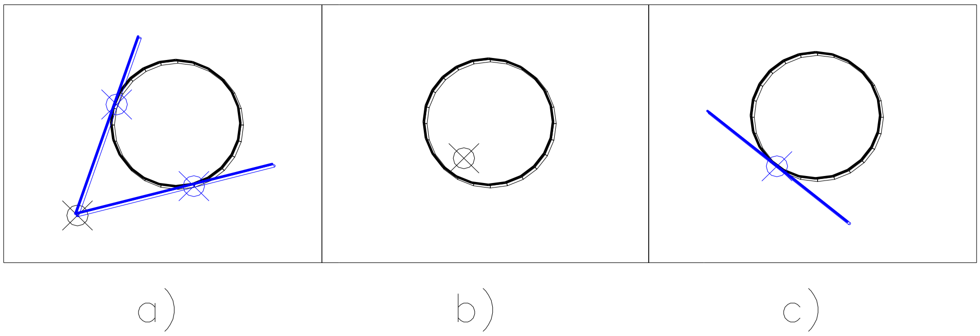
The resolution method consists in:
1. Calculating the dimensions of the cone (the radius R of the basis): its vertex will be any point of the line r (point A in the figure), the height of the cone (h) will be the Z coordinate.
2. Drawing the basis of the cone: circumference of radius R and center A'.
3. Calculating the horizontal trace of the line r: point B.
4. Drawing from B the tangent lines to the basis of the cone: horizontal lines m and n.
5. The two solutions of the general case are the planes α = r+m, and β = r+n.

Possibilities:
a) If the slope of the requested planes is greater than the slope of the line, there are two solution planes (two planes will be tangent to the cone).
b) If the slope of the requested planes is smaller than the slope of the line, there is no solution (there is no tangent plane to the cone).
c) If the slope of the requested planes is the same as the slope of the line, there is one solution (one plane will be tangent to the cone).



6.5. Mutual examples of both subjects

►***Example 45 (A)***

Find the plane that contains the line $r\colon\begin{cases} x-z=3 \\ 3y+z=9 \end{cases}$ and forms an angle of 45º with the plane XOY.

Solution:

Using the concept of sheaf of planes, it is known that the plane that contains the line r is:

$\pi_1\colon x-z-3+\alpha\,(3y+z-9)=0$, being its normal vector $n_{\pi_1}=(1{,}3\alpha,\alpha-1)$.

On the other hand, the normal vector of the plane $\pi_2\colon z=0$ is $n_{\pi_2}=(0{,}0{,}1)$. As the angle between two planes is the angle between their normal vectors:

$$\cos 45° = \frac{|(1{,}3\alpha,\alpha-1)(0{,}0{,}1)|}{\sqrt{1^2+(3\alpha)^2+(\alpha-1)^2}\ \sqrt{1}} = \frac{|\alpha-1|}{\sqrt{10\alpha^2-2\alpha+2}} = \frac{1}{\sqrt{2}} \quad \rightarrow$$

$$2(\alpha-1)^2 = 10\alpha^2 - 2\alpha + 2 \rightarrow \;\; 4\alpha^2 + \alpha = 0 \;\rightarrow \begin{cases} \alpha_1 = 0 \\ \alpha_2 = -\frac{1}{4} \end{cases}$$

By substituting these values in the plane π_1, the following two planes are obtained:

$\pi_{1a}: x - z - 3$ and $\pi_{1b}: 4x - 3y - 5z - 3 = 0$

►*Example 45* (G)

Find and draw the planes that contain the line r and form an angle of 45º with the plane PH.

Solution: There are two planes, α and β, that satisfy the requested conditions.