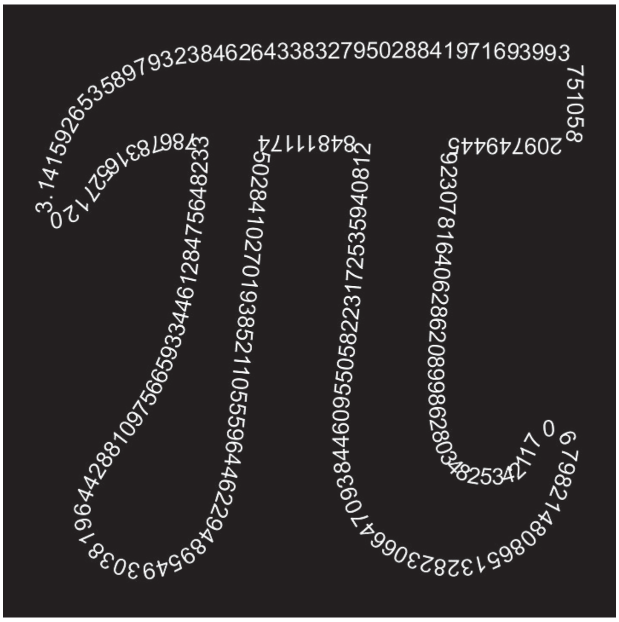
Universidad del País Vasco
Euskal Herriko Unibertsitatea

MATHS BASIC COURSE FOR UNDERGRADUATES



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STATEMENTS: 8th SUBJECT. ELEMENTARY LINEAR ALGEBRA

Exercise 1. *Prove that, if K is a field and n is a natural number and we consider the usual structure of K-vector space on K^n, determined by the operations $(a_1, \ldots, a_n) + (b_1, \ldots, b_n) = (a_1 + b_1, \ldots, a_n + b_n)$ and $\lambda(a_1, \ldots, a_n) = (\lambda a_1, \ldots, \lambda a_n)$, then the K-dimension of K^n is n.*

Exercise 2. *Calculate the following determinant:*

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 2 & 0 & 3 \\ 0 & -8 & 2 & 0 \\ 1 & 2 & 0 & 2 \end{vmatrix}.$$

Exercise 3. *Let K be a field, and let $a_1, \ldots, a_n \in K$.*

(i) *Prove that*

$$\begin{vmatrix} 1 & 1 & \ldots & 1 \\ a_1 & a_2 & \ldots & a_n \\ \vdots & \vdots & & \vdots \\ a_1^{n-1} & a_2^{n-1} & \ldots & a_n^{n-1} \end{vmatrix} = \prod_{i>j}(a_i - a_j)$$

(ii) *With the hypotheses as above, deduce for $i \in \mathbb{Z}^+$ the value of*

$$\begin{vmatrix} a_1^i & a_2^i & \ldots & a_n^i \\ a_1^{i+1} & a_2^{i+1} & \ldots & a_n^{i+1} \\ \vdots & \vdots & & \vdots \\ a_1^{i+n-1} & a_2^{i+n-1} & \ldots & a_n^{i+n-1} \end{vmatrix}.$$

Exercise 4. *Find the inverse of the following A matrix:*

$$A = \begin{pmatrix} 2 & 0 & 2 \\ 3 & 1 & 6 \\ 2 & 1 & 2 \end{pmatrix}.$$