

CAMPUS OF INTERNATIONAL EXCELLENCE

Example 7.2

Autocorrelation

Pilar González and Susan Orbe

Dpt. Applied Economics III (Econometrics and Statistics)

Questions.

Load the file chicken.gdt.

- a. Estimate a regression model of consumption of chicken on disposable income, the prices of chicken, pork and beef and the avian flu epidemic. Write down the Sample Regression Function.
- b. Is the effect of the avian flu epidemic statistically significant?
- c. Under the assumption that the error term is autocorrelated, use a statistic robust to autocorrelation to test whether the effect of the avian flu epidemic is statistically significant.
- d. Comment the time series plot of the OLS residuals.

(日) (四) (日) (日) (日)

Questions.

- e. Perform the Durbin-Watson test to analyse whether the error term follows a first order autoregressive process.
- f. Perform the Breusch-Godfrey test to analyse whether the error term follows a first order autoregressive process.
- g. Given the results obtained in these two autocorrelation tests, how would you test the individual significance of the variable avian flu epidemic? Justify your answer in terms of the properties of the estimators.
- h. Interpret the results.

The results of estimating the model proposed in item a. by OLS are shown below.

gretl: model 2								×
<u>File Edit T</u> ests	<u>S</u> ave <u>G</u> rapł	ns <u>A</u> nalysis	; <u>L</u> aTeX					8
Model 2: OLS Dependent va	, using ob riable: Y	servatio	ns 1990)-2012 (T = 23)			<u>^</u>
	coefficie	nt st	d. erro	or t-r	atio	p-value		
const X2	32.9461	340 0	41690	9.	642 583	2.63e-08	***	
X3	-10.6778	3.	71893	-2.	871	0.0106	**	
X4 X5	1.16019	1.	36528 07269	1.	870 082	0.2945		E
AVIAN	2.74647	0.	927321	2.	962	0.0087	***	
Mean depende	nt var 3	9.66957	S.D.	depende	nt var	7.3729	50	
Sum squared	resid 4	4.76912	S.E.	of regr	ession	1.62279	99	
R-squared	0	.962565	Adjus	sted R-s	quared	0.95155	55	
F(5, 17)	8	7.42504	P-val	lue (F)		1.62e-1	1	
Log-likeliho	od -4	0.29487	Akail	ce crite	rion	92.5897	74	
Schwarz crit	erion 9	9.40270	Hanna	an-Quinn		94.3031	18	
rho	0	.370274	Durbi	in-Watso	n	1.16950	01	
								-

It is possible to test the individual significance of the avian flu epidemic effect using these results, if all the assumptions of the Multiple Regression Model are satisfied.

Pilar González and Susan Orbe | OCW 2014

To estimate the covariance matrix of the OLS estimators robust to autocorrelation, select the option <u>Robust standard errors</u> in the **specify model** dialog box.



Mark Select from regular HCCME options in the dialog box.



And select HAC to use the Newey-West estimator for the covariance matrix of the OLS estimators.

General	Network	Programs	HCCME	Manuals			
		F	or cross-	ectional d	lata	HC0	-
			For tin	ne-series d	lata	HAC	-
			F	or panel d	lata	Arellano	-
			For GARC	H estimat	ion [QML	-

Notice that the only difference in the OLS estimation results comes from the new standard errors robust to autocorrelation. As a consequence the t-ratios and the p-values are different as well.

