Solution to Task T5.

The Multiple Regression Model. Estimation.

Task T5.1. Change of units of measurement.

Beach umbrella rental

a. Estimate model (1) clicking:

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Model -->  Ordinary Least Squares ...
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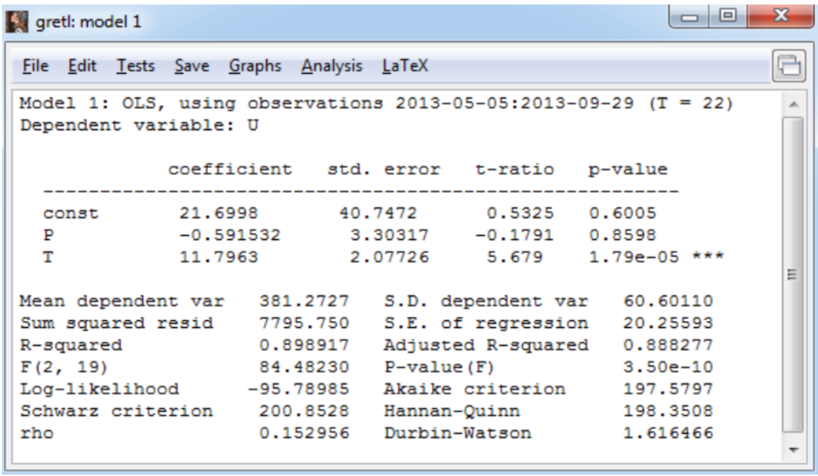

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gretl: model 1
File Edit Tests Save Graphs Analysis LaTeX
Model 1: OLS, using observations 2013-05-05:2013-09-29 (T = 22)
Dependent variable: U

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  const       21.6998      40.7472       0.5325   0.6005
  P           -0.591532     3.30317     -0.1791   0.8598
  T           11.7963       2.07726      5.679    1.79e-05 ***

Mean dependent var   381.2727   S.D. dependent var   60.60110
Sum squared resid    7795.750   S.E. of regression   20.25593
R-squared            0.898917   Adjusted R-squared   0.888277
F(2, 19)             84.48230   P-value(F)           3.50e-10
Log-likelihood      -95.78985   Akaike criterion     197.5797
Schwarz criterion    200.8528   Hannan-Quinn         198.3508
rho                  0.152956   Durbin-Watson        1.616466
```

Sample Regression Function:

$$\hat{U}_t = 21.6998 - 0.591532\ P_t + 11.7963\ T_t \qquad t = 1, 2, \ldots, 22$$

$$SSR = 7795.750 \qquad R^2 = 0.898917$$

$\hat{\beta}_1$: It is estimated that the number of umbrellas rented when there is no charge for the rental and the temperature is 0°C is 21.6998.

$\hat{\beta}_2$: It is estimated that the number of rented umbrellas decreases by 0.591532 units when the price increases by €1, holding the temperature constant.

$\hat{\beta}_3$: It is estimated that the number of rented umbrellas increases by 11.7963 units when the temperature increases by 1°C, holding the price constant.

b. Generate a new variable price measured in dollars: PD, price in dollars.

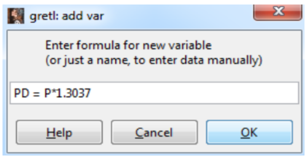
Click in the menu bar

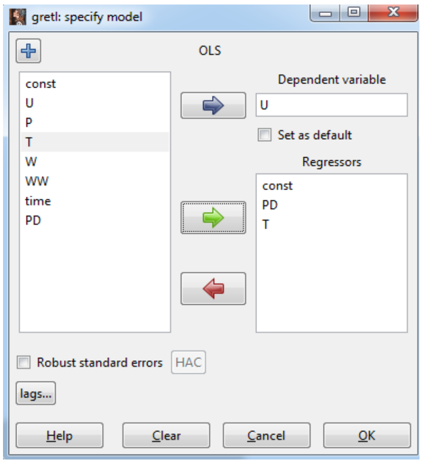
```
Add -->  Define new variable ...
```

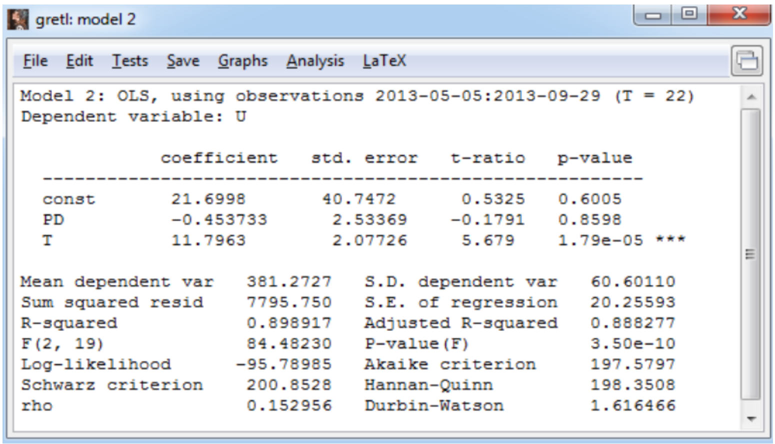
and write down the formula to convert euros into dollars in the dialog box. Once the new variable is generated, click

`Model --> Ordinary Least Squares ...`

in order to estimate the model. In the dialog box, choose the dependent variable (U) and the regressors price in dollars (PD) and temperature in degrees Celsius (T).






gretl: model 2

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Model 2: OLS, using observations 2013-05-05:2013-09-29 (T = 22)
Dependent variable: U

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  const        21.6998      40.7472       0.5325   0.6005
  PD           -0.453733     2.53369     -0.1791   0.8598
  T            11.7963       2.07726      5.679    1.79e-05 ***

Mean dependent var   381.2727   S.D. dependent var   60.60110
Sum squared resid    7795.750   S.E. of regression   20.25593
R-squared            0.898917   Adjusted R-squared   0.888277
F(2, 19)             84.48230   P-value(F)           3.50e-10
Log-likelihood      -95.78985   Akaike criterion     197.5797
Schwarz criterion    200.8528   Hannan-Quinn         198.3508
rho                  0.152956   Durbin-Watson        1.616466
```

