Solution to Task T5.

The Multiple Regression Model. Estimation.

Task T5.1. Change of units of measurement.

Beach umbrella rental

a. Estimate model (1) clicking:

Model --> Ordinary Least Squares ...

📓 gretl: model 1									x
<u>F</u> ile <u>E</u> dit <u>T</u> ests	<u>Save G</u>	raphs <u>A</u> na	alysis	<u>L</u> aTeX					2
Model 1: OLS Dependent va	, using riable:	observa U	tion	s 201:	3-05-05:2013	3-09-29	(T =	= 22)	*
	coeffic	ient	std.	erro	r t-ratio	p-va	lue		
const	21.699	8	40.	7472	0.5325	0.60	05		
T	11.796	53	2.	07726	5.679	1.79	e-05	***	=
Mean depende	nt var	381.27	27	S.D.	dependent v	var 6	0.601	10	
R-squared	resid	7795.7	50 17	S.E. Adju	of regressi sted R-squar	ced 0	.8882	93 277	
F(2, 19) Log-likeliho	od	84.482	30 85	P-vai Akail	lue(F) ke criterior	3	.50e-	-10 797	
Schwarz crit	erion	200.85	28	Hanna	an-Quinn	1	98.35	508	
rno		0.1529	56	Durb:	in-watson	1	.6164	66	-

Sample Regression Function:

 $\hat{U}_t = 21.6998 - 0.591532 P_t + 11.7963 T_t$ $t = 1, 2, \dots, 22$ SSR = 7795.750 $R^2 = 0.898917$

- $\hat{\beta}_1$: It is estimated that the number of umbrellas rented when there is no charge for the rental and the temperature is 0°C is 21.6998.
- $\hat{\beta}_2$: It is estimated that the number of rented umbrellas decreases by 0.591532 units when the price increases by $\in 1$, holding the temperature constant.
- $\hat{\beta}_3$: It is estimated that the number of rented umbrellas increases by 11.7963 units when the temperature increases by 1°C, holding the price constant.
- b. Generate a new variable price measured in dollars: PD, price in dollars.

Click in the menu bar

Add --> Define new variable ...

and write down the formula to convert euros into dollars in the dialog box. Once the new variable is generated, click

Model --> Ordinary Least Squares ...

in order to estimate the model. In the dialog box, choose the dependent variable (U) and the regressors price in dollars (PD) and temperature in degrees Celsius (T).



Sample Regression Function:

$$\hat{U}_t = 21.6998 - 0.453733 \ PD_t + 11.7963 \ T_t \qquad t = 1, 2, \dots, 22$$

$$SSR = 7795.750 \qquad R^2 = 0.898917$$

c. Generate a new variable temperature measured in degrees Fahrenheit: TF, temperature in degrees Fahrenheit.

Click in the menu bar

Add --> Define new variable ...

and write down the formula to convert degrees Celsius into degrees Fahrenheit in the dialog box. Once the new variable is generated, click

Model --> Ordinary Least Squares

in order to estimate the model. In the dialog box, choose the dependent variable (U) and the regressors price in euros (P) and temperature in degrees Fahrenheit (TF).

	📓 gretl: specify model	
	e 0	LS
	const U P T W WW time PD TF	Dependent variable U U Set as default Regressors const P TF TF
🛐 gretl: add var		
Enter formula for new variable (or just a name, to enter data manually)	Robust standard errors HAC	
TF = 1.8*T+32 Help Cancel QK	lags Help <u>C</u> lear	<u>Cancel</u> <u>O</u> K

