
Practica nº4: Interpolación polinomial

Uso del comando interno de *Mathematica*

En esta práctica estudiaremos cómo construir el polinomio interpolador de un conjunto de datos. *Mathematica* posee un comando muy flexible y versátil,

InterpolatingPolynomial[*nube*,*var*],

que permite construir el polinomio correspondiente:

```
nube = {{0, 2}, {2, 5}, {-2, 1}};  
p[x_] = InterpolatingPolynomial[nube, x]
```

$$2 + \left(\frac{3}{2} + \frac{1}{4} (-2 + x) \right) x$$

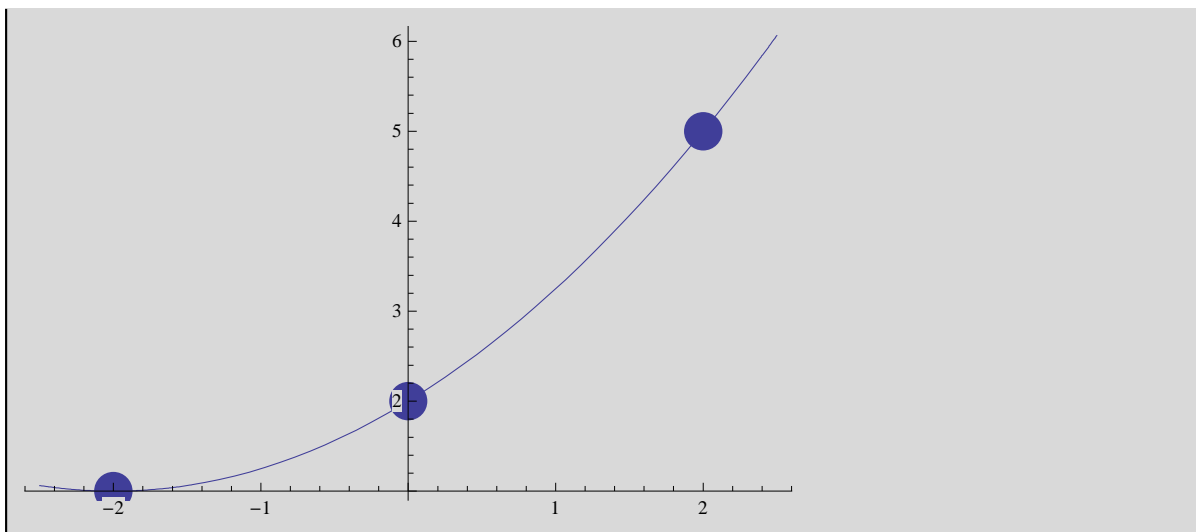
Observe que el resultado se ofrece en forma de Newton. Para obtener la forma estándar, basta expandir el polinomio:

```
Expand[p[x]]
```

$$2 + x + \frac{x^2}{4}$$

Veamos el resultado gráficamente:

```
pt = ListPlot[nube, PlotStyle -> {PointSize[0.05]}];
pol = Plot[p[x], {x, -2.5, 2.5}];
Show[pol, pt]
```



Problema 1:

a) Tabule los valores de la función $g(x) = \text{Log}[x + 1 / x]$ en los nodos $x = 0.1, 0.2, 0.3, 0.4, 0.5$.

b) Interpole la nube de puntos obtenida mediante el comando Interpolating Polynomial y halle una aproximación de $g(0.26)$.

c) Estime el error de interpolación en ese punto. restando al valor exacto el valor aproximado.

d) dibujar la funcion diferencia $g(x) - p(x)$.

```
puntos = Table[{i, Log[(i + 1) / i]}, {i, 0.1, 0.5, 0.1}]
```

```
{{0.1, 2.3979}, {0.2, 1.79176}, {0.3, 1.46634}, {0.4, 1.25276}, {0.5, 1.09861}}
```

```
q[x_] = InterpolatingPolynomial[puntos, x]
```

```
1.09861 +
(-3.24821 + (7.04792 + (-23.2925 + 48.5168 (-0.2 + x)) (-0.3 + x)) (-0.1 + x))
(-0.5 + x)
```

```
Expand[q[x]]
```

$$3.57005 - 15.7938 x + 47.903 x^2 - 76.6609 x^3 + 48.5168 x^4$$

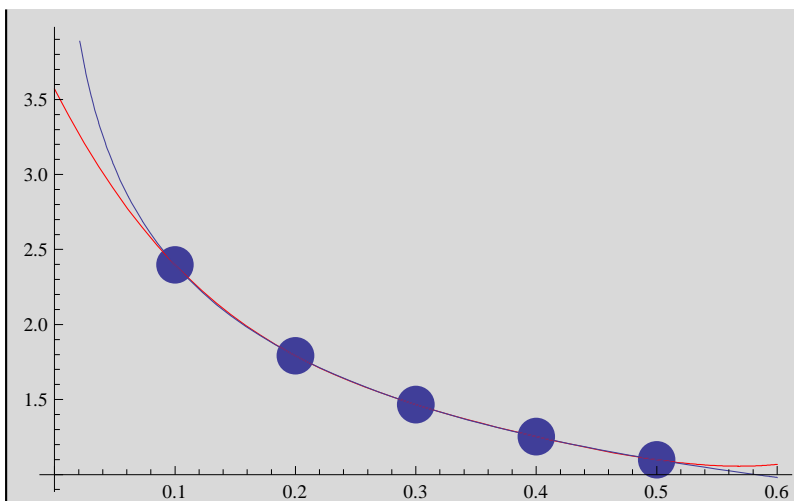
```
q[x] /. x -> 0.26
```

1.57624

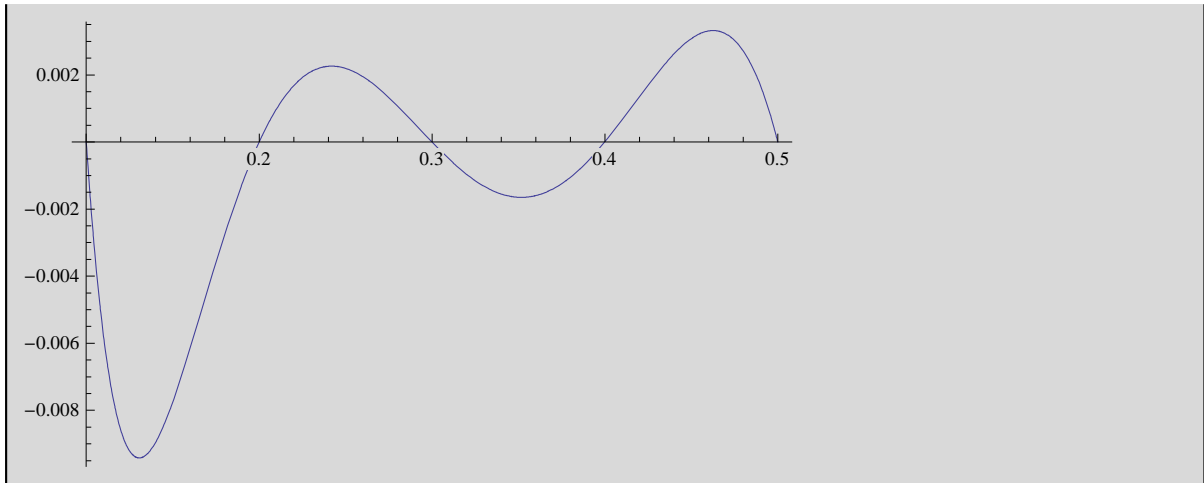
```
Log[(0.26 + 1) / 0.26]
```

1.57819

```
pt = ListPlot[puntos, PlotStyle -> {PointSize[0.05]}];
pol = Plot[q[x], {x, 0, 0.6}, PlotStyle -> {RGBColor[1, 0, 0]}];
fun = Plot[Log[(x + 1) / x], {x, 0, 0.6}];
Show[{pt, pol, fun}]
```



```
Plot[Log[(x + 1) / x] - q[x], {x, 0.1, 0.5}]
```



Problema 2 :Polinomio de Lagrange. Construcción de las funciones de base de Lagrange

En caso de necesidad, podemos construir directamente los polinomios básicos de Lagrange

$$L_i(x) = \prod_{j \neq i} \frac{(x - x_j)}{(x_i - x_j)}$$

usando las capacidades del *Mathematica* para operar con listas:

Sea $f(x)=\text{Log}[10,\text{Tan}(x)]$. A partir de los valores: $f(1.00)=0.1924$, $f(1.05)=0.2414$, $f(1.10)=0.2933$ y $f(1.15)=0.3492$

se pide:

1º.-Calcular el polinomio de interpolación de Lagrange. Para ello, seguir los siguientes pasos.

- Generar la lista de nodos x_i
- Plantear una lista L con elementos unidad donde se almacenarán los $L_i(x)$
- Calcular los $L_i(x)$ y almacenarlos
- crear un vector y con los valores $f(x_i)$

El polinomio sera el producto escalar de los vectores y y L

2º.-Utilizar dicho polinomio para obtener un valor aproximado de $f(1.09)$

4º.-Representar el error cometido, graficamente.

```
f[x_] := Log[10, Tan[x]]
```

```
Datos = Table[{i, f[i]}, {i, 1., 1.15, 0.05}]
```

```
{ {1., 0.192402}, {1.05, 0.241376}, {1.1, 0.293309}, {1.15, 0.34918} }
```

```
nodos = Table[Datos[[i, 1]], {i, 1, Length[Datos]}]
```

```
{1., 1.05, 1.1, 1.15}
```

```
L = Table[1, {Length[Datos]}]
```

```
{1, 1, 1, 1}
```

```
Do[ If[ j ≠ i, L[[i]] = L[[i]] *  $\frac{(x - \text{nodos}[[j]])}{(\text{nodos}[[i]] - \text{nodos}[[j]])}$  ; ],  
  {i, 1, Length[L]}, {j, 1, Length[L]} ]
```

```
Expand[L]
```

```
{1771. - 4836.67 x + 4400. x^2 - 1333.33 x^3, -5060. + 14060. x - 13000. x^2 + 4000. x^3,
 4830. - 13630. x + 12800. x^2 - 4000. x^3, -1540. + 4406.67 x - 4200. x^2 + 1333.33 x^3}
```

```
y = Table[Datos[[i, 2]], {i, 1, 4}]
```

```
{0.192402, 0.241376, 0.293309, 0.34918}
```

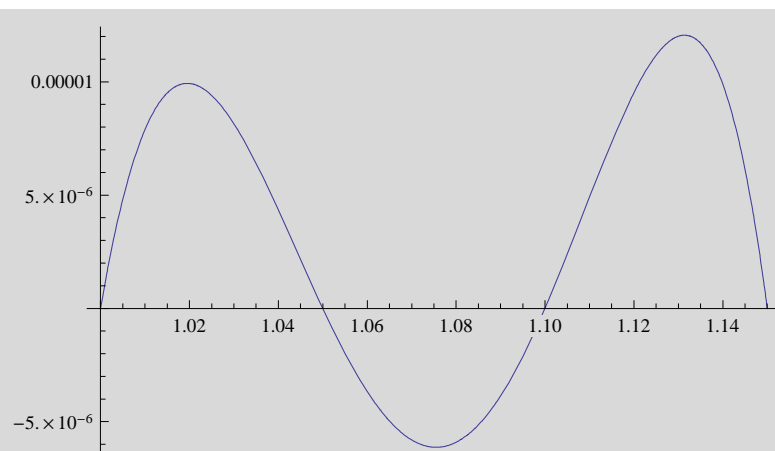
```
p[x_] = Expand[L.y]
```

```
-1.67004 + 4.07114 x - 3.51133 x^2 + 1.30264 x^3
```

```
p[1.09]
```

```
0.282639
```

```
Plot[p[x] - Log[10, Tan[x]], {x, 1., 1.15}]
```



Problema 3: Polinomios interpoladores de Newton

1.- Calcular el polinomio interpolador de Newton para la misma función, mediante diferencias finitas.

-Calcular las diferencias finitas.

- Plantear el polinomio interpolador de newton.

2.- Calcular el error dibujando la función diferencia

```
dif = Table[Datos[[i, 2]], {i, 1, Length[Datos]}]
```

```
{0.192402, 0.241376, 0.293309, 0.34918}
```

```
For[j = 2, j ≤ Length[dif],  
  For[k = Length[dif], k ≥ j, dif[[k]] = dif[[k]] - dif[[k - 1]]; k--]; j++]
```

```
dif
```

```
{0.192402, 0.0489736, 0.00295987, 0.000976978}
```

Polinomio

$$q[t_] = \sum_{i=1}^{\text{Length}[\text{Datos}]} \text{dif}[[i]] * \text{Binomial}[t, i - 1]$$

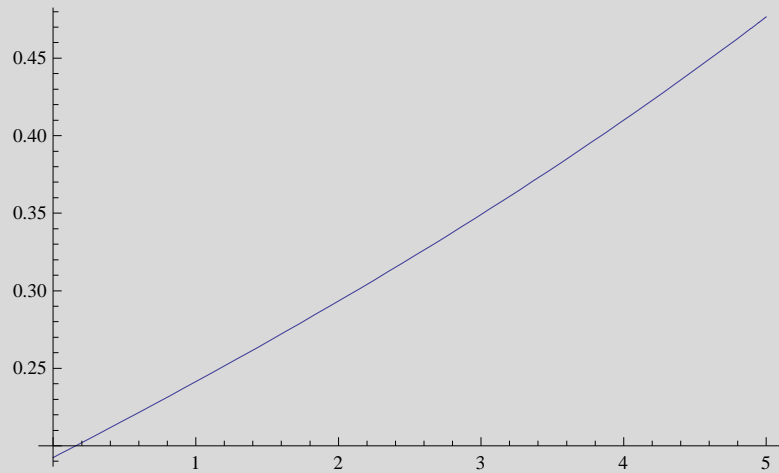
```
0.192402 + 0.0489736 t + 0.00147993 (-1 + t) t + 0.00016283 (-2 + t) (-1 + t) t
```

```
Clear[t]
```

```
q[t]
```

```
0.192402 + 0.0489736 t + 0.00147993 (-1 + t) t + 0.00016283 (-2 + t) (-1 + t) t
```

```
Plot[q[t], {t, 0, 5}]
```



```
h = 0.05; x0 = 1;
```

```
Clear[t]
```

```
x0
```

```
1
```

```
h
```

```
0.05
```

```
t = (x - x0) / h
```

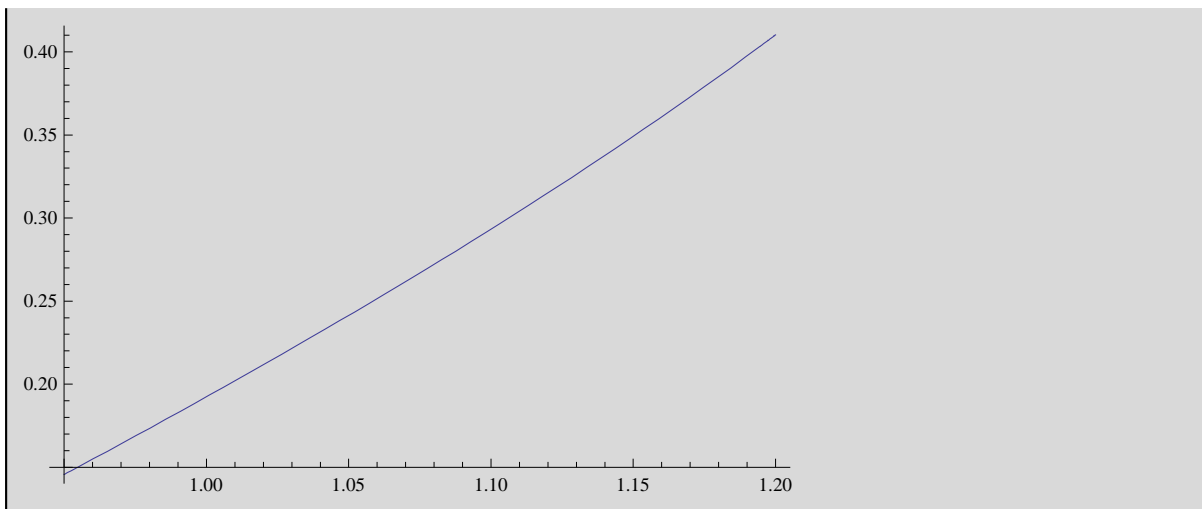
```
20. (-1 + x)
```

```
p[x_] = q[t] /. t -> (x - x0) / h
```

```
0.192402 + 0.979472 (-1 + x) + 0.0295987 (-1 + 20. (-1 + x)) (-1 + x) +  
0.00325659 (-2 + 20. (-1 + x)) (-1 + 20. (-1 + x)) (-1 + x)
```



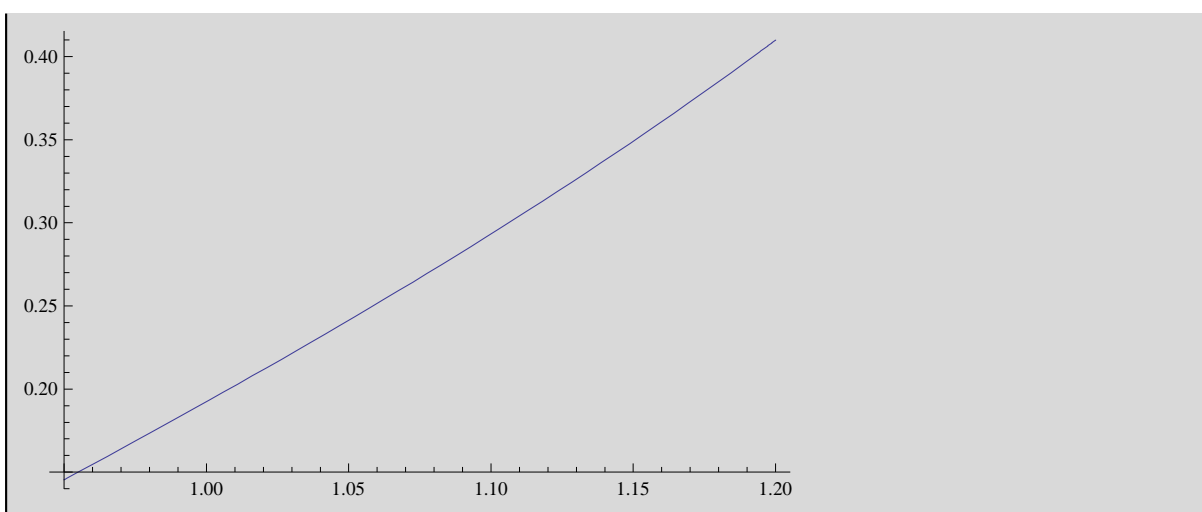
```
Plot[f[x], {x, 0.95, 1.2}]
```



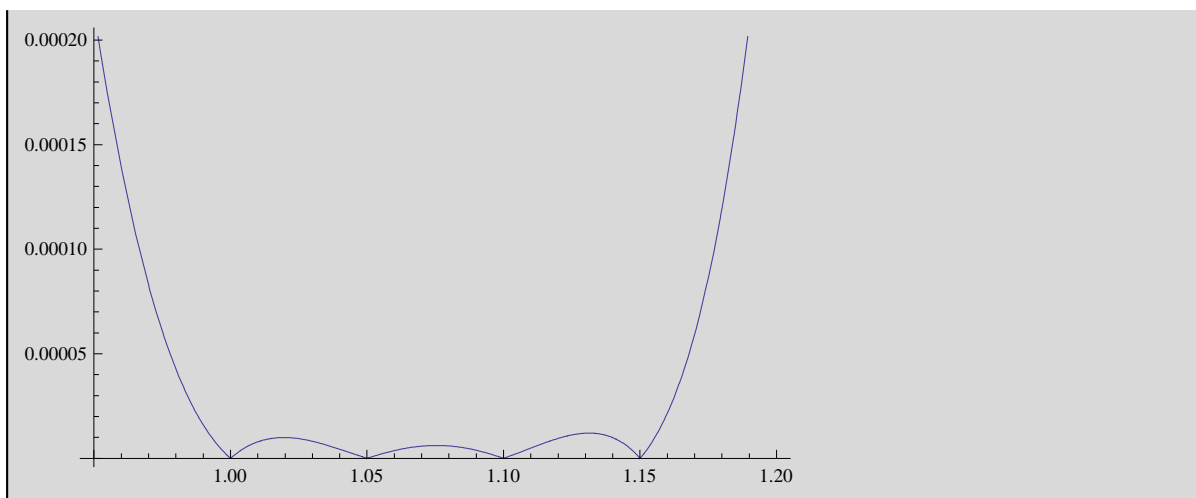
```
Expand[p[x]]
```

```
-1.67004 + 4.07114 x - 3.51133 x2 + 1.30264 x3
```

```
Plot[p[x], {x, 0.95, 1.2}]
```



```
Plot[Abs[f[x] - p[x]], {x, 0.95, 1.2}]
```