📓 gretl: model 3						×
<u>F</u> ile <u>E</u> dit <u>T</u> ests	<u>S</u> ave <u>G</u> raphs <u>A</u>	nalysis <u>L</u> aTeX				8
Model 3: OLS	, using observ	ations 1990-	2012 (T = 23)		*
HAC standard	errors, bandw	idth 2 (Bart	lett kernel)			
	coefficient	std. error	t-ratio	p-value		
const	32.9461	3.65126	9.023	6.83e-08	***	
X2	0.00645340	0.00372001	1.735	0.1009		
X3	-10.6778	3.21690	-3.319	0.0041	***	
X4	3.91841	1.13074	3.465	0.0030	***	=
X5	1.16019	0.789811	1.469	0.1601		
AVIAN	2.74647	0.515741	5.325	5.58e-05	***	
Mean depende	nt var 39.66	957 S.D. d	ependent var	7.3729	50	
Sum squared	resid 44.76	912 S.E. o	f regression	1.6227	99	
R-squared	0.962	565 Adjust	ed R-squared	0.9515	55	
F(5, 17)	78.13	359 P-valu	e(F)	4.04e-	11	
Log-likeliho	od -40.29	487 Akaike	criterion	92.589	74	
Schwarz crit	erion 99.40	270 Hannan	-Quinn	94.303	18	
rho	0.370	274 Durbin	-Watson	1.1695	01	-

Plot of the OLS residuals against time.



Regression residuals (= observed - fitted Y)

The Durbin-Watson statistic is always displayed in the last row of the estimation output window when dealing with time series data.

📓 gretl: model 2						×
<u>F</u> ile <u>E</u> dit <u>T</u> ests <u>S</u> ave <u>G</u>	<u>a</u> raphs <u>A</u> nalysi	s <u>L</u> aTeX				8
Model 2: OLS, using Dependent variable:	observatio Y	ons 1990-20)12 (T = 23	;)		^
coeffi	cient st	d. error	t-ratio	p-value		
const 32.94	61 3.	41690	9.642	2.63e-08	***	
X3 -10.67	78 3.	.71893	-2.871	0.0106	**	
X4 3.91 X5 1.16	841 1. 019 1.	.36528 .07269	2.870	0.0106 0.2945	**	=
AVIAN 2.74	647 0.	927321	2.962	0.0087	***	
Mean dependent var	39.66957	S.D. dep S.F. of	endent var	7.3729	50	
R-squared	0.962565	Adjusted	l R-squared	0.9515	55	
F(5, 17) Log-likelihood	87.42504 -40.29487	P-value(Akaike c	(F) riterion	1.62e- 92.589	11 74	
Schwarz criterion rho	99.40270 0.370274	Hannan-Q Durbin-W)uinn Jatson	94.303	18	
						-

To perform the Durbin-Watson test we need its p-value. Select

```
Tests -> Durbin-Watson p-value
```

The null hypothesis of no autocorrelation is rejected if the p-value is smaller than the level of significance chosen.

📓 gretl: mo	del 2			x	
<u>F</u> ile <u>E</u> dit	<u>T</u> ests <u>Save</u> <u>G</u> raphs <u>A</u> nalysis	<u>L</u> aTeX		8	
Model 2: Depender	Omit variables Add variables Sum of coefficients Linear restrictions	1990-2012 (T = 23) error t-ratio	p-value	-	
const X2 X3 X4	Non-linearity (squares) Non-linearity (<u>l</u> ogs) <u>R</u> amsey's RESET	.690 9.642 407790 1.583 .893 -2.871 528 2.870	2.63e-08 *** 0.1320 0.0106 ** 0.0106 **		
X5 AVIAN Mean dep Sum squa R-square	Heteroskedasticity <u>N</u> ormality of residual Influential observations <u>C</u> ollinearity <u>C</u> how test	269 1.082 7321 2.962 S.D. dependent var S.E. of regression Adjusted R-squared	0.2945 0.0087 *** 7.372950 1.622799 0.951555	Е	🛐 grett: Durbin-Watson 💿 🖬 🗾
F(5, 17) Log-like Schwarz rho	<u>A</u> utocorrelation <u>D</u> urbin-Watson p-value A <u>R</u> CH <u>Q</u> LR test	P-value(F) Akaike criterion Hannan-Quinn Durbin-Watson	1.62e-11 92.58974 94.30318 1.169501	•	Durbin-Watson statistic = 1.1695 p-value = 0.000838601

Pilar González and Susan Orbe | OCW 2014

To perform the Breusch-Godfrey autocorrelation test, use the **Tests** pulldown menu.

