Solution to Exercise 1: Description of models SIP and SIPD for marine disease transmission through contact with environmental pathogens

In this document, solution of Exercise 1 is provided. The equation systems of the SIP and SIPD models are described. These models describe marine disease transmission through contact with environmental pathogens released by infected animals and dead infected animals, respectively. MATLAB codes for these models with plots for model simualtions are also facilitated in matlab files.

Model	Transmission	Applicable systems
SIP	Contact with infective particles or released by infected individuals	Black-band disease and Aspergillosis in corals; Whithering syndrome (WS) in abalone; transmission of trematode cercariae.
SIPD	Contact with infective particles or fomites released by dead infected individuals	Black-band disease and Aspergillosis in corals through breakdown of decaying tissue; abalone with WS and shrimp with White-Spot disease shed particles during decay and scavenging processes.

Table 1: Models, model characteristics, and example disease potentially applicable. The disease list is not meant to be comprehensive, nor does a unique mention of a disease imply restriction of the disease to that particular model.

As in the previous lesson, R_0 is formulated for each model and details for getting R_0 of these models by applying the next-generation matrix (NGM) method are described in lessons 4 and 5.

SIP model

In this model the disease is not transmitted from infected animals to susceptible animals by contact between individuals. Infected animals release infectious particles into the environment (P) and these waterborne pathogens can contact the susceptible animals thereby transmitting the disease. We consider a version of a model proposed to study the population dynamics of microparasitic infections.

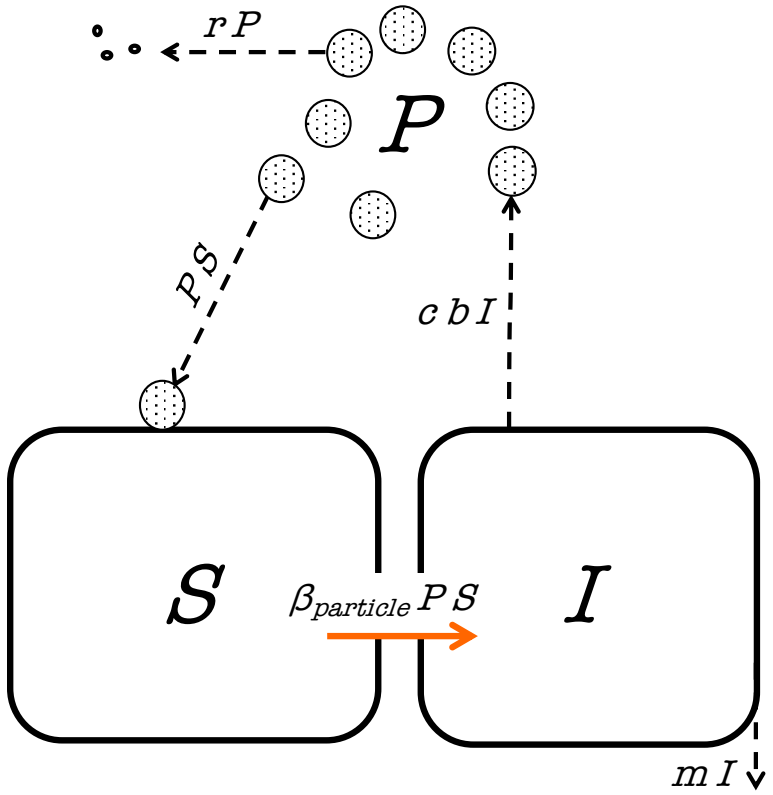

Figure 1: Flow diagram for the SIP model. The variables (compartments) for each model are represented by upper letters: susceptible animals S, infected animals I, waterborne pathogens or environmental particles P. The model parameters are represented by lower letters described in Table 2. Orange solid arrows represent the transmission processes and dashed black arrows represent other main processes.

