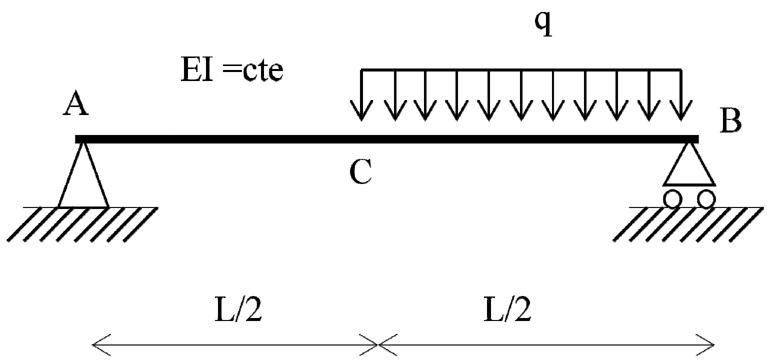
Ejercicio 2: estructura isostática

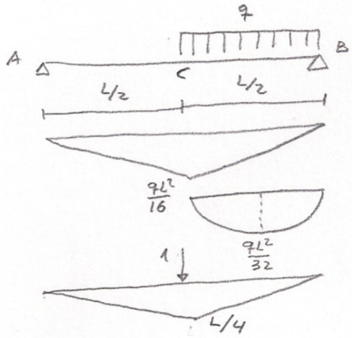


Calcular la flecha en C considerando la viga deformable y también considerando el tramo CB indeformable, empleando en cada caso:

a) El método de la carga unitaria
b) El método de la viga conjugada
c) En 2º Teorema de Castigliano
d) Los Teoremas de área de momentos

Solución:

Por carga unitaria



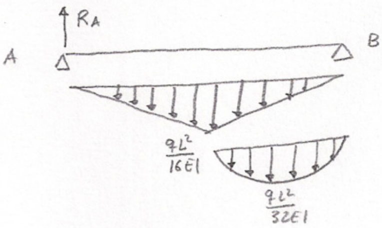
a) Considerando todo deformable

$$y_C\downarrow = \frac{1}{3EI}\cdot\frac{qL^2}{16}\cdot\frac{L}{4}\cdot L + \frac{1}{3EI}\cdot\frac{L}{4}\cdot\frac{qL^2}{32}\cdot\frac{L}{2} = \frac{5qL^4}{768EI}$$

b) Considerando CB indeformable

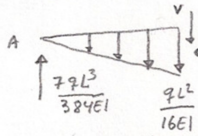
$$y_C\downarrow = \frac{1}{3EI}\cdot\frac{qL^2}{16}\cdot\frac{L}{4}\cdot\frac{L}{2} = \frac{qL^4}{384EI}$$

Por viga conjugada



a) Considerando todo deformable

$$\sum M_B = 0 \rightarrow R_A\cdot L = \frac{qL^2}{16EI}\cdot\frac{L}{2}\cdot\frac{L}{2} + 2\cdot\frac{L}{2}\left(\frac{qL^2}{32EI}\right)\frac{1}{3}\cdot\frac{L}{4} \rightarrow R_A = \frac{7qL^3}{384EI}\uparrow$$

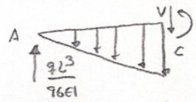


$\Sigma M_C = 0$

$$\frac{7qL^3}{384EI}\cdot\frac{L}{2} - \frac{qL^2}{16EI}\cdot\frac{L}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{L}{2} = m_C = \frac{5qL^4}{768EI} = y_C\downarrow$$

b) Considerando CB indeformable

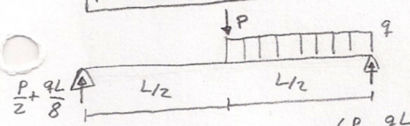
$$\sum M_B = 0 \rightarrow R_A\cdot L = \frac{qL^2}{16EI}\cdot\frac{L}{2}\cdot\frac{1}{2}\left(\frac{L}{2}+\frac{L}{6}\right) \Rightarrow R_A = \frac{qL^3}{96EI}$$

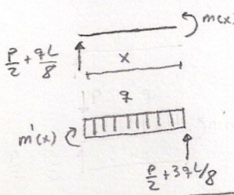


$\Sigma M_C = 0$

$$\frac{qL^3}{96EI}\cdot\frac{L}{2} - \frac{qL^2}{16EI}\cdot\frac{L}{2\cdot 2}\cdot\frac{1}{3}\cdot\frac{L}{2} = m_C = \frac{qL^4}{384EI} = y_C\downarrow$$

Por Castigliano





$$m(x) = \left(\frac{P}{2}+\frac{qL}{8}\right)x$$

$$\frac{\partial m(x)}{\partial P} = -x/2$$

$$m'(x) = \left(\frac{P}{2}+\frac{3qL}{8}\right)x - \frac{qx^2}{2}$$

$$\frac{\partial m'(x)}{\partial P} = \frac{x}{2}$$

Desplazamiento de P por deformación de AC cuando P=0

$$\left[\int_0^{\frac{L}{2}}\frac{\left(\frac{P}{2}+\frac{qL}{8}\right)\frac{x^2}{2}dx}{EI}\right]_{P=0} = \frac{qL^4}{384EI}$$

Desplazamiento de P por deformación de C-B cuando P=0

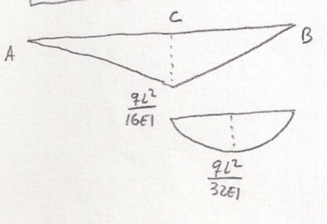
$$\int_0^{\frac{L}{2}}\frac{\left[\left(\frac{3qL}{8}\cdot x\right)-\frac{qx^2}{2}\right]\frac{x}{2}dx}{EI} = qL^2/256EI$$

a) Considerando todo deformable:

$$\frac{qL^4}{384EI} + \frac{qL^4}{256EI} = \frac{5qL^4}{768EI}$$

b) Considerando CB indeformable $= \frac{qL^4}{384EI}$

Por Área de momentos



a) Considerando todo deformable

$[A,B] \rightarrow y_B = y_A + \theta_A L_{AB} - A_{AB}\cdot X_{GB}$

$$0 = 0 + \theta_A L - \left[\frac{qL^2}{16EI}\cdot\frac{L}{2}\cdot\frac{L}{2} + \frac{2L}{2}\cdot\frac{qL^2}{32EI}\cdot\frac{1}{3}\frac{L}{4}\right] \rightarrow \theta_A = \frac{7qL^3}{384EI}$$

$$[A-C] \rightarrow y_C = 0 + \frac{7qL^3}{384EI}\cdot\frac{L}{2} - \left[\frac{qL^2}{16EI}\cdot\frac{L}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{L}{2}\right] = \frac{5qL^4}{768EI}$$

b) Considerando CB indeformable

$$[A-B] \rightarrow 0 = 0 + \theta_A L - \left[\frac{qL^2}{16EI}\cdot\frac{L}{2}\cdot\frac{1}{2}\cdot\left(\frac{L}{2}+\frac{1}{3}\cdot\frac{L}{2}\right)\right] \rightarrow \theta_A = \frac{qL^3}{96EI}$$

$$[A-C] \rightarrow y_C = 0 + \frac{qL^3}{96EI}\cdot\frac{L}{2} - \left[\frac{qL^2}{16EI}\cdot\frac{L}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{L}{2}\right] = \frac{qL^4}{384EI}$$