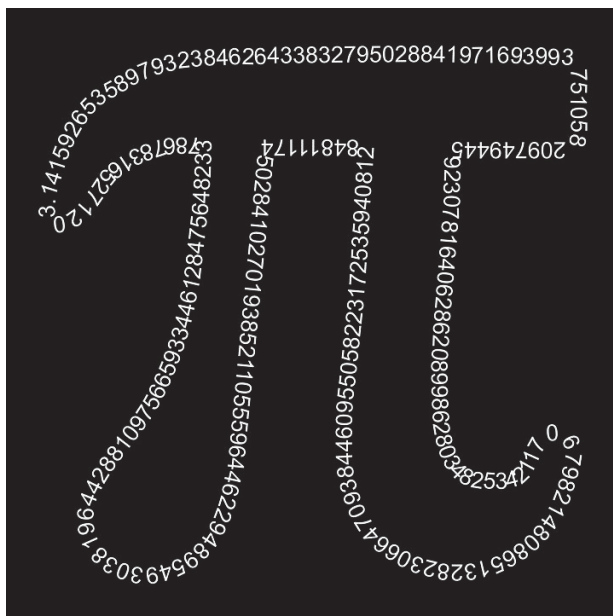


MATHS BASIC COURSE FOR UNDERGRADUATES



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STATEMENTS: 3rd SUBJECT. FUNCTIONS

Exercise 1. Let f be a function from \mathbb{R} to \mathbb{R} given by the formula $f(x) = x^2$, for any $x \in \mathbb{R}$. Calculate the following image sets and inverse images by the function f : $\text{im} f$, $f([0, 2])$, $f([2, +\infty))$, $f((-\infty, -1) \cup [2, +\infty))$, $f^{-1}(1)$, $f^{-1}(-1)$, $f^{-1}([-1, 0])$ and $f^{-1}((1, +\infty))$.

Exercise 2. Analyze in which domains and codomains the function f given by the expression $f(x) = x^2$ is bijective or not, and in the case it is possible, calculate which is its corresponding inverse function.

Exercise 3. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined by parts as follows:

$$f(x) = \begin{cases} x + 3, & \text{when } x < -1 \\ x^2 + 1, & \text{when } -1 \leq x \leq 1 \\ x + 1, & \text{when } x > 1 \end{cases}$$

- (i) Is the function f injective? Is f surjective?
- (ii) Calculate $f([0, 2])$ and $f^{-1}([0, 2])$.
- (iii) Consider the function g defined by the expression $g(x) = |x| + 1$. Calculate the composition function $f \circ g$, and latter the image set $(f \circ g)(\mathbb{R})$.

Exercise 4. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ be two functions, given by the expressions $f(x) = x^2$ and $g(x) = x + 2$, respectively. In the case it is possible, calculate the following composition functions: $f \circ f$, $f \circ g$, $g \circ f$ and $g \circ g$.