Thus, the model assumes that susceptibles are infected with a rate $\beta_{particle}PS$ (Equations **??** and **??**). The release rate of infective particles by infected individuals occurs

at rate c and the pathogens in the water are inactivated at a rate r (Equation **??**) by dilution, transport downstream, or by reduction of infectiousness by inactivation or death.

The model can be described by the following system:

$$\frac{d\,S}{dt} = -\beta_{particle}\,P\,S; \tag{1}$$

$$\frac{d\,I}{dt} = \beta_{particle}\,P\,S - m\,I; \tag{2}$$

$$\frac{d\,P}{dt} = c\,b\,I - r\,P. \tag{3}$$

The basic reproduction number is defined as:

$$R_0 = \sqrt{\frac{\beta_{particle}\,N}{m}\frac{c\,b}{r}}. \tag{4}$$

The response on R_0 due to changes in initial population N has a nonlinear increasing trend (Figure 2). In this model (Equation **??**), large populations are less vulnerable to epizootics in conditions of relatively high r (i.e. a short pathogen life span in the water and/or rapid dilution) with respect to the particle release rate c. As the pathogen release rate rises with respect to the inactivation rate of infective particles in the water r, for a given m, the probability of a disease outbreak increases substantially even at low transmission rates for small populations. Death of infected individuals effectively terminates particle release; thus, a high mortality rate m can limit epizootic development even if the body burden of pathogens b in the infected individuals is high.

SIPD model

Arguably, in marine systems, waterborne or free-living pathogens (P) are released more commonly, or at least more abruptely, by dead infected animals (D) instead of live infected individuals (I), and the disease is transmitted by contact of susceptible

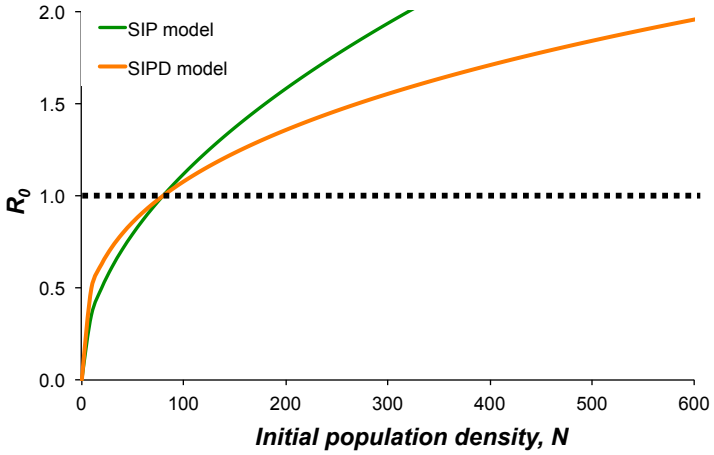

Figure 2: Theoretical estimations of R_0 for models SIP and SIPD, for increasing population density N. Using as examples marine host-pathogen systems described in Table 1, the following values of the parameters were used: $\beta_{particle} = 1 \times 10^{-5}$, $m = d = 1 \times 10^{-2}$, $c = 1 \times 10^{-3}$, $b = 1 \times 10^{4}$, $r = 8 \times 10^{-1}$.

animals (S) with the released free-living pathogens (P).

