Solution to Exercise E7.

Heteroskedasticity and Autocorrelation.

Exercise E7.1 Beach umbrella rental

Part I. Simple Linear Regression Model.

a. Regression model: $U_t = \alpha + \beta T_t + u_t \quad t = 1, \ldots, 22$

Model 1: OLS, using observations 2013-05-05–2013-09-29 (T = 22)
Dependent variable: U

	Coefficient	Std. Error	t-ratio	p-value
const	27.0692	26.9160	1.0057	0.3266
T	11.4595	0.860078	13.3238	0.0000

Mean dependent var	381.2727	S.D. dependent var	60.60110
Sum squared resid	7808.908	S.E. of regression	19.75969
R^2	0.898747	Adjusted R^2	0.893684
$F(1, 20)$	177.5241	P-value(F)	2.09e–11
Log-likelihood	−95.80841	Akaike criterion	195.6168
Schwarz criterion	197.7989	Hannan–Quinn	196.1308
$\hat{\rho}$	0.127938	Durbin–Watson	1.661439

SRF: $\hat{S}_t = 27.0692 + 11.4595\ T_t$

b. White's test.

$$H_0 : \sigma_t^2 = \sigma^2 \qquad\qquad LM = TR^2 \overset{H_0,a}{\sim} \chi^2(2)$$
$$H_a : \text{Heteroskedasticity}$$

Auxiliary regression:

$$\hat{u}_t^2 = \alpha_0 + \alpha_1 T_t + \alpha_2 T_t^2 + w_t$$

Decision rule: $LM = 3.293854 < 5.99146 = \chi^2_{0.05}(2)$

The null hypothesis is not rejected at the 5% significance level. The variance of the error term is constant throughout the sample.

c. Durbin-Watson test.

$$\begin{cases} H_0 : \rho = 0 & \text{(No first order autocorrelation)} \\ H_a : u_t = \rho u_{t-1} + v_t \quad \rho > 0 & \text{(Positive first order autoregressive process.)} \end{cases}$$

Test statistic: $$DW = \frac{\sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{T} \hat{u}_t^2}$$

Decision rule: $DW = 1.661439 > 1.4289 = d_U$, the null hypothesis is not rejected at the 5% significance level. There is no evidence of a first order autocorrelation process in the error term.

d. Given the results of the tests, the assumptions of the Multiple Regression Model on the error term are satisfied. Therefore, conditional on X, the OLS estimator is linear and unbiased and it has the smallest variance in the class of all linear and unbiased estimators.

e. Test the statistical significance of the variable temperature.

$$H_0 : \beta = 0 \qquad t = \frac{\hat{\beta} - 0}{\hat{\sigma}_{\hat{\beta}}} \overset{H_0}{\sim} t(T-k)$$
$$H_a : \beta \neq 0$$

Since $\mid t \mid = 13.3238 > 2.08596 = t_{0.025}(20)$, the null hypothesis is rejected at the 5% significance level. Therefore, the variable temperature is statistically significant.

Part II. General Linear Regression Model.

a. Regression model: $U_t = \beta_1 + \beta_2 T_t + \beta_3 P_t + \beta_4 WW_t + v_t \quad t = 1, \ldots, 22$

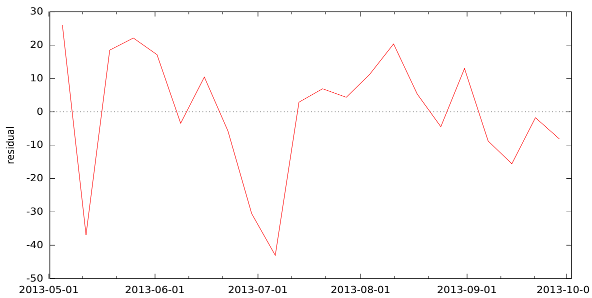
Model 2: OLS, using observations 2013-05-05–2013-09-29 ($T = 22$)
Dependent variable: U

	Coefficient	Std. Error	t-ratio	p-value
const	44.5693	45.7575	0.9740	0.3429
T	11.0471	2.18129	5.0645	0.0001
P	−0.0524909	3.32631	−0.0158	0.9876
WW	−10.5245	9.73983	−1.0806	0.2942