📓 gretl: model 3					
<u>File E</u> dit <u>T</u> ests <u>Save G</u>	raphs <u>A</u> nalysis	<u>L</u> aTeX			6
Model 3: OLS, using Dependent variable:	observation U	s 2013-05-05	:2013-09-:	29 (T =	22) 🔺
coeffic	cient std.	error t-r	atio p-	value	
const -188.03	13 76.	7043 -2.	451 0.	0241	**
P -0.59	91532 3.	30317 -0.	.1791 0.0	8598	
TF 6.55	5353 1.	15403 5.	679 1.	79e-05	*** E
Mean dependent var	381.2727	S.D. depend	lent var	60.601	10
Sum squared resid	7795.750	S.E. of reg	gression	20.255	93
R-squared	0.898917	Adjusted R-	-squared	0.8882	77
F(2, 19)	84.48230	P-value(F)		3.50e-	10
Log-likelihood	-95.78985	Akaike crit	erion	197.57	97
Schwarz criterion	200.8528	Hannan-Quin	ın	198.35	08
rho	0.152956	Durbin-Wats	son	1.6164	66 🚽

Sample Regression Function:

$$\hat{U}_t = -188.013 - 0.591532 P_t + 6.55353 TF_t$$
 $t = 1, 2, \dots, 22$
 $SSR = 7795.750$ $R^2 = 0.898917$

d. Generate a new variable, number of rented umbrellas measured in hundreds (UH) and estimate a regression model choosing (UH) as the dependent variable and price in euros (P) and temperature in degrees Celsius (T) as regressors.

	📓 gretl: specify model		
	4	OLS	
	const		Dependent variable
	U P		UH
	T		Set as default
	W		Regressors
	time		const p
	PD		T
	UH		
gretl: add var			
Enter formula for new variable			
(or just a name, to enter data manually)	Robust standard errors	HAC	
UH = U/100	lags		
Help Cancel OK	Help Cle	ar C	ancel <u>O</u> K

gretl: model 5							×
<u>F</u> ile <u>E</u> dit <u>T</u> ests	<u>S</u> ave <u>G</u> ra	phs <u>A</u> naly	sis <u>L</u> aTeX				8
Model 5: OLS Dependent va	, using c riable: U	bservat: JH	ions 201	3-05-05:20	013-09-29 (1	[= 22)	^
	coeffici	ent st	td. erro	r t-rat:	io p-value	2	
const	0.21699	8 0	.407472	0.53	25 0.6005		
T	0.11796	53 0	.0207726	5.67	9 1.79e-()5 ***	E
Mean depende Sum squared R-squared F(2, 19) Log-likeliho Schwarz crit rho	nt var resid od erion -	3.81272' 0.77957 0.89891' 84.48230 5.523890 1.774652 0.152950	7 S.D. 5 S.E. 7 Adju 0 P-va 0 Akai 2 Hann 6 Durb	dependent of regres sted R-squ lue(F) ke criter: an-Quinn in-Watson	t var 0.60 ssion 0.20 uared 0.88 3.50 ion -5.04 -4.2 1.61	06011 02559 38277 0e-10 47780 76729 L6466	

Sample Regression Function:

$$\widehat{UH}_t = 0.216998 - 0.00591532 P_t + 0.117963 T_t$$
 $t = 1, 2, \dots, 22$
 $SSR = 0.779575$ $R^2 = 0.898917$

e. Estimate a regression model choosing the number of rented umbrellas (U) as a dependent variable and the price in dollars (PD) and the temperature in degrees Fahrenheit (TF) as regressors.



gretl: model 6						- 0	X
<u>File Edit Tests</u>	s <u>S</u> ave <u>G</u> raj	ohs <u>A</u> nalysis	<u>L</u> aTeX				Ð
Model 6: OLS Dependent va	5, using o ariable: U	bservatio	ns 2013	-05-05:2013-	-09-29 (T	= 22)	<u> </u>
	coeffici	ent std	. error	t-ratio	p-value		
const PD TF	-188.013 -0.453 6.553	76 733 2 53 1	.7043 .53369 .15403	-2.451 -0.1791 5.679	0.0241 0.8598 1.79e-05	**	=
Mean depende Sum squared R-squared F(2, 19) Log-likeliho Schwarz crit rho	ent var resid bod - cerion	381.2727 7795.750 0.898917 84.48230 95.78985 200.8528 0.152956	S.D. S.E. Adjus P-val Akaik Hanna Durbi	dependent va of regressio ted R-square ue(F) e criterion n-Quinn n-Watson	ar 60.60 on 20.25 ed 0.888 3.50e 197.5 198.3 1.616	110 593 277 -10 797 508 466	Ţ