Efecto runge

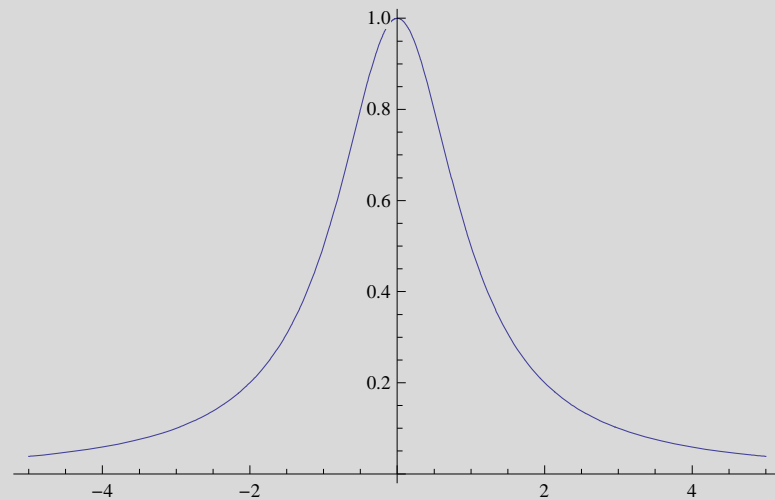
-
- Interpolacion polinómica de la función $f(x) = \frac{1}{(1+x^2)}$ en el intervalo $[-5, 5]$ utilizando once puntos igualmente espaciados, mediante diferencias finitas.
 - Dibujar la función $f(x)$
 - Dibujar simultaneamente $f(x)$ y $p(x)$

```
Clear[x]
```

```
Clear[t]
```

```
f[x_] := 1 / (1 + x^2)
```

```
grafico = Plot[f[x], {x, -5, 5}]
```



```
lista = Table[{x, f[x]}, {x, -5, 5}]
```

```
{ {-5, 1/26}, {-4, 1/17}, {-3, 1/10}, {-2, 1/5}, {-1, 1/2},
  {0, 1}, {1, 1/2}, {2, 1/5}, {3, 1/10}, {4, 1/17}, {5, 1/26} }
```

```
dif = Table[lista[[i, 2]], {i, 1, Length[lista]}]
```

```
{ 1/26, 1/17, 1/10, 1/5, 1/2, 1, 1/2, 1/5, 1/10, 1/17, 1/26 }
```

```
For[j = 2, j ≤ Length[dif],
  For[k = Length[dif], k ≥ j, dif[[k]] = dif[[k]] - dif[[k - 1]]; k--]; j++]
```

```
dif
```

```
{ 1/26, 9/442, 23/1105, 42/1105, 114/1105, -54/221, -180/221, 6048/1105, -19152/1105, 9072/221, -18144/221 }
```

```
coef = dif
```

```
{ 1/26, 9/442, 23/1105, 42/1105, 114/1105, -54/221, -180/221, 6048/1105, -19152/1105, 9072/221, -18144/221 }
```

```
Binomial[t, 10]
```

```
Binomial[t, 10]
```