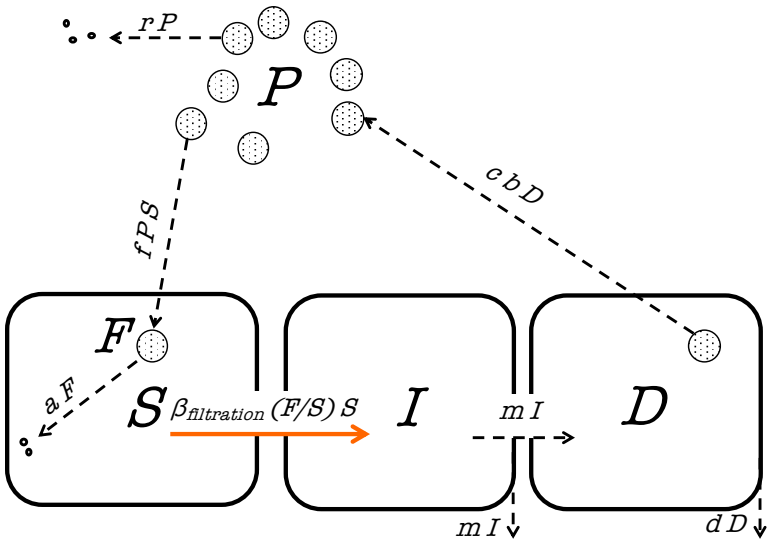

Figure 3: Flow diagram for the SIPD model. The variables (compartments) for each model are represented by upper letters: susceptible animals S, infected animals I, dead infeced animals D, and waterborne pathogens or environmental particles P. The model parameters are represented by lower letters described in Table 2.

When this occurs, a large number of infective particles may be released in a short

time. The release of pathogens can occur during the natural decomposition process of dead animals or by the action of scavengers. The SIPD model incorporates the dynamics of organisms (S, I, and D) and pathogens (P) in the environment. Pathogens infect hosts by contact with susceptible animals (Equations **??** and **??**) and infected hosts die due to disease (Equation **??**). Internal or attached pathogens b are released from dead animals at rate c. Similarly to the SIP model, pathogens in the water are inactivated at a rate r by natural death, or removed from the system by dilution or advection (Equation **??**).

The governing equations are:

$$\frac{d\,S}{dt} = -\beta_{particle}\,P\,S; \tag{5}$$

$$\frac{d\,I}{dt} = \beta_{particle}\,P\,S - m\,I; \tag{6}$$

$$\frac{d\,D}{dt} = m\,I - d\,D; \tag{7}$$

$$\frac{d\,P}{dt} = c\,b\,D - r\,P. \tag{8}$$

The basic reproduction number is

$$R_0 = \sqrt[3]{\frac{\beta_{particle}\,N}{d}\,\frac{c\,b}{r}}. \tag{9}$$

In this model, the infection process is regulated by the removal of dead animals by the action of scavengers or natural decomposition d (Equation **??**), instead of the mortality of infected individuals m (Equation **??**), and cb refers to the body burden of infective particles in the dead animal tissue. The SIPD model is less sensitive than the SIP model to changes in parameter values resulting in an inherently slower transmission process (Figure 2). However, the release rate of pathogens from decaying tissue c is commonly much faster than from live infected animals, the body burden b of infective particles is higher in dead tissue than in the average living

animal, and the removal rate of dead animals d is also much faster than the disease mortality rate m. The rapidity of tissue decay releasing particles means that the particle loss rate r must be high to limit epizootic development either through high flow and rapid water exchange rates or through the very rapid mortality of infective particles (Figure 2).

Variables, Parameters	Definition	Units
S	Susceptible hosts in the population	Number of individuals
I	Infected hosts in the population	Number of individuals
D	Dead infected hosts in the population	Number of individuals
P	Waterborne pathogens in the enviornmet (i.e. local pool)	Number of particles
N	Susceptible hosts in the initial population	Number of individuals
R_0	Basic reproduction number	Nondimensional
	Individual $^{-1}$ day^{-1}	
$\beta_{particle}$	Disease transmission rate by contact between suceptibles and waterborne pathogens.	Particle $(water)^{-1}$ day^{-1}
m	Disease mortality rate	day^{-1}
d	Removal rate of dead individuals by scavengers or bacteria (decay)	day^{-1}
c	Release rate of pathogens from infected or dead animals	day^{-1}
b	Average body burden of pathogens in infected or dead animals	Number of particles
r	Loss rate of waterborne pathogens from the local pool	day^{-1}

Table 2: Description of variables and parameters. The last column identifies the models in which the variable or parameter is used. An asterisk identifies the use of the variable in the R_0 formulation for that model. Note that all models have an implicit surface area (m^{-2}) or volume (m^{-3}) for individuals and waterborne pathogens respectively.