Sample Regression Function:

$$\hat{U}_t = -188.013 - 0.453733 \ PD_t + 6.55353 \ TF_t \qquad t = 1, 2, \dots, 22$$

$$SSR = 7795.750 \qquad R^2 = 0.898917$$

- f. Conclusions.
- A. Coefficients.

In item b. the units of measurement of the regressor ${\cal P}$ have been changed by multiplying by 1.3037.

- The estimate of coefficient β_2 is divided by 1.3037 because it is necessary to multiply the prices by 1.3037 to convert euros into dollars. Thus, if prices are measured in euros $\hat{\beta}_2 = -0.591532$, and if they are measured in dollars $\hat{\beta}_2 = -0.591532/1.3037 = -0.453733$.
- Note that the information provided by both estimates about the variation in the number of rented umbrellas generated by a unit increase in prices is the same.

In item c. the units of measurement of the regressor T have been changed by multiplying by 1.8 and adding 32.

- Both the estimate of the coefficient of the variable temperature $(\hat{\beta}_3)$ and the estimate of the intercept $(\hat{\beta}_1)$ change. Only the estimate of the coefficient of the variable price remains the same.
- Remember that to convert degrees Celsius into degrees Fahrenheit you have first to multiply by 1.8 and then add 32.

The estimate of the coefficient of variable temperature is affected only by the multiplication factor. Therefore, if the temperature is measured in degrees Celsius $\hat{\beta}_3 = 11.7963$, and if the temperature is measured in degrees Fahrenheit $\hat{\beta}_3 = 11.7963/1.8 = 6.55353$.

The constant that is added to the variable temperature (32) implies a change in the average level of this variable, therefore it only changes the estimate of the intercept.

• Note that the information provided by both estimates about the variation in the number of rented umbrellas generated by a unit increase in temperature is the same.

In item d. the units of measurement of the dependent variable (U) have been changed by dividing by 100.

- As it can be observed, the estimates of all the coefficients have changed: they are divided by 100.
- Note that the information provided by the estimates about the variation in the number of rented umbrellas generated either by a unit increase in prices or a unit increase in temperature is the same.

In item e. you have modified the units of measurement of the two regressors. Therefore, the estimates of all the slope coefficients change. **B.** Coefficient of determination.

The coefficient of determination does never change when you modify the units of measurement. The result is always the same: 89.898917% of the sample variable in the number of rented umbrellas is explained by the variations in prices and temperature.

C. Sum of squared residuals.

The value of the sum of squared residuals depends on the units of measurement of the dependent variable. Therefore, the sum of squared residuals only changes when the units of measurement of the dependent variable are modified (item d.).

To measure the number of rented umbrellas in hundreds, it is necessary to divide the original data by 100. Therefore, the sum of squared residuals in item d. will be divided by $100^2 = 10000$. Thus, if the model is estimated with the original units the SSR is 7795.750 and if the model is estimated with the number of umbrellas measured in hundreds is 0.779575.

Task T5.2. Cobb-Douglas production function.

To transform the variables into logarithms, select variables capital, labor and output and click

Add --> Logs of selected variables

The main window shows three new variables. To estimate the log-log model (2), click

Model --> Ordinary Least Squares ...

Choose the dependent variable l_{-q} and the regressors l_{-k} and l_{-l} .