Mean dependent var	381.2727	S.D. dependent var	60.60110
Sum squared resid	7320.861	S.E. of regression	20.16716
R^2	0.905075	Adjusted R^2	0.889254
$F(3, 18)$	57.20762	P-value(F)	2.11e–09
Log-likelihood	−95.09850	Akaike criterion	198.1970
Schwarz criterion	202.5612	Hannan–Quinn	199.2251
$\hat{\rho}$	0.107465	Durbin–Watson	1.685968

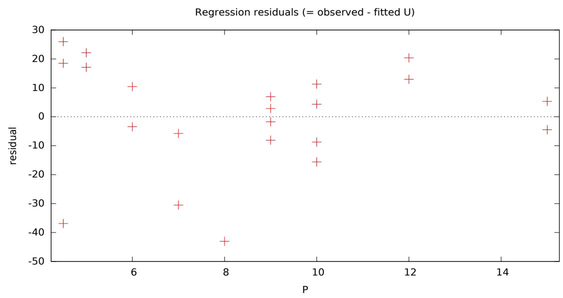
SRF: $\hat{U}_t = 44.5693 + 11.0471\, T_t - 0.0524909\, P_t - 10.5245\, WW_t \quad t = 1, \ldots, 22$

b. Residuals time series plot.



There is not a regular pattern in the residuals and the variance seems to be constant over the whole sample. Nevertheless, the two deep troughs observed in the residuals would have to be analysed.

b. Graph of the residuals against P.



It seems that the variance of the residuals decreases with prices. In order to test this hypothesis it is necessary to perform a heteroskedasticity test.

c. White's test.

$$H_0 : \sigma_t^2 = \sigma^2 \qquad\qquad LM = TR^2 \overset{H_0,a}{\sim} \chi^2(8)$$
$$H_a : \text{Heteroskedasticity}$$

Auxiliary regression:

$$\hat{u}_t^2 = \alpha_0 + \alpha_1 T_t + \alpha_2 T_t^2 + \alpha_3 P_t + \alpha_4 P_t^2 + \alpha_5\, WW_t + \alpha_6\, T_t P_t + \alpha_7\, T_t\, WW_t + \alpha_8\, P_t\, WW_t + w_t$$

Decision rule: $LM = 9.098285 < 15.5073 = \chi^2_{0.05}(8)$

the null hypothesis is not rejected at the 5% significance level. Therefore, there is no evidence of heteroskedasticity in the sample.

d. Durbin-Watson test.

$$\begin{cases} H_0 : \rho = 0 \quad \text{(No first order autocorrelation)} \\ H_a : u_t \;=\; \rho\, u_{t-1} + v_t \quad \rho > 0 \quad \text{(First order autoregressive process (positive parameter))} \end{cases}$$

Test statistic: $$DW = \frac{\sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{T} \hat{u}_t^2}$$

Decision rule: $DW = 1.685968 > 1.6640 = d_U$, the null hypothesis is not rejected at the 5% significance level. Therefore, there is no evidence in the sample of first order autocorrelation in the error term.

e. Given that the assumptions on the error term are satisfied, the inference based on the OLS estimator is valid and the tests performed in Exercise 6 are valid. Thus, it may be concluded that the variables price and *windy week* are not statistically significant and that the appropriate model to determine the number of rented umbrellas is model (1).

Exercise E7.2 Holiday cottages

Model A

a. Regression model: $RP_i = \alpha_1 + \alpha_2\, NR_i + \alpha_3\, BP_i + u_i$

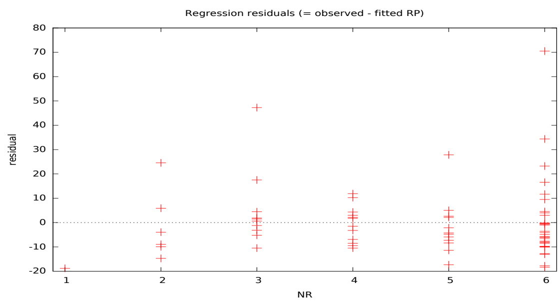
Model 1: OLS, using observations 1–75
Dependent variable: RP

	Coefficient	Std. Error	t-ratio	p-value
const	38.4321	7.22899	5.3164	0.0000
NR	2.26766	1.20082	1.8884	0.0630
BP	1.49558	1.09746	1.3628	0.1772