Sample Regression Function:

$$\hat{U}_t = 21.6998 \; - \; 0.453733 \; PD_t \; + \; 11.7963 \; T_t \qquad t = 1, 2, \ldots, 22$$

$$SSR = 7795.750 \qquad R^2 = 0.898917$$

c. Generate a new variable temperature measured in degrees Fahrenheit: TF, temperature in degrees Fahrenheit.

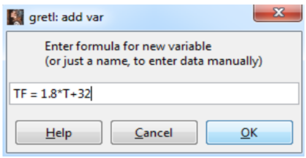
Click in the menu bar

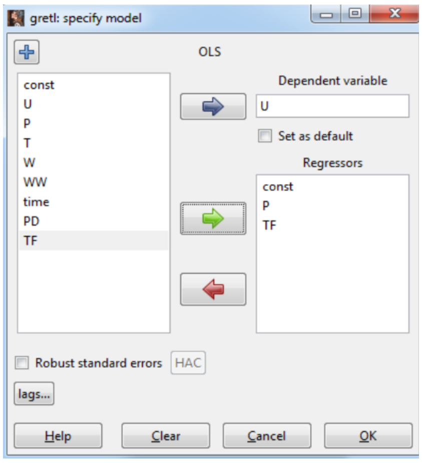
`Add --> Define new variable ...`

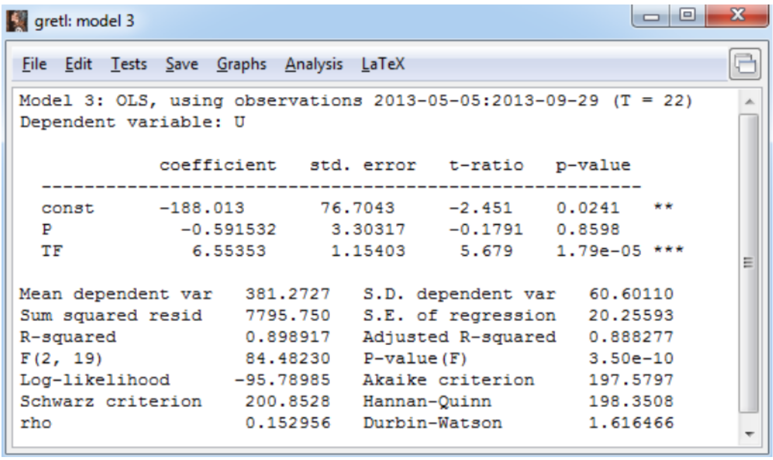
and write down the formula to convert degrees Celsius into degrees Fahrenheit in the dialog box. Once the new variable is generated, click

`Model --> Ordinary Least Squares`

in order to estimate the model. In the dialog box, choose the dependent variable (U) and the regressors price in euros (P) and temperature in degrees Fahrenheit (TF).