📓 gretl: specify model		
+	OLS	
const k I		Dependent variable Lq Set as default
4 _k _ _q		Regressors const I_k I_I
	4	
Robust standard errors	HC1	
Help <u>C</u> le	ar	Cancel <u>O</u> K

📓 gretl: model 1					×
<u>F</u> ile <u>E</u> dit <u>T</u> ests	<u>Save</u> <u>G</u> raphs <u>A</u>	<u>A</u> nalysis <u>L</u> aTeX			8
Model 1: OLS, Dependent var	using obser iable: l_q	vations 1-33			Â
	coefficient	std. error	t-ratio	p-value	
const	-0.128673	0.546132	-0.2356	0.8153	
lk	0.487731	0.703872	0.6929	0.4937	
1_1	0.558992	0.816438	0.6847	0.4988	-
Mean depender	nt var 2.99	9473 S.D. de	ependent va	ar 0.375902	=
Sum squared n	esid 1.40	9389 S.E. of	f regressio	on 0.216748	
R-squared	0.68	8303 Adjuste	ed R-square	ed 0.667523	
F(2, 30)	33.1	2368 P-value	e(F)	2.55e-08	
Log-likelihoo	d 5.20	5330 Akaike	criterion	-4.410660	
Schwarz crite	erion 0.07	8862 Hannan-	-Quinn	-2.900072	
Log-likelihoo	od for $q = -9$	3.7773			-

Sample Regression Function:

 $\widehat{l_{-}q_{i}} = -0.128673 + 0.487731 \ l_{-}k_{i} + 0.558992 \ l_{-}l_{i} \qquad i = 1, 2, \dots, 33.$

Since model (2) is a log-log model, its coefficients are interpreted as elasticities:

- $\hat{\beta}_2 = 0.487731$: it is estimated that the output increases by 0.487731% when capital increases by 1%, holding labour constant.
- $\hat{\beta}_3 = 0.558992$: it is estimated that the output increases by 0.558992% when labour increases by 1%, holding capital constant.

ONE WAY of estimating model (2) subject to constant returns to scale by Restricted Least Squares is:

1 step. Derive the restricted model including the restriction $(\beta_2 + \beta_3 = 1)$ in model (2):

$$\ln q_{i} = \beta_{1} + \beta_{2} \ln k_{i} + (1 - \beta_{2}) \ln l_{i} + u_{i}$$
$$\ln q_{i} - \ln l_{i} = \beta_{1} + \beta_{2} (\ln k_{i} - \ln l_{i}) + u_{i}$$
$$\ln(q/l)_{i} = \beta_{1} + \beta_{2} \ln(k/l)_{i} + u_{i} \quad (3)$$

2 step. Estimate the restricted model (3) by OLS.

To estimate the restricted model (3), it is necessary to generate the new dependent variable $\ln(q/l)$ and the new regressor $\ln(k/l)$ clicking

Add --> Define new variable ...

🙀 gretl: add var	🕅 gretl: add var
Enter formula for new variable (or just a name, to enter data manually)	Enter formula for new variable (or just a name, to enter data manually)
ql = q/l	kl = k/l
Help Cancel QK	Help Cancel OK

Then, you have to obtain the logarithms of these new variables, highlighting them in the main window and clicking

Add --> Logs of selected variables

Once the logarithms have bee generated, click

Model --> Ordinary Least Squares

and choose the dependent variable, $l_{-}ql$, and the regressor, $l_{-}kl$.

gretl: specify model		
+	OLS	
const k I q		Dependent variable
Lk Ll Lq ql kl		Regressors const I_kl
l_qi l_ki	4	
Robust standard errors	HC1	
<u>H</u> elp <u>C</u> le	ar (<u>C</u> ancel <u>O</u> K