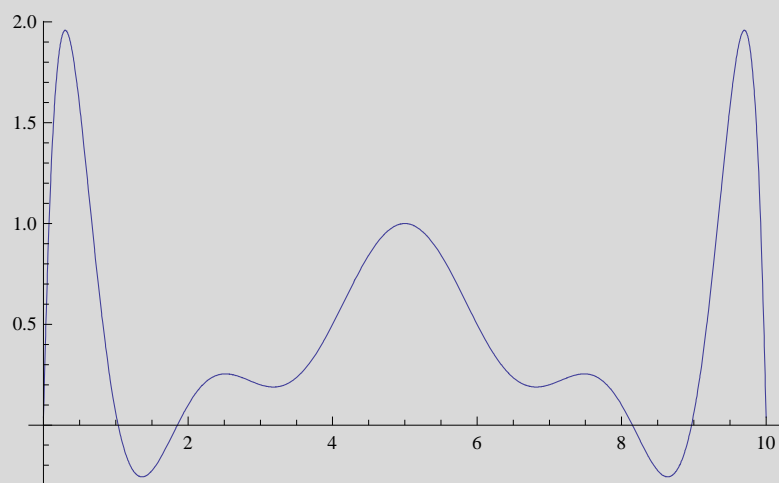
$$p[t_] = \sum_{i=1}^{\text{Length}[lista]} \text{coef}[[i]] * \text{Binomial}[t, i - 1]$$

$$\begin{aligned} & \frac{1}{26} + \frac{9t}{442} + \frac{23(-1+t)t}{2210} + \frac{7(-2+t)(-1+t)t}{1105} + \frac{19(-3+t)(-2+t)(-1+t)t}{4420} - \\ & \frac{9(-4+t)(-3+t)(-2+t)(-1+t)t}{4420} - \frac{180}{221} \text{Binomial}[t, 6] + \frac{6048 \text{Binomial}[t, 7]}{1105} - \\ & \frac{19152 \text{Binomial}[t, 8]}{1105} + \frac{9072}{221} \text{Binomial}[t, 9] - \frac{18144}{221} \text{Binomial}[t, 10] \end{aligned}$$

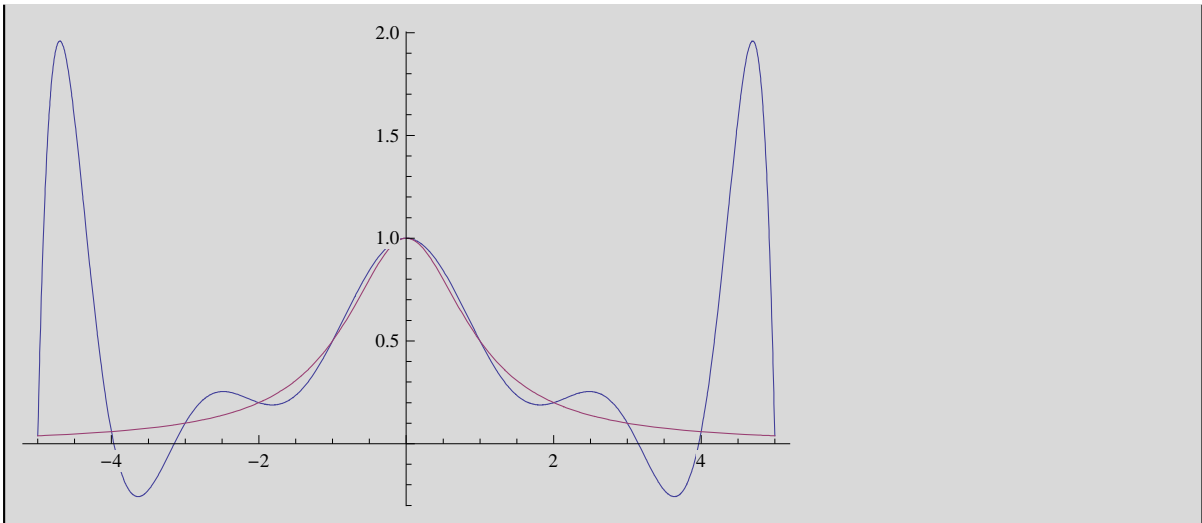
```
Factor[p[t]]
```

$$\begin{aligned} & \frac{1}{4420} (170 - 230t + 621t^2 - 401t^3 + 109t^4 - 9t^5 - \\ & 3600 \text{Binomial}[t, 6] + 24192 \text{Binomial}[t, 7] - 76608 \text{Binomial}[t, 8] + \\ & 181440 \text{Binomial}[t, 9] - 362880 \text{Binomial}[t, 10]) \end{aligned}$$