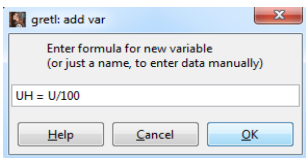


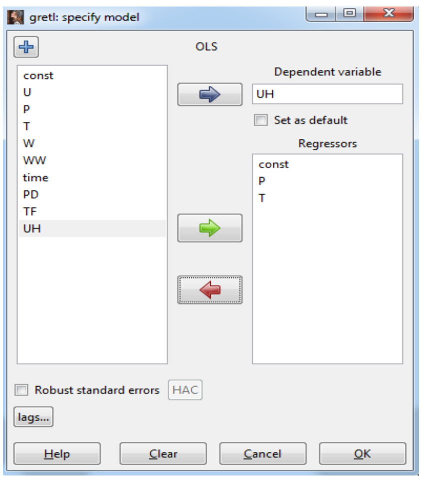
Sample Regression Function:

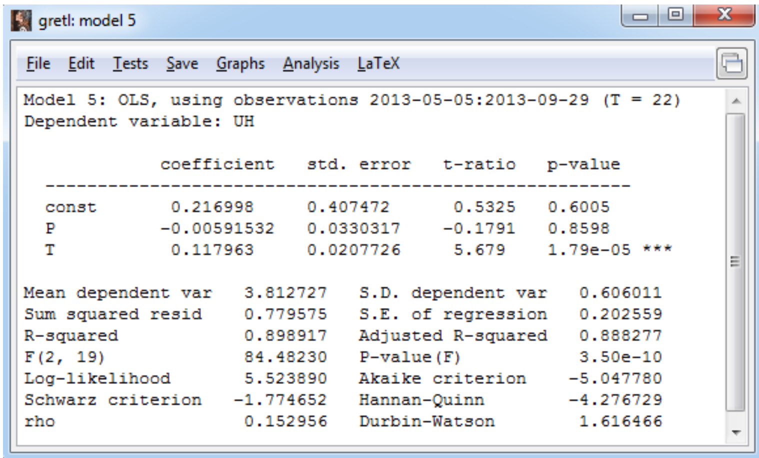
$$\hat{U}_t = -188.013 \; - \; 0.591532 \; P_t \; + \; 6.55353 \; TF_t \qquad t = 1, 2, \ldots, 22$$

$$SSR = 7795.750 \qquad R^2 = 0.898917$$

d. Generate a new variable, number of rented umbrellas measured in hundreds (UH) and estimate a regression model choosing (UH) as the dependent variable and price in euros (P) and temperature in degrees Celsius (T) as regressors.





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Model 5: OLS, using observations 2013-05-05:2013-09-29 (T = 22)
Dependent variable: UH

              coefficient    std. error    t-ratio    p-value
  -----------------------------------------------------------
  const        0.216998      0.407472       0.5325    0.6005
  P           -0.00591532    0.0330317     -0.1791    0.8598
  T            0.117963      0.0207726      5.679     1.79e-05 ***

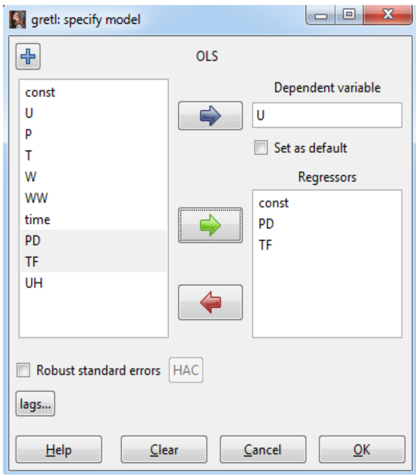
Mean dependent var    3.812727   S.D. dependent var    0.606011
Sum squared resid     0.779575   S.E. of regression    0.202559
R-squared             0.898917   Adjusted R-squared    0.888277
F(2, 19)              84.48230   P-value(F)            3.50e-10
Log-likelihood        5.523890   Akaike criterion     -5.047780
Schwarz criterion    -1.774652   Hannan-Quinn         -4.276729
rho                   0.152956   Durbin-Watson         1.616466
```

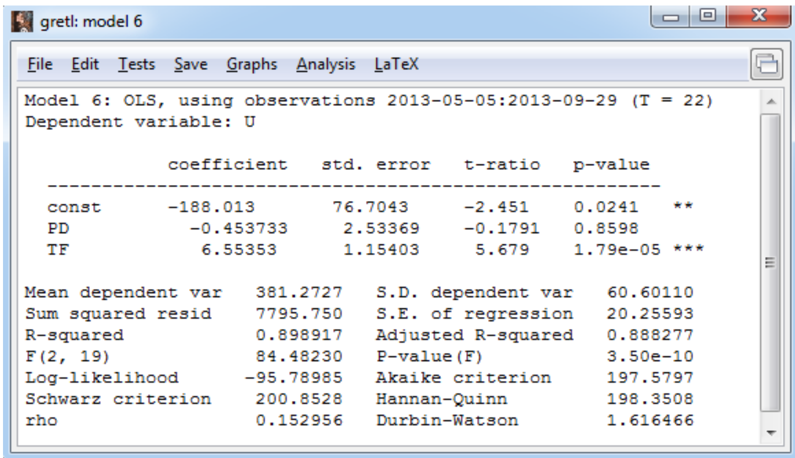
Sample Regression Function:

$$\widehat{UH}_t = 0.216998 \;-\; 0.00591532\; P_t \;+\; 0.117963\; T_t \qquad t = 1, 2, \ldots, 22$$

$$SSR = 0.779575 \qquad R^2 = 0.898917$$

e. Estimate a regression model choosing the number of rented umbrellas (U) as a dependent variable and the price in dollars (PD) and the temperature in degrees Fahrenheit (TF) as regressors.



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Model 6: OLS, using observations 2013-05-05:2013-09-29 (T = 22)
Dependent variable: U

             coefficient    std. error    t-ratio    p-value
  -----------------------------------------------------------
  const      -188.013       76.7043       -2.451     0.0241    **
  PD           -0.453733     2.53369      -0.1791    0.8598
  TF            6.55353      1.15403       5.679     1.79e-05 ***

Mean dependent var   381.2727   S.D. dependent var   60.60110
Sum squared resid    7795.750   S.E. of regression   20.25593
R-squared            0.898917   Adjusted R-squared   0.888277
F(2, 19)             84.48230   P-value(F)           3.50e-10
Log-likelihood      -95.78985   Akaike criterion     197.5797
Schwarz criterion    200.8528   Hannan-Quinn         198.3508
rho                  0.152956   Durbin-Watson        1.616466
```

Sample Regression Function:

$$\hat{U}_t = -188.013 \;-\; 0.453733\; PD_t \;+\; 6.55353\; TF_t \qquad t = 1, 2, \ldots, 22$$

$$SSR = 7795.750 \qquad R^2 = 0.898917$$

f. Conclusions.

A. Coefficients.

In item b. the units of measurement of the regressor P have been changed by multiplying by 1.3037.

- The estimate of coefficient β_2 is divided by 1.3037 because it is necessary to multiply the prices by 1.3037 to convert euros into dollars. Thus, if prices are measured in euros $\hat{\beta}_2 = -0.591532$, and if they are measured in dollars $\hat{\beta}_2 = -0.591532/1.3037 = -0.453733$.
- Note that the information provided by both estimates about the variation in the number of rented umbrellas generated by a unit increase in prices is the same.

In item c. the units of measurement of the regressor T have been changed by multiplying by 1.8 and adding 32.

- Both the estimate of the coefficient of the variable temperature ($\hat{\beta}_3$) and the estimate of the intercept ($\hat{\beta}_1$) change. Only the estimate of the coefficient of the variable price remains the same.
- Remember that to convert degrees Celsius into degrees Fahrenheit you have first to multiply by 1.8 and then add 32.

  The estimate of the coefficient of variable temperature is affected only by the multiplication factor. Therefore, if the temperature is measured in degrees Celsius $\hat{\beta}_3 = 11.7963$, and if the temperature is measured in degrees Fahrenheit $\hat{\beta}_3 = 11.7963/1.8 = 6.55353$.

  The constant that is added to the variable temperature (32) implies a change in the average level of this variable, therefore it only changes the estimate of the intercept.
- Note that the information provided by both estimates about the variation in the number of rented umbrellas generated by a unit increase in temperature is the same.

In item d. the units of measurement of the dependent variable (U) have been changed by dividing by 100.

- As it can be observed, the estimates of all the coefficients have changed: they are divided by 100.
- Note that the information provided by the estimates about the variation in the number of rented umbrellas generated either by a unit increase in prices or a unit increase in temperature is the same.

In item e. you have modified the units of measurement of the two regressors. Therefore, the estimates of all the slope coefficients change.

B. Coefficient of determination.

The coefficient of determination does never change when you modify the units of measurement. The result is always the same: 89.898917% of the sample variable in the number of rented umbrellas is explained by the variations in prices and temperature.

C. Sum of squared residuals.

The value of the sum of squared residuals depends on the units of measurement of the dependent variable. Therefore, the sum of squared residuals only changes when the units of measurement of the dependent variable are modified (item d.).

To measure the number of rented umbrellas in hundreds, it is necessary to divide the original data by 100. Therefore, the sum of squared residuals in item d. will be divided by $100^2 = 10000$. Thus, if the model is estimated with the original units the SSR is 7795.750 and if the model is estimated with the number of umbrellas measured in hundreds is 0.779575.

Task T5.2. Cobb-Douglas production function.

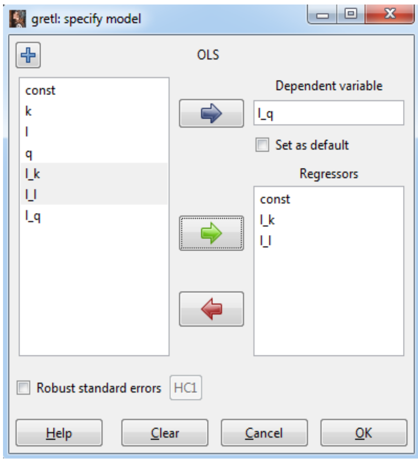
To transform the variables into logarithms, select variables capital, labor and output and click

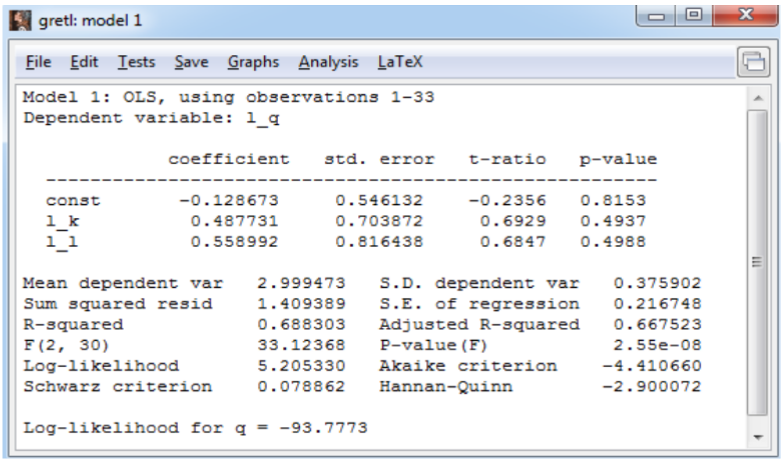
`Add --> Logs of selected variables`

The main window shows three new variables. To estimate the log-log model (2), click

`Model --> Ordinary Least Squares ...`

Choose the dependent variable l_q and the regressors l_k and l_l.





Sample Regression Function:

$$\widehat{l\_q}_i = -0.128673 + 0.487731\ l\_k_i + 0.558992\ l\_l_i \qquad i = 1, 2, \ldots, 33.$$

Since model (2) is a log-log model, its coefficients are interpreted as elasticities:

$\hat{\beta}_2 = 0.487731$: it is estimated that the output increases by 0.487731% when capital increases by 1%, holding labour constant.

$\hat{\beta}_3 = 0.558992$: it is estimated that the output increases by 0.558992% when labour increases by 1%, holding capital constant.

ONE WAY of estimating model (2) subject to constant returns to scale by Restricted Least Squares is:

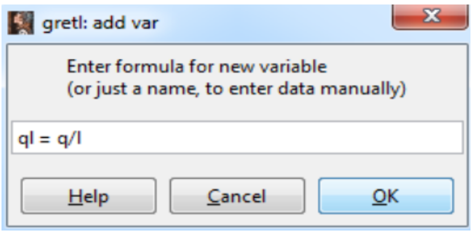
1 step. Derive the restricted model including the restriction ($\beta_2 + \beta_3 = 1$) in model (2):

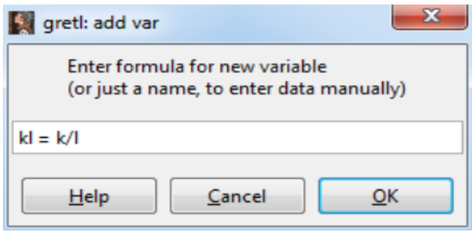
$$
\begin{aligned}
\ln q_i &= \beta_1 + \beta_2 \ln k_i + (1 - \beta_2) \ln l_i + u_i \\
\ln q_i - \ln l_i &= \beta_1 + \beta_2 (\ln k_i - \ln l_i) + u_i \\
\ln(q/l)_i &= \beta_1 + \beta_2 \ln(k/l)_i + u_i \qquad (3)
\end{aligned}
$$

2 step. Estimate the restricted model (3) by OLS.

To estimate the restricted model (3), it is necessary to generate the new dependent variable $\ln(q/l)$ and the new regressor $\ln(k/l)$ clicking

`Add -->  Define new variable ...`





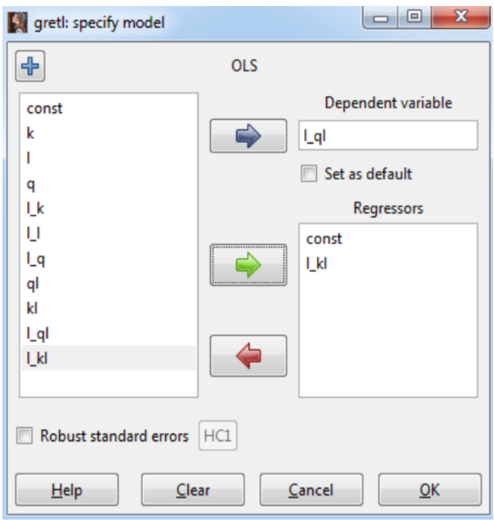
Then, you have to obtain the logarithms of these new variables, highlighting them in the main window and clicking

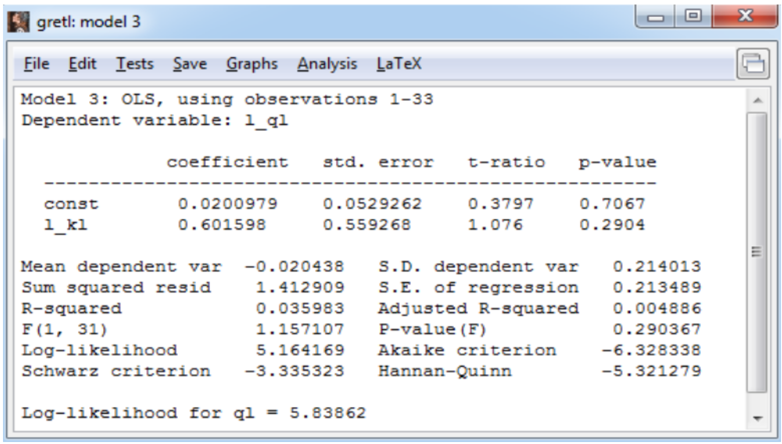
`Add -->  Logs of selected variables`

Once the logarithms have bee generated, click

`Model -->  Ordinary Least Squares`

and choose the dependent variable, l_ql, and the regressor, l_kl.






```
Model 3: OLS, using observations 1-33
Dependent variable: l_ql

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  const       0.0200979    0.0529262    0.3797    0.7067
  l_kl        0.601598     0.559268     1.076     0.2904

Mean dependent var   -0.020438   S.D. dependent var    0.214013
Sum squared resid     1.412909   S.E. of regression    0.213489
R-squared             0.035983   Adjusted R-squared    0.004886
F(1, 31)              1.157107   P-value(F)            0.290367
Log-likelihood        5.164169   Akaike criterion     -6.328338
Schwarz criterion    -3.335323   Hannan-Quinn         -5.321279

Log-likelihood for ql = 5.83862
```

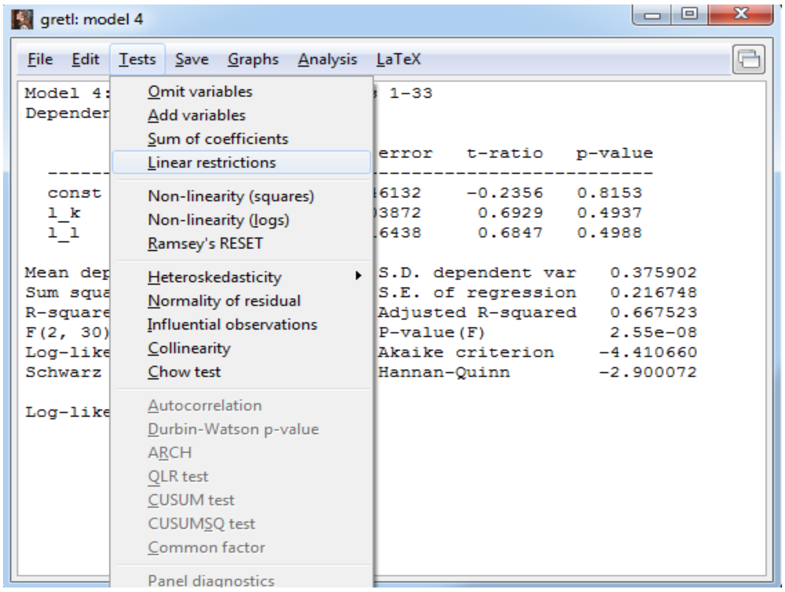
The results of estimating the restricted model are:

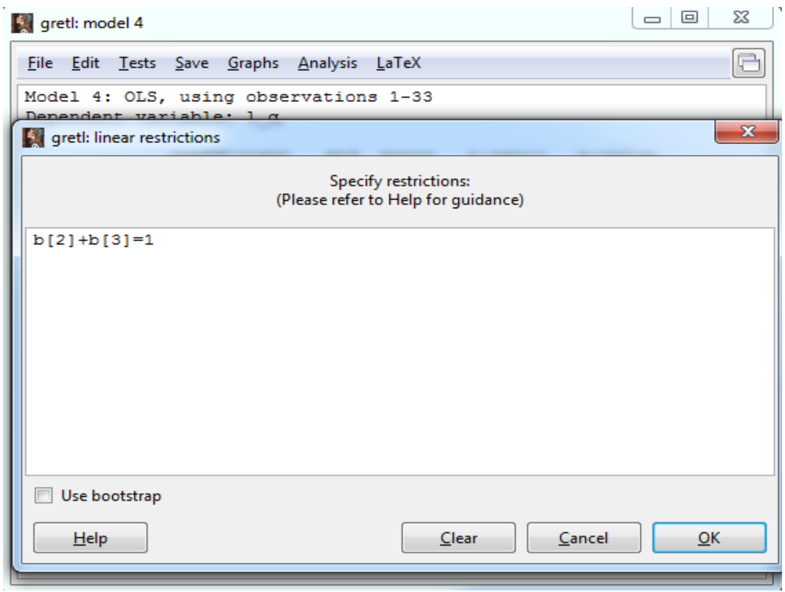
$$\widehat{\ln(q/l)}_i = 0.0200979 + 0.601598 \ln(k/l)_i \qquad i = 1, 2, \ldots, 33$$

The results of estimating model (2) by Restricted Least Squares are:

$$\widehat{\ln q_i} = 0.0200979 + 0.601598 \ln k_i + 0.398402 \ln l_i \qquad i = 1, 2, \ldots, 33$$

ANOTHER WAY of estimating model (2) subject to constant returns to scale by Restricted Least Squares is:





gretl: linear restrictions

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Restriction:
 b[l_k] + b[l_l] = 1

Test statistic: F(1, 30) = 0.074932, with p-value = 0.786163

Restricted estimates:

             coefficient   std. error   t-ratio   p-value
  -------------------------------------------------------
  const       0.0200979    0.0529262    0.3797    0.7067
  l_k         0.601598     0.559268     1.076     0.2904
  l_l         0.398402     0.559268     0.7124    0.4816
```

The results obtained are the same.