

## Hoja de problemas I.1

### SOLUCIÓN DEL EJERCICIO 18

$$\Psi(x) = \begin{cases} x+1 & x \bmod 2 = 0 \\ x-1 & x \bmod 2 \neq 0 \end{cases}$$

- DOMINIO Y RANGO:

$$\text{dom } (\Psi) = \{x : x \bmod 2 = 0\} \cup \{x : x \bmod 2 \neq 0\} = \mathbb{N}$$

$$\begin{aligned} \text{ran } (\Psi) = & \{x+1 : x \bmod 2 = 0\} \cup \{x-1 : x \bmod 2 \neq 0\} = \{x : x \bmod 2 \neq 0\} \cup \\ & \cup \{x : x \bmod 2 = 0\} = \mathbb{N} \end{aligned}$$

- TOTAL: SI

$$\forall x \in \mathbb{N} \quad \Psi(x) \downarrow$$

- INYECTIVA: SI

Sean  $x, y \in \sum^*$ , siendo  $x \neq y$

caso 1)  $x, y$  pares  $\Rightarrow \Psi(x) = x+1 \neq y+1 = \Psi(y)$  (porque  $x \neq y$ )

caso 2)  $x, y$  impares  $\Rightarrow \Psi(x) = x-1 \neq y-1 = \Psi(y)$  (porque  $x \neq y$ )

caso 3)  $x$  par  $\wedge y$  impar  $\Rightarrow \Psi(x) = x+1 \neq y-1 = \Psi(y)$

(porque  $x+1$  será impar e  $y-1$  será par)

- SOBREYECTIVA: SI

Sea  $z \in \mathbb{N}$

caso 1)  $z$  par:  $z+1$  impar  $\Rightarrow \Psi(z+1) = z$

caso 2)  $z$  impar:  $z-1$  par  $\Rightarrow \Psi(z-1) = z$

- BIYECTIVA: SI

Por ser total, inyectiva y sobreyectiva

- FUNCIÓN INVERSA:

$$\Psi^{-1}(z) \equiv \left\{ \begin{array}{l} z-1 \quad (z-1) \bmod 2 = 0 \\ z+1 \quad (z+1) \bmod 2 \neq 0 \end{array} \right\} \cong \left\{ \begin{array}{l} z-1 \quad z \bmod 2 \neq 0 \\ z+1 \quad z \bmod 2 = 0 \end{array} \right\} \cong \Psi(z)$$

$$\theta(x) \cong \begin{cases} x & x \bmod 10 \neq 1 \\ 9 + 2 * x & \text{c.c.} \end{cases}$$

- DOMINIO Y RANGO:

$$\text{dom}(\theta) = \mathbb{N}$$

$$\text{ran}(\theta) = \{x : x \bmod 10 \neq 1\} \cup \{9+2*x : x \bmod 10 = 1\} =$$

$$\{x : x \bmod 10 \neq 1\} \cup \{9+2*(10*n+1) : n \geq 0\} =$$

$$\{x : x \bmod 10 \neq 1\} \cup \{(2*n+1)*10+1 : n \geq 0\} =$$

$$\sum^* - \{20*n+1 : n \geq 0\}$$

- TOTAL: SI

$$\forall x \in \mathbb{N} \theta(x) \downarrow$$

- INYECTIVA: SI

Sean  $x, y \in \sum^*$ , siendo  $x \neq y$

$$\underline{\text{caso 1}}) \quad x \bmod 10 \neq 1 \wedge y \bmod 10 \neq 1 \Rightarrow \theta(x) = x \neq y = \theta(y)$$

$$\underline{\text{caso 2}}) \quad x \bmod 10 = y \bmod 10 = 1 \Rightarrow \theta(x) = 9+2*x \neq 9+2*y = \theta(y)$$

$$\underline{\text{caso 3}}) \quad x \bmod 10 \neq 1 \wedge y \bmod 10 = 1 \Rightarrow y = 10*n+1 \Rightarrow$$

$$\Rightarrow \theta(y) = 9+2*y = 11+20*n \Rightarrow \theta(y) \bmod 10 = 1 \Rightarrow$$

$$\Rightarrow \theta(y) \neq x = \theta(x)$$

- SOBREYECTIVA: NO

$$\forall x \in \mathbb{N} \neg \theta(x) = 1 \quad (\text{es falso } 1 \bmod 10 \neq 1, \text{ y es imposible } 9+2*y = 1)$$