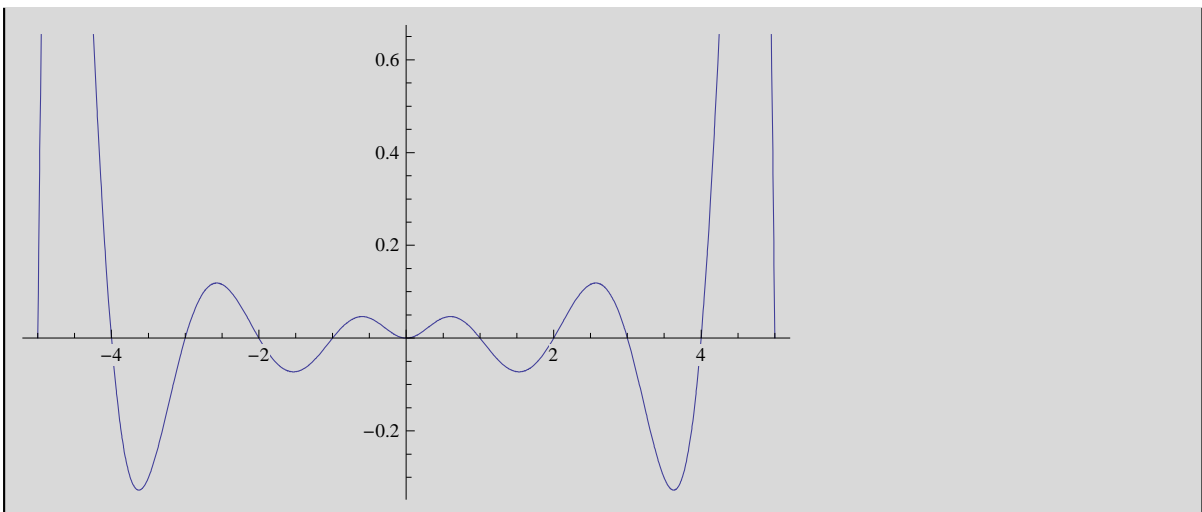
```
Plot[p[t], {t, 0, Length[lista] - 1}]
```



```
Plot[{p[t] /. t ->  $\frac{x+5}{1}$ , f[x]}, {x, -5, 5}]
```



```
Plot[{(p[t] /. t ->  $\frac{x+5}{1}$ ) - f[x]}, {x, -5, 5}]
```



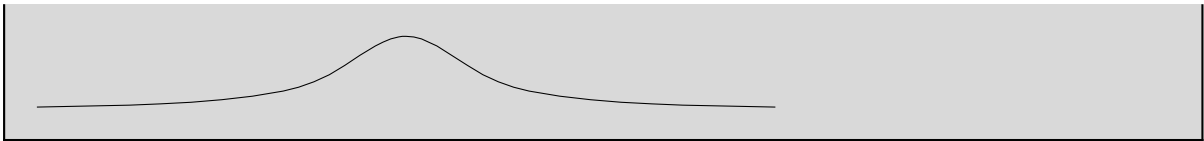
Aproximación mediante splines cúbicos

```
Needs["Splines`"]
```

```
lista = Table[{x, f[x]}, {x, -5, 5}]
```

```
{ {-5,  $\frac{1}{26}$ }, {-4,  $\frac{1}{17}$ }, {-3,  $\frac{1}{10}$ }, {-2,  $\frac{1}{5}$ }, {-1,  $\frac{1}{2}$ },  
  {0, 1}, {1,  $\frac{1}{2}$ }, {2,  $\frac{1}{5}$ }, {3,  $\frac{1}{10}$ }, {4,  $\frac{1}{17}$ }, {5,  $\frac{1}{26}$ }}
```

```
ej2 = Show[Graphics[Spline[lista, Cubic]]]
```



```
Show[grafico, ej2]
```

