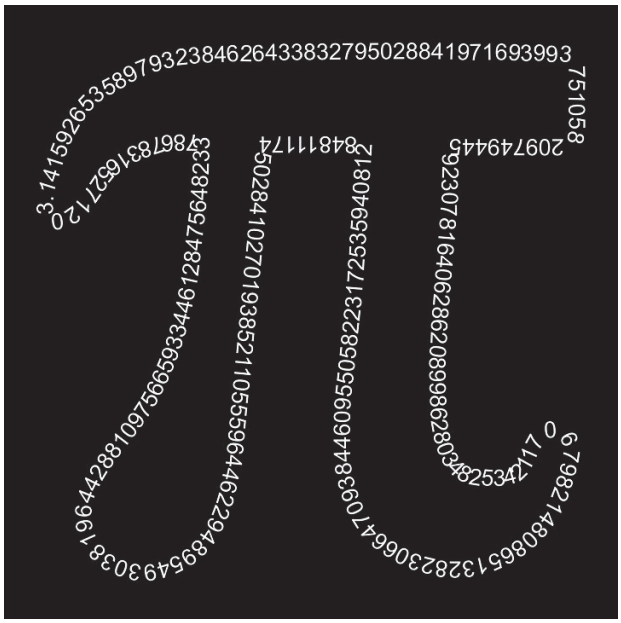
MATHS BASIC COURSE FOR UNDERGRADUATES



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TEST 2. STATEMENTS

Exercise 1. Prove using the induction method that for any natural number n, the following statement fulfills:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

Exercise 2. Let $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{2} + i\sqrt{2}$ be two complex numbers. Calculate the modulus and the argument of each of them. Compute also the cubic roots of the complex number $z = z_1 z_2$.

Exercise 3. Using the Euclidean extended algorithm, calculate the greatest common divisor of the numbers 486 and 2012, and express it as a combination of those numbers using the Bezout's identity.

Exercise 4. Using Sieve of Eratosthenes, find all the prime numbers between 150 and 219.

Exercise 5. Calculate for the following system of linear congruences a particular solution, and latter, determine the set of all possible solutions of that system.

$$\begin{array}{rcl} 10x & \equiv & 3 \pmod{23} \\ 5x & \equiv & 4 \pmod{27} \end{array}$$

Exercise 6. First, remind Fermat's Little Theorem. If p is a prime number, prove also that the following statement fulfills:

$$1^{p-1} + 2^{p-1} + \cdots + (p-1)^{p-1} \equiv -1 (\text{mod } p).$$

Exercise 7. Let $p(x) = x^4 + 2x^3 - x^2 - 2x + 1$ be a polynomial.

(i) Calculate $\gcd(p(x), p'(x))$, being $p'(x)$ the derivative polynomial of the polynomial $p(x)$.

(ii) Conclude from item (i) which are the multiple roots of the polynomial $p(x)$.

Exercise 8. Decompose the following rational function as a partial fraction decomposition.

$$\frac{3x+1}{x^4 + 2x^3 + x^2}.$$

Exercise 9. Solve the polynomial inequation: $|x-1| + |x+4| > 10$.