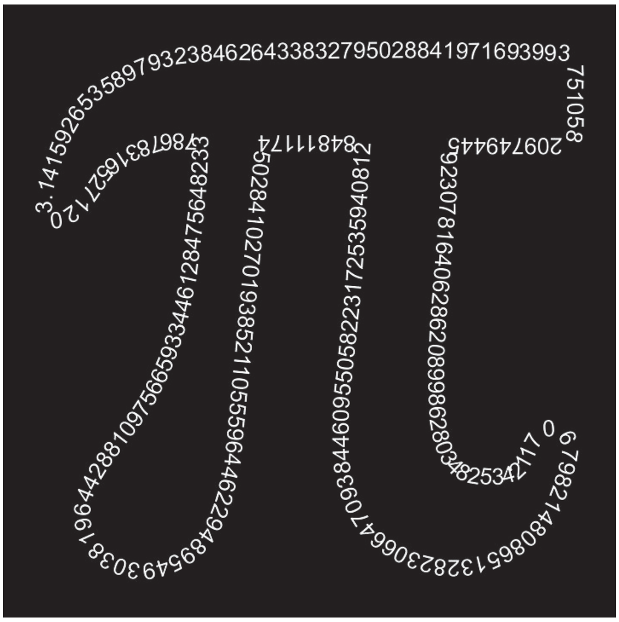
MATHS BASIC COURSE FOR UNDERGRADUATES



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STATEMENTS: 6th SUBJECT. POLYNOMIALS

Exercise 1. *Prove that the polynomial* $f(x) = x^3 - 1$ *has an unique root over the field* $\mathbb{R}$.

Exercise 2. *Calculate* $gcd(x^5 - 1, x^3 + x - 2)$.

Exercise 3. *Let* $f(x) \in K[x]$ *be a polynomial such that* $dg(f(x)) = 2$ *or* 3. *Prove that* $f(x)$ *is irreducible over* K *if and only if the polynomial* $f(x)$ *does not have roots over the field* K.

Exercise 4. *Let* $f(x) = a_0 + a_1x + \cdots + a_nx^n \in \mathbb{Z}[x]$ *be a polynomial with coefficients over* $\mathbb{Z}$. *Assume that* $dg(f(x)) \geq 2$. *If the polynomial* $f(x)$ *admits a rational root, prove that this root must be of type* $\frac{r}{s}$, *satisfying that* $r \mid a_0, s \mid a_n$, $r, s \in \mathbb{Z}$ *and* $gcd(r, s) = 1$.

Exercise 5. *Prove that the polynomial* $f(x) = 2x^3 - x^2 + 8x + 1 \in \mathbb{Z}[x]$ *does not have rational roots.*

Exercise 6. *Decompose the polynomial* $f(x) = x^4 - 2x^2 + 8x + 1 \in \mathbb{Z}[x]$ *as a product of irreducible factors.*

Exercise 7. *Decompose the polynomial* $f(x) = x^6 - 25x^5 + 3x^2 + 12 \in \mathbb{Z}[x]$ *as a product of irreducible factors.*