Universidad del País Vasco · Euskal Herriko Unibertsitatea

CAMPUS OF INTERNATIONAL EXCELLENