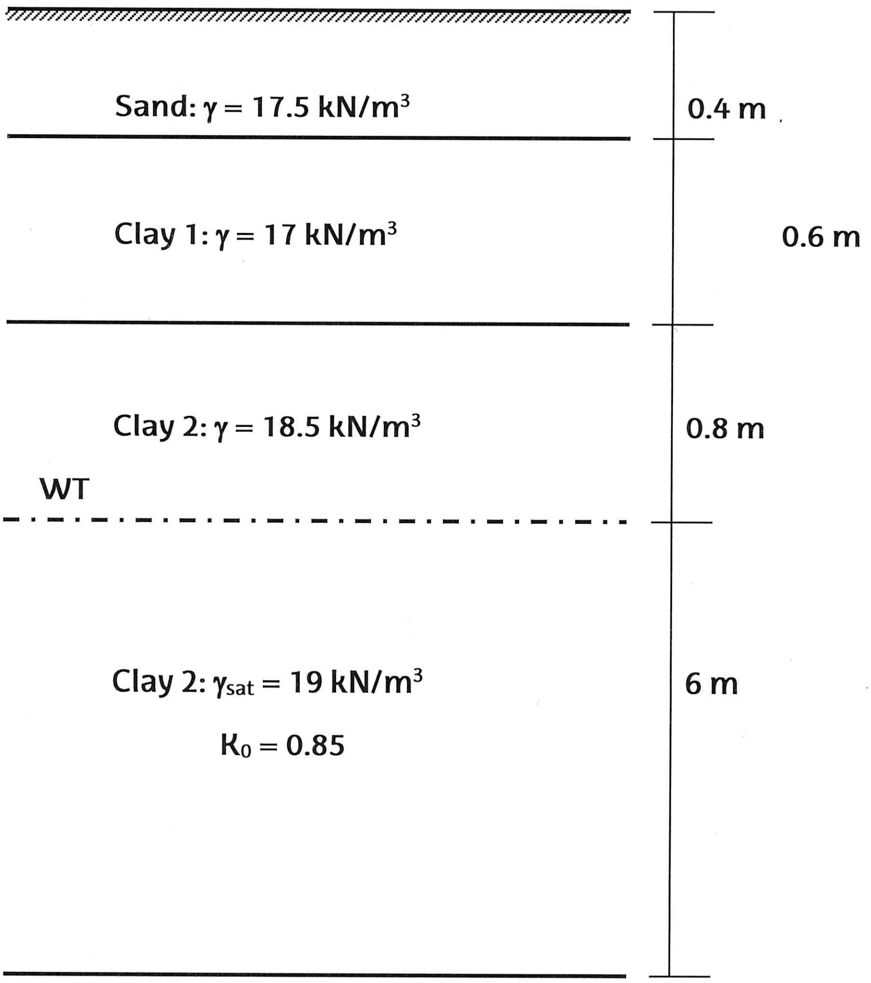
EXERCISE 7

A civil engineering project is to be constructed in an area where the ground surface is horizontal (see figure). A rectangular foundation located 3 m below the ground surface will be used to transfer the construction load uniformly.

1) At the pre-construction phase, calculate and plot Mohr circles of total and effective stresses at any point of the soil mass 3 m below the ground surface.

Assuming that a 3 m by 3.4 m rectangular foundation will carry a uniform load of 115 kN/m^2, determine, at the post-construction phase:

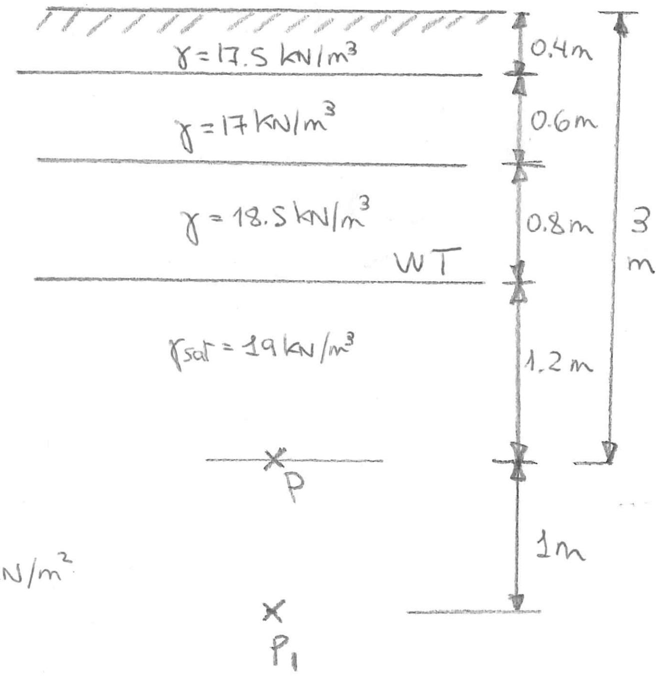
2) Total vertical stress at points 0 m and 1 m below the centre of the foundation.



Answers:

1) $\sigma_v = 54.8$ kN/m^2; u = 11.76 kN/m^2; $\sigma'_v = 43.04$ kN/m^2; $\sigma'_H = 36.58$ kN/m^2; $\sigma_H = 48.34$ kN/m^2. 2) σ_v (0 m) = 115 kN/m^2; σ_v (1 m) = 126.78 kN/m^2.

1) Pre-construction phase.



In order to determine Mohr circles at point P, it is necessary to calculate σ_V, u, σ'_V, σ'_H and σ_H.

At this point, slides 6 to 9 of the lecture material will be used.

$$\sigma_V = \Sigma \gamma_i h_i = 17.5 \cdot 0.4 + 17 \cdot 0.6 + 18.5 \cdot 0.8 + 19 \cdot 1.2 = 54.8\ kN/m^2$$

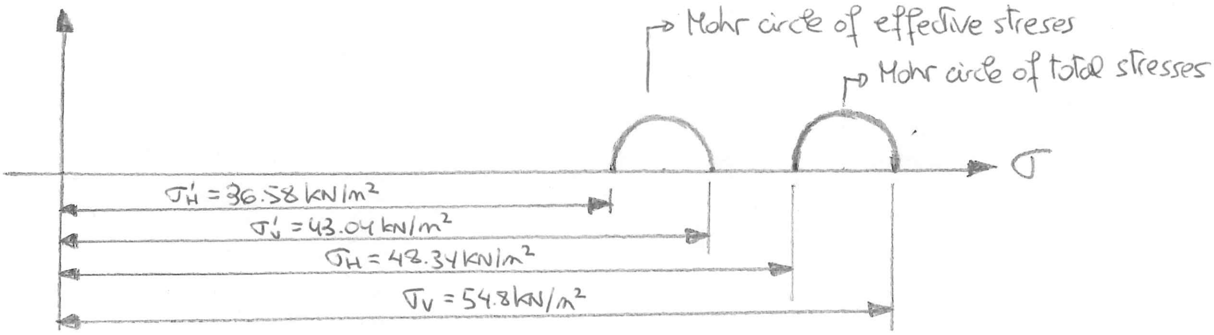
$$u = \gamma_w \cdot h_{WT} = 9.8\,(3-(0.4+0.6+0.8)) = 11.76\ kN/m^2$$

$$\sigma'_V = \sigma_V - u = 54.8 - 11.76 = 43.04\ kN/m^2$$

$$\sigma'_H = K_0 \cdot \sigma'_V = 0.85 \cdot 43.04 = 36.58\ kN/m^2$$

$$\sigma_H = \sigma'_H + u = 36.58 + 11.76 = 48.34\ kN/m^2$$

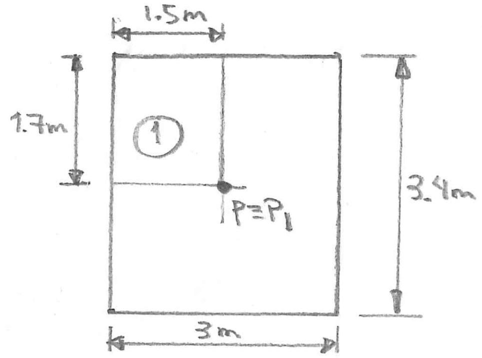
These values are taken to plot Mohr circles.



2) Rectangular footing: $B = 3m$, $L = 3.4m$, $q = 115\ kN/m^2$. Located 3 m below the ground surface. Then, foundation is located onto the plane where point P is.

$\left.\begin{array}{l}\sigma_V(P)\\ \sigma_V(P_1)\end{array}\right\}$ at the post-construction phase are asked, where points P and P_1 are below the centre of the foundation.

After the load has been applied, the vertical stress increment at points P and P_1 will be



(PLAN VIEW)

$$\Delta\sigma_v(P) = q_{NET} \cdot I_P$$

$$q_{NET} = q - \gamma \cdot H_{exc} = 115 - [17.5 \cdot 0.4 + 17 \cdot 0.6 + 18.5 \cdot 0.8 + 19 \cdot 1.2] = 60.2 \text{ kN/m}^2$$

$$I_P = 4 \cdot I_P^{(1)}$$

$I_P^{(1)}$: $a = 1.7$, $b = 1.5$, $z = 0 \Rightarrow m = \frac{a}{z} = \frac{1.7}{0} = \infty$; $n = \frac{b}{z} = \frac{1.5}{0} = 0 \Rightarrow$

$\Rightarrow I_P^{(1)} = 0.25$ (in chart slide 15)

$\Rightarrow I_P = 4 \cdot I_P^{(1)} = 4 \cdot 0.25 = 1 \Rightarrow \Delta\sigma_v(P) = 60.2 \cdot 1 = 60.2 \text{ kN/m}^2$

$\Delta\sigma_v(P_1) = q_{NET} \cdot I_{P_1}$, where q_{NET} is unique, so $q_{NET} = 60.2 \text{ kN/m}^2$

$$I_{P_1} = 4\, I_{P_1}^{(1)}$$

$I_{P_1}^{(1)}$: $a = 1.7$, $b = 1.5$, $z = 1 \Rightarrow m = \frac{a}{z} = \frac{1.7}{1} = 1.7$; $n = \frac{b}{z} = \frac{1.5}{1} = 1.5 \Rightarrow$

$\Rightarrow I_{P_1}^{(1)} \approx 0.220$ (approximately, in chart or table, slides 15 and 16)

$\Rightarrow I_{P_1} = 4 \cdot I_{P_1}^{(1)} = 4 \cdot 0.220 = 0.88 \Rightarrow \Delta\sigma_v(P_1) = 60.2 \cdot 0.88 = 52.98 \frac{\text{kN}}{\text{m}^2}$

$\sigma_{v_{TOTAL}}(P) = \sigma_v(P) + \Delta\sigma_v(P) = 54.8 + 60.2 = \underline{\underline{115 \text{ kN/m}^2}}$

at the pre-construction phase

$\sigma_v(P_1) = \Sigma \gamma_i \cdot h_i = 17.5 \cdot 0.4 + 17 \cdot 0.6 + 18.5 \cdot 0.8 + 19 \cdot (1.2 + 1) = 73.8 \text{ kN/m}^2$

$\sigma_{v_{TOTAL}}(P_1) = \sigma_v(P_1) + \Delta\sigma_v(P_1) = 73.8 + 52.98 = 126.78 \text{ kN/m}^2$