Mean dependent var	56.13893	S.D. dependent var	14.98446
Sum squared resid	15263.15	S.E. of regression	14.55982
R^2	0.081392	Adjusted R^2	0.055875
$F(2, 72)$	3.189724	P-value(F)	0.047064
Log-likelihood	−305.7595	Akaike criterion	617.5189
Schwarz criterion	624.4714	Hannan–Quinn	620.2950

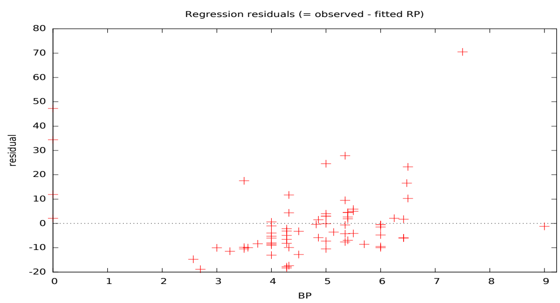
SRF: $\widehat{RP}_i = 38.4321 + 2.26766\, NR_i + 1.49558\, BP_i$

b. Plot of the residuals against the number of rooms.



It seems that there is not a regular pattern in the residuals. It does not seem that the variance of the residuals may depend on the number of rooms.

Plot of the residuals against the price of breakfast.



It seems that the variance of the residuals may increase with the variable price of breakfast.

c. Goldfeld-Quandt test.

$$H_0 : \sigma_t^2 = \sigma^2$$
$$H_a : \text{Heteroskedasticity increasing with BP}$$

Given the plot of the residuals against BP, we sort the sample by the variable price of breakfast.

First sample (first 25 observations):

$$\widehat{RP}_i = 56.5307 + 2.68329\, NR_i - 5,49373\, BP_i \quad SSR_1 = 2987.026$$

Second sample (last 25 observations):

$$\widehat{RP}_i = -15.4501 + 4.35148\, NR_i + 9.23536\, BP_i \quad SSR_1 = 5183.993$$

Test statistic: $GQ = \dfrac{SSR_2/N_2 - k_2}{SSR_1/N_1 - k_1} \overset{H_0}{\sim} \mathcal{F}(N_2 - k_2, N_1 - k_1)$

Decision rule: $GQ = \frac{5183.993}{2987.026} \times \frac{25-3}{25-3} = 1.735503 < 2.04777 = \mathcal{F}_{0.05}(22, 22)$

The null hypothesis of homoskedasticity is not rejected at the 5% significance level. The variance of the error term does not depend on the price of breakfast.

d. Given the previous result, the standard distributions of the test statistics used to make inference based on the OLS estimator are valid.

Test the statistical significance of the variable number of rooms.

$$H_0 : \alpha_2 = 0 \qquad t = \frac{\hat{\alpha}_2 - 0}{\hat{\sigma}_{\hat{\alpha}_2}} \overset{H_0}{\sim} t(T-k)$$
$$H_a : \alpha_2 \neq 0$$

Since $\mid t \mid = 1.888 < 1.99346 = t_{0.025}(72)$, the null hypothesis is not rejected at the 5% significance level. Therefore, the variable number of rooms is not statistically significant.

Model B

a. Regression model:

$$RP_i = \lambda_1 + \lambda_2\, NR_i + \lambda_3\, BP_i + \lambda_4\, WIFIF_i + \lambda_5\, WIFIP_i + \lambda_6\, LOCC_i + u_i$$

Model 2: OLS, using observations 1–75
Dependent variable: RP

	Coefficient	Std. Error	t-ratio	p-value
const	40.5761	7.39661	5.4858	0.0000
NR	1.94192	1.21303	1.6009	0.1140
BP	0.559911	1.21918	0.4593	0.6475
WIFIF	6.98544	3.65362	1.9119	0.0600
WIFIP	−5.75696	12.0827	−0.4765	0.6352
LOCC	2.11170	5.43209	0.3887	0.6987

Mean dependent var	56.13893	S.D. dependent var	14.98446
Sum squared resid	14429.98	S.E. of regression	14.46133
R^2	0.131536	Adjusted R^2	0.068604
$F(5, 69)$	2.090130	P-value(F)	0.077001
Log-likelihood	−303.6545	Akaike criterion	619.3089
Schwarz criterion	633.2138	Hannan–Quinn	624.8610