Tests -> Autocorrelation

Write down the lag order for the test, in this case 1.

📓 gretl: mod	del 2	
<u>F</u> ile <u>E</u> dit	<u>T</u> ests <u>S</u> ave <u>G</u> raphs <u>A</u> nalysis	EaTeX
Model 2: Depender	<u>O</u> mit variables <u>A</u> dd variables Sum of coefficients Linear restrictions	error Lag order for test: 1
const X2 X3 X4	Non-linearity (squares) Non-linearity (<u>l</u> ogs) <u>R</u> amsey's RESET	690 407790 893 528 2.870 0.0106 **
X5 AVIAN Mean dep Sum squa R-square F(5, 17)	Heteroskedasticity Normality of residual Influential observations Collinearity Chow test	 269 1.082 0.2945 7321 2.962 0.0087 *** S.D. dependent var 7.372950 S.E. of regression 1.622799 Adjusted R-squared 0.951555 P-value (F) 1.62e-11
Log-like Schwarz rho	<u>A</u> utocorrelation <u>D</u> urbin-Watson p-value A <u>R</u> CH <u>Q</u> LR test	Akaike criterion 92.88974 Hannan-Quinn 94.30318 Durbin-Watson 1.169501

The results of estimating the Breusch-Godfrey auxiliary regression are shown below.

📓 gretl: autocorre	elation					x
7 8 F q	. 🖯 🗶					
Breusch-God OLS, using Dependent v	frey test for f observations 19 ariable: uhat	first-order au 990-2012 (T =	tocorrelati 23)	ion		*
	coefficient	std. error	t-ratio	p-value		
Const X2 X3 X4 X5 AVIAN uhat_1	0.0780902 0.00296443 2.23397 -0.543116 -1.11627 0.0230535 0.596444	3.20167 0.00414833 3.69102 1.31298 1.17486 0.868924 0.325105	0.02439 0.7146 0.6052 -0.4137 -0.9501 0.02653 1.835	0.9808 0.4851 0.5535 0.6846 0.3562 0.9792 0.0852	*	ш
Unadjuste	d R-squared = 0	.173802				
Test statis with p-value	tic: LMF = 3.36 e = P(F(1,16) >	5825, 3.36582) = (.0852			
Alternative with p-value	statistic: TR [^] e = P(Chi-squar	2 = 3.997453, e(1) > 3.9974	15) = 0.0450	5		
Ljung-Box Q with p-valu	' = 2.2193, e = P(Chi-squar	e(1) > 2.2193	3) = 0.136			-

Results (I).

Sample regression function:

$$\widehat{Y}_t = 32.9461 + 0.006345340 X 2_t - 10.6778 X 3_t + 3.91841 X 4_t + 1.16019 X 5_t + 2.74647 A VIA N_t \quad t = 1990, \dots, 2012$$

Individual significance test for the avian flu epidemic explanatory variable.

$$\begin{array}{l} H_0: \beta_6 = 0 \\ H_0: \beta_6 \neq 0 \end{array} \qquad t = \frac{\hat{\beta}_6 - 0}{\hat{\sigma}_{\hat{\beta}_6}} \stackrel{H_0}{\sim} t(N-k) \end{array}$$

where $\hat{\sigma}_{\hat{\beta}_6}$ is computed using the estimate of the covariance matrix of the OLS estimators give by $\hat{V}(\hat{\beta}) = \hat{\sigma}(X'X)^{-1}$.

$$|t| = |2.962| > 2.10982 = t_{0.05/2}(40 - 7)$$

Therefore, the null hypothesis is rejected at a 5 % significance level and it may be concluded that the avian flu epidemic is a statistically significant variable.