- BIYECTIVA: NO

Por no ser sobreyectiva

- FUNCIÓN INVERSA:

$$\theta^{-1}(z) \cong \left\{ \begin{array}{ll} z & z \bmod 10 \neq 1 \\ \frac{z-9}{2} & z-9 \text{ par} \wedge ((z-9)/2) \bmod 10 = 1 \\ \perp & \text{c.c.} \end{array} \right\} \cong \left\{ \begin{array}{ll} z & z \bmod 10 \neq 1 \\ \frac{z-9}{2} & z \bmod 10 = 1 \wedge \\ & \wedge z/10 \text{ impar} \\ \perp & \text{c.c.} \end{array} \right\}$$

$$\phi(x) \equiv \begin{cases} 2*x & x < 10 \\ | \\ x/2 & x \geq 10 \wedge x \text{ mod } 2 = 0 \\ | \\ \perp & \text{c.c.} \end{cases}$$

- DOMINIO Y RANGO:

$$\text{dom } (\phi) = \{x : x < 10\} \cup \{x : x \geq 10 \wedge x \text{ par}\} = \sum^* - \{x : x > 10 \wedge x \text{ impar}\}$$

$$\text{ran } (\phi) = \{2*x : x < 10\} \cup \{x/2 : x \geq 10 \wedge x \text{ par}\} = \\ \{2*x : x < 10\} \cup \{x : x \geq 5\} = \mathbb{N} - \{1, 3\}$$

- TOTAL: NO

$$\phi(11) \uparrow$$

- INYECTIVA: NO

$$\phi(4) = \phi(16)$$

- SOBREYECTIVA: NO

$$\forall x \in \mathbb{N} \neg \phi(x) = 1 \quad (\text{es imposible } 2*x = 1 \text{ y } x/2 = 1 \text{ con } x \geq 10)$$

- BIYECTIVA: NO

$$\chi(x) \cong \begin{cases} x/3 & x \bmod 3 \neq 0 \\ \perp & \text{c.c} \end{cases}$$

- DOMINIO Y RANGO:

$$\text{dom } (\chi) = \{x : x \bmod 3 \neq 0\}$$

$$\text{ran } (\chi) = \{x/3 : x \bmod 3 \neq 0\}$$

- TOTAL: NO

$$\chi(3) \uparrow$$

- INYECTIVA: NO

$$\chi(4) = \chi(5)$$

- SOBREYECTIVA: SI

Sea  $z \in \prod$

Se verifica  $\chi(3*z+1) = z$

- BIYECTIVA: NO

## Composiciones

$$\Psi(\chi(x)) \equiv \begin{cases} x/3 + 1 & x \bmod 3 \neq 0 \wedge x/3 \text{ par} \\ x/3 - 1 & x \bmod 3 \neq 0 \wedge x/3 \text{ impar} \\ \perp & \text{c.c.} \end{cases}$$

$$\chi(\theta(x)) \equiv \begin{cases} x/3 & x \bmod 10 \neq 1 \wedge x \bmod 3 \neq 0 \\ \perp & x \bmod 10 \neq 1 \wedge x \bmod 3 = 0 \\ (9+2*x)/3 & x \bmod 10 = 1 \wedge (9+2*x) \bmod 3 \neq 0 \\ \perp & x \bmod 10 = 1 \wedge (9+2*x) \bmod 3 = 0 \end{cases} \quad \equiv$$

$$\begin{cases} x/3 & x \bmod 10 \neq 1 \wedge x \bmod 3 \neq 0 \\ (9+2*x)/3 & x \bmod 10 = 1 \wedge x \bmod 3 \neq 0 \\ \perp & \text{c.c.} \end{cases}$$

$$\Psi(\Psi(x)) \equiv x$$

$$\phi(\phi(x)) \equiv \begin{cases} 4*x & x < 10 \wedge 2*x < 10 \\ x & x < 10 \wedge 2*x \geq 10 \wedge 2*x \bmod 2 = 0 \\ \perp & x < 10 \wedge 2*x \geq 10 \wedge 2*x \bmod 2 \neq 0 \\ 2*(x/2) & x \geq 10 \wedge x \bmod 2 = 0 \wedge x/2 < 10 \\ x/4 & x \geq 10 \wedge x \bmod 2 = 0 \wedge x/2 \geq 10 \wedge x/2 \bmod 2 = 0 \\ \perp & x \geq 10 \wedge x \bmod 2 = 0 \wedge x/2 \geq 10 \wedge x/2 \bmod 2 \neq 0 \\ \perp & x \geq 10 \wedge x \bmod 2 \neq 0 \end{cases} \quad \equiv$$

$$\begin{cases} 4*x & x < 5 \\ x & 5 \leq x < 10 \vee (10 \leq x < 20 \wedge x \text{ par}) \\ x/4 & x \geq 20 \wedge x \bmod 4 = 0 \\ \perp & \text{c.c.} \end{cases}$$