SRF:

$$\begin{aligned}\widehat{RP}_i &= 40.5761 + 1.94192\, NR_i + 0.559911\, BP_i + 6.98544\, WIFIF_i - \\ &- 5.75696\, WIFIP_i + 2.11170\, LOCC_i\end{aligned}$$

White's test.

$$\begin{array}{ll} H_0 : \sigma_i^2 = \sigma^2 & \\ H_a : \text{Heteroskedasticity} & LM = TR^2 \overset{H_{0,a}}{\sim} \chi^2(14) \end{array}$$

Decision rule: $LM = 23.284126 < 23.6848 = \chi^2_{0.05}(14)$.

The null hypothesis of homoskedasticity is not rejected at the 5% significance level. Nevertheless, it should be noted that the sample value of the statistic and the χ^2 quantile are very close.

If we choose the second option of the White's test offered by Gretl that includes only the squares in the auxiliary regression, we get:

$LM = 14.924290 > 14.0671 = \chi^2_{0.05}(7)$ and the null hypothesis of homoskedasticity is rejected at the 5% significance level.

b. Given the result of the White's test, it would be recommendable to estimate the model by OLS but using to make inference the estimated covariance matrix of the OLS estimator $\hat{\beta}$ robust to heteroskedasticity proposed by White.

The estimation results are:

Model 2: OLS, using observations 1–75
Dependent variable: RP
Heteroskedasticity-robust standard errors, variant HC0

	Coefficient	Std. Error	t-ratio	p-value
const	40.5761	12.0497	3.3674	0.0012
NR	1.94192	1.11954	1.7346	0.0873
BP	0.559911	2.11458	0.2648	0.7920
WIFIF	6.98544	2.17754	3.2079	0.0020
WIFIP	−5.75696	8.48536	−0.6785	0.4998
LOCC	2.11170	3.02680	0.6977	0.4877

Mean dependent var	56.13893	S.D. dependent var	14.98446
Sum squared resid	14429.98	S.E. of regression	14.46133
R^2	0.131536	Adjusted R^2	0.068604
$F(5, 69)$	3.905635	P-value(F)	0.003553
Log-likelihood	−303.6545	Akaike criterion	619.3089
Schwarz criterion	633.2138	Hannan–Quinn	624.8610

c. One-sided test.

$$H_0 : \lambda_2 \leq 0 \qquad\qquad t = \frac{\hat{\lambda}_2 - 0}{\hat{\sigma}_{\hat{\lambda}_2}} \overset{H_0,a}{\sim} N(0,1)$$
$$H_a : \lambda_2 > 0$$

Since $t = 1.735 > 1.64 = N(0,1)_{0.05}$, the null hypothesis is rejected at the 5% significance level. Therefore, the variable number of rooms affects positively the prices.

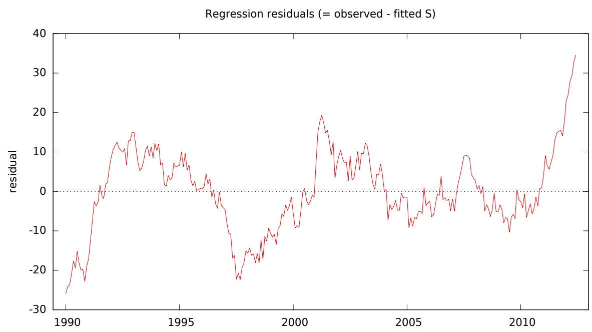
Exercise E7.3 Soy milk

Part I. General Linear Regression Model.

a. Regression model: $S_t = \beta_1 + \beta_2\, P_t + \beta_3\, AE_t + \beta_4\, AE_t^2 + u_t$

SRF: $\hat{S}_t = 57.5860 - 1.38907\, P_t + 3.77879\, AE_t - 0.0166627\, AE_t^2$

b. Residuals against time.