Pilar González and Susan Orbe | OCW 2014

Results (II).

Individual significance test for avian flu epidemic using Newey-West robust to autocorrelation estimate of the covariance matrix.

$$\begin{array}{l} H_0: \beta_6 = 0 \\ H_0: \beta_6 \neq 0 \end{array} \qquad t = \frac{\hat{\beta}_6 - 0}{\hat{\sigma}^R_{\hat{\beta}_6}} \overset{H_0,a}{\sim} N(0,1) \end{array}$$

where $\hat{\sigma}_{\hat{\beta}_6}^R$ is computed using the estimator of the covariance matrix of the OLS estimators robust to autocorrelation proposed by Newey-West.

Since $t = |5.325| > 1.96 = N_{0.05/2}(0, 1)$, the null hypothesis is rejected at a 5% significance level and it may be concluded that the avian flu epidemic is a statistically significant variable.

The **residuals time series plot** shows a group of negative residuals at the beginning of the sample followed by a group of positive residuals and so on. These clusters of residuals, positive and negative, may suggest the presence of a first order autoregressive process in the error term. Therefore, the assumption of no autocorrelation in the error term may not hold. The presence of autocorrelation should be checked using some autocorrelation tests.

Results (III).

Durbin-Watson test.

$$\begin{cases} H_0: \rho = 0 \quad (\text{No autocorrelation}) \\ H_a: u_t = \rho \, u_{t-1} + v_t \quad \rho > 0 \quad (\text{Positive autocorrelation}) \end{cases}$$

Test statistic:
$$DW = rac{{\sum\limits_{t = 2}^{r} {{({{\hat u}_t} - {{\hat u}_{t - 1}})^2}} }}{{\sum\limits_{t = 1}^{T} {{{\hat u}_t}^2} }}}$$

τ

Decision rule: $DW = 1.169501 \in (d_L = 0.8949, d_U = 1.9196)$

Therefore, at the 5 % significance level, it is not possible to reach any conclusion using the critical values of the Durbin-Watson tables.

Since the Durbin-Watson p-value is 0.0008, it may concluded that the null hypothesis of no autocorrelation is rejected at the 5 % significance level, that is, the error term follows a first order autoregressive process.

Results (IV).

Breusch-Godfrey test.

 H_0 : No autocorrelation of order 1

 H_a : First order autocorrelation (p = 1)

Auxiliary regression:

 $\hat{u}_{t} = \alpha_{1} + \alpha_{2} X 2_{t} + \alpha_{3} X 3_{t} + \alpha_{4} X 4_{t} + \alpha_{5} X 5_{t} + \alpha_{6} A V I A N_{t} + \alpha_{7} \hat{u}_{t-1} + w_{t}$

Test statistic: $BG = TR^2 \stackrel{H_{0,a}}{\sim} \chi^2(p=1)$

Decision rule: $BG = 3.997453 > 3.84 = \chi^2_{0.05}(1)$

Therefore, the null hypothesis of no autocorrelation is rejected at the 5 % significance level, that is, the error term follows a first order autoregressive process.

Pilar González and Susan Orbe | OCW 2014

Results (V).

Conclusions.

The autocorrelation tests conclude that the error term is autocorrelated so the assumption of no autocorrelation is not satisfied.

As a consequence, the OLS estimator of the vector of coefficients is, conditional on X, linear and unbiased but it has not the smallest variance in the class of linear and unbiased estimators.

The estimator of the variance of the error term, $\hat{\sigma}^2 = SSR/(N-k)$ and the standard estimator of the covariance matrix of the OLS estimator are biased. Therefore the inference based on the t-statistic constructed using these estimators is not valid.

It is possible to perform significance tests valid for large samples using an estimator of the covariance matrix of the OLS estimators robust to autocorrelation, for instance the Newey-West estimator. Therefore, the second test performed to check the significance of the avian flu epidemic is the appropriate one.

э