🙀 gretl: model 3					
<u>F</u> ile <u>E</u> dit <u>T</u> ests <u>S</u> ave (araphs <u>A</u> nalysis	<u>L</u> aTeX			8
Model 3: OLS, using Dependent variable:	observation: l_ql	s 1-33			Â
coeffi	cient std.	error	t-ratio	p-value	
const 0.020 l_kl 0.601	0979 0.05 598 0.55	29262 9268	0.3797 1.076	0.7067 0.2904	
Mean dependent var Sum squared resid R-squared F(1, 31) Log-likelihood Schwarz criterion	-0.020438 1.412909 0.035983 1.157107 5.164169 -3.335323	S.D. dep S.E. of Adjusted P-value(Akaike o Hannan-Q	endent va regressic l R-square F) riterion Quinn	ar 0.2140 on 0.2134 ed 0.0048 0.2903 -6.3283 -5.3212	013 E 899 886 867 338 279
Log-likelihood for	ql = 5.83862				-

The results of estimating the restricted model are:

$$\ln(q/l)_i = 0.0200979 + 0.601598 \ln(k/l)_i$$
 $i = 1, 2, \dots, 33$

The results of estimating model (2) by Restricted Least Squares are:

 $\widehat{\ln q_i} = 0.0200979 + 0.601598 \ln k_i + 0.398402 \ln l_i \qquad i = 1, 2, \dots, 33$

ANOTHER WAY of estimating model (2) subject to constant returns to scale by Restricted Least Squares is:

<u>F</u> ile <u>E</u> dit		
	<u>T</u> ests <u>Save</u> <u>G</u> raphs <u>A</u> nalysis	LaTeX
Model 4: Depender	<u>O</u> mit variables <u>A</u> dd variables Sum of coefficients Linear restrictions	1-33 error t-ratio p-value
const l_k l_l	Non-linearity (squares) Non-linearity (logs) <u>R</u> amsey's RESET	6132 -0.2356 0.8153 3872 0.6929 0.4937 6438 0.6847 0.4988
Mean dep Sum squa R-square F(2, 30) Log-like Schwarz	Heteroskedasticity Normality of residual Influential observations Collinearity Chow test	S.D. dependent var 0.375902 S.E. of regression 0.216748 Adjusted R-squared 0.667523 P-value(F) 2.55e-08 Akaike criterion -4.410660 Hannan-Quinn -2.900072
Log-like	<u>A</u> utocorrelation <u>D</u> urbin-Watson p-value A <u>R</u> CH <u>Q</u> LR test <u>C</u> USUM test CUSUM <u>S</u> Q test <u>C</u> ommon factor	
	Panel diagnostics	
gretl: mod	del 4	
<u>File</u> <u>E</u> dit	<u>T</u> ests <u>Save</u> <u>G</u> raphs <u>A</u> nalysis	LaTeX
<u>File</u> <u>E</u> dit Model 4:	<u>Tests</u> <u>Save</u> <u>Graphs</u> <u>Analysis</u> OLS, using observation	LaTeX
<u>File</u> <u>Edit</u> Model 4: Dependen	<u>Tests Save Graphs Analysis</u> OLS, using observation t variable: 1 of ear restrictions	LaTeX
Eile Edit Model 4: Dependen	Tests Save Graphs Analysis OLS, using observation t variable: 1 of ear restrictions Speci (Please refer	LaTeX
<u>t</u> ile <u>k</u> dit Model 4: Depender ∭ gretl: lin b[2]+b[Iests Save Graphs Analysis OLS, using observation t variable: 1 α ear restrictions Speci (Please refer 3]=1	LaTeX
<u>t</u> ile <u>k</u> dit Model 4: Depender grett: lin b[2]+b[Use bo	Iests Save Graphs Analysis OLS, using observation t variable: 1 α ear restrictions Speci (Please refer 3]=1	LaTeX

📓 gretl: linear re	strictions				X
Restriction: b[1_k] + b[1_1] = 1					^
Test statistic: $F(1, 30) = 0.074932$, with p-value = 0.786163					
Restricted estimates:					E
	coefficient	std. error	t-ratio	p-value	
const	0.0200979	0.0529262	0.3797	0.7067	
1_k	0.601598	0.559268	1.076	0.2904	
1_1	0.398402	0.559268	0.7124	0.4816	
					Ψ.

The results obtained are the same.