The residuals do not oscillate randomly around zero. There are groups of positive and negative residuals. This fact suggests that the error term might follow a first order autoregressive process with positive parameter.

c. Durbin-Watson test.

$$\begin{cases} H_0 : \rho = 0 & \text{(No first order autocorrelation)} \\ H_a : u_t = \rho\, u_{t-1} + v_t \quad \rho > 0 & \text{(First order autoregressive process (positive parameter))} \end{cases}$$

Test statistic: $$DW = \frac{\sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{T} \hat{u}_t^2}$$

Decision rule: $DW = 0.084094 < 1.7781 = d_L$, the null hypothesis is rejected at the 5% significance level. Therefore, there is evidence in the sample of the presence of a first order autoregressive process (positive parameter) in the error term.

d. Breusch-Godfrey test for autocorrelation of order 1.

H_0 : No autocorrelation of order $p = 1$
H_a : Autocorrelation of order $p = 1$:

The output of Gretl is:

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Breusch-Godfrey test for first-order autocorrelation
OLS, using observations 1990:01-2012:06 (T = 270)
Dependent variable: uhat

             coefficient    std. error    t-ratio    p-value
  ------------------------------------------------------------
  const      15.6772        14.2305        1.102     0.2716
  P          -0.0260848      0.0262868    -0.9923    0.3219
  AE         -0.181503       0.198601     -0.9139    0.3616
  sq_AE       0.000623372    0.000746968   0.8345    0.4047
  uhat_1      0.966433       0.0203928    47.39      2.07e-131 ***

  Unadjusted R-squared = 0.894460

Test statistic: LMF = 2245.889133,
with p-value = P(F(1,265) > 2245.89) = 2.07e-131

Alternative statistic: TR^2 = 241.504118,
with p-value = P(Chi-square(1) > 241.504) = 1.85e-054

Ljung-Box Q' = 233.861,
with p-value = P(Chi-square(1) > 233.861) = 8.58e-053
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Test statistic: $BG = TR^2 \overset{H_0,a}{\sim} \chi^2(1)$

Decision rule: $BG = 241.504118 > 3.84 = \chi^2_{0.05}(1)$

The null hypothesis of no autocorrelaton is rejected at the 5% significance level. Therefore, the error term shows first order autocorrelation.

e. Breusch-Godfrey test for autocorrelation up to order 12.

H_0 : No autocorrelation of order $p = 12$

H_a : Autocorrelation up to order $p = 12$

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Breusch-Godfrey test for autocorrelation up to order 12
OLS, using observations 1990:01-2012:06 (T = 270)
Dependent variable: uhat

             coefficient    std. error    t-ratio    p-value
  ------------------------------------------------------------
  const      10.8369        14.2821        0.7588    0.4487
  P          -0.0243133      0.0255800    -0.9505    0.3428
  AE         -0.110300       0.199156     -0.5538    0.5802
  sq_AE       0.000354715    0.000748152   0.4741    0.6358
  uhat_1      0.827753       0.0627962    13.18      1.39e-030 ***
  uhat_2      0.356879       0.0813707     4.386     1.69e-05  ***
  uhat_3     -0.154457       0.0844110    -1.830     0.0684    *
  uhat_4     -0.199465       0.0847859    -2.353     0.0194    **
  uhat_5      0.0301600      0.0854870     0.3528    0.7245
  uhat_6      0.203399       0.0857590     2.372     0.0184    **
  uhat_7     -0.0258866      0.0857166    -0.3020    0.7629
  uhat_8     -0.0944013      0.0860638    -1.097     0.2737
  uhat_9      0.0917464      0.0852713     1.076     0.2830
  uhat_10     0.0402079      0.0849897     0.4731    0.6366
  uhat_11    -0.0872901      0.0818427    -1.067     0.2872
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  uhat_12    -0.0391187      0.0645283     -0.6062   0.5449

  Unadjusted R-squared = 0.905770

Test statistic: LMF = 203.460617,
with p-value = P(F(12,254) > 203.461) = 9.99e-123

Alternative statistic: TR^2 = 244.557855,
with p-value = P(Chi-square(12) > 244.558) = 1.86e-045

Ljung-Box Q' = 1388.38,
with p-value = P(Chi-square(12) > 1388.38) = 4.45e-290
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Test statistic: $BG = TR^2 \overset{H_0,a}{\sim} \chi^2(12)$

Decision rule: $BG = 244.557855 > 21.0261 = \chi^2_{0.05}(12)$

The null hypothesis of no autocorrelation is rejected at the 5% significance level. Therefore, the error term shows autocorrelation of order 12.

f. The assumptions on the error term are not satisfied: the error term is autocorrelated. As a consequence, conditional on X, the OLS estimators are linear and unbiased but they have not the smallest variance in the class of all linear and unbiased estimators. Furthermore, it is necessary to use an estimator of the covariance matrix of the OLS estimators robust to autocorrelation for the inference performed using the OLS estimator to be valid.

The estimation results in this context are:

Model 2: OLS, using observations 1990:01–2012:06 (T = 270)
Dependent variable: S
HAC standard errors, bandwidth 4 (Bartlett kernel)

	Coefficient	Std. Error	t-ratio	p-value
const	57.5860	131.172	0.4390	0.6610
P	−1.38907	0.197105	−7.0474	0.0000
AE	3.77879	1.77454	2.1294	0.0341
sq_AE	−0.0166627	0.00648700	−2.5686	0.0108

Mean dependent var	120.8475	S.D. dependent var	16.87912
Sum squared resid	28697.27	S.E. of regression	10.38674
R^2	0.625554	Adjusted R^2	0.621331
$F(3, 266)$	30.68774	P-value(F)	4.55e–17
Log-likelihood	−1013.042	Akaike criterion	2034.083
Schwarz criterion	2048.477	Hannan–Quinn	2039.863
$\hat{\rho}$	0.965596	Durbin–Watson	0.084094

Part II. Trend.

a. Regression model:

$$S_t = \beta_1 + \beta_2\, P_t + \beta_3\, AE_t + \beta_4\, AE_t^2 + \beta_5\, time + \beta_6\, time^2 + \beta_7\, time^3 + u_t$$

Model 3: OLS, using observations 1990:01–2012:06 (T = 270)
Dependent variable: S

	Coefficient	Std. Error	t-ratio	p-value
const	−501.568	24.8240	−20.2050	0.0000
P	4.17251	0.129212	32.2919	0.0000
AE	1.38658	0.279551	4.9600	0.0000
sq_AE	−0.00572334	0.000946580	−6.0463	0.0000
time	0.945285	0.0558900	16.9133	0.0000
sq_time	0.00288812	0.000336880	8.5731	0.0000
timecubic	−1.44749e–005	9.72801e–007	−14.8796	0.0000

Mean dependent var	120.8475	S.D. dependent var	16.87912
Sum squared resid	2594.895	S.E. of regression	3.141102
R^2	0.966141	Adjusted R^2	0.965369
$F(6, 263)$	1250.769	P-value(F)	3.7e–190
Log-likelihood	−688.6021	Akaike criterion	1391.204
Schwarz criterion	1416.393	Hannan–Quinn	1401.319
$\hat{\rho}$	0.587180	Durbin–Watson	0.827546

SRF:

$$\begin{aligned}\hat{S}_t &= -501.568 + 4.17251\, P_t + 1.38658\, AE_t - 0.00572334\, AE_t^2 + \\ &+ 0.945285\, time + 0.00288812\, time^2 - 1.44749 10^{-05}\, time^3\end{aligned}$$

c. Breusch-Godfrey test for autocorrelation up to order 12.

H_0 : No autocorrelation of order $p = 12$

H_a : Autocorrelation up to order $p = 12$

The ouput of Gretl is:

```
Breusch-Godfrey test for autocorrelation up to order 12
OLS, using observations 1990:01-2012:06 (T = 270)
Dependent variable: uhat

              coefficient    std. error    t-ratio    p-value
  -------------------------------------------------------------
  const       -10.5985       19.9236       -0.5320    0.5952
  P             0.0106923     0.0949042     0.1127    0.9104
  AE            0.143322      0.241628      0.5932    0.5536
  sq_AE        -0.000494112   0.000815623  -0.6058    0.5452
  time         -0.0189710     0.0457003    -0.4151    0.6784
  sq_time       0.000126489   0.000264940   0.4774    0.6335
  timecubic    -2.39991e-07   7.24508e-07  -0.3312    0.7407
  uhat_1        0.477442      0.0632936     7.543     8.40e-013 ***
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uhat_2        0.360458      0.0701919     5.135     5.65e-07  ***
uhat_3       -0.161536      0.0736684    -2.193     0.0292    **
uhat_4       -0.339565      0.0744734    -4.560     8.02e-06  ***
uhat_5       -0.128674      0.0762109    -1.688     0.0926    *
uhat_6        0.263132      0.0769407     3.420     0.0007    ***
uhat_7       -0.0160528     0.0777490    -0.2065    0.8366
uhat_8       -0.213685      0.0771087    -2.771     0.0060    ***
uhat_9        0.0429044     0.0749392     0.5725    0.5675
uhat_10       0.102201      0.0742724     1.376     0.1700
uhat_11      -0.0182295     0.0716443    -0.2544    0.7994
uhat_12      -0.0736954     0.0664680    -1.109     0.2686

  Unadjusted R-squared = 0.486911

Test statistic: LMF = 19.849505,
with p-value = P(F(12,251) > 19.8495) = 3.55e-030

Alternative statistic: TR^2 = 131.466022,
with p-value = P(Chi-square(12) > 131.466) = 3.13e-022
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Test statistic: $BG = TR^2 \overset{H_{0},a}{\sim} \chi^2(12)$

Decision rule: $BG = 131.466022 > 21.0261 = \chi^2_{0.05}(12)$

The null hypothesis of no autocorrelation is rejected at the 5% significance level. Therefore, the error term shows autocorrelation up to order 12.

d. The Newey-West estimator of the covariance matrix robust to autocorrelation would have to be used in order to perform valid inference using the OLS estimator.

The estimation results in this context are:

Model 4: OLS, using observations 1990:01–2012:06 (T = 270)
Dependent variable: S
HAC standard errors, bandwidth 4 (Bartlett kernel)

	Coefficient	Std. Error	t-ratio	p-value
const	−501.568	33.7535	−14.8598	0.0000
P	4.17251	0.187066	22.3050	0.0000
AE	1.38658	0.444510	3.1193	0.0020
sq_AE	−0.00572334	0.00158227	−3.6172	0.0004
time	0.945285	0.0872819	10.8303	0.0000
sq_time	0.00288812	0.000458242	6.3026	0.0000
timecubic	−1.44749e–005	1.33226e–006	−10.8649	0.0000

Mean dependent var	120.8475	S.D. dependent var	16.87912
Sum squared resid	2594.895	S.E. of regression	3.141102
R^2	0.966141	Adjusted R^2	0.965369
$F(6, 263)$	485.1589	P-value(F)	4.3e–139
Log-likelihood	−688.6021	Akaike criterion	1391.204
Schwarz criterion	1416.393	Hannan–Quinn	1401.319
$\hat{\rho}$	0.587180	Durbin–Watson	0.827546

Test whether the trend component is statistically significant.

$$\begin{array}{l} H_0 : \beta_5 = \beta_6 = \beta_7 = 0 \\ H_a : \exists \beta_i \neq 0 \end{array} \qquad F = \frac{SSR_R - SSR_{UR}}{SSR_{UR}} \frac{T-k}{q} \overset{H_0}{\sim} \mathcal{F}(q, T-k)$$

Since $F = 351.353 > 2.63893 = \mathcal{F}_{0.05}(3, 263)$, the null hypothesis is rejected at the 5% significance level. Therefore, the trend component is statistically significant.

e. Is the trend linear?

$$\begin{array}{l} H_0 : \beta_6 = \beta_7 = 0 \\ H_a : \exists \beta_i \neq 0 \end{array} \qquad F = \frac{SSR_R - SSR_{UR}}{SSR_{UR}} \frac{T-k}{q} \overset{H_0}{\sim} \mathcal{F}(q, T-k)$$

Since $F = 70.3769 > 3.03012 = \mathcal{F}_{0.05}(2, 263)$, the null hypothesis is rejected at the 5% significance level. Therefore, the trend component is not linear: the order of the polynomial should be higher.

Is the trend quadratic?

$$\begin{array}{l} H_0 : \beta_7 = 0 \\ H_a : \beta_7 \neq 0 \end{array} \qquad F = \frac{SSR_R - SSR_{UR}}{SSR_{UR}} \frac{T-k}{q} \overset{H_0}{\sim} \mathcal{F}(q, T-k)$$

Como $F = 118.047 > 3.87706 = \mathcal{F}_{0.05}(1, 263)$, the null hypothesis is rejected at the 5% significance level. Therefore, the trend component is not quadratic: the order of the polynomial should be higher.

Is the trend cubic? Yes, given the results of the last two tests, the trend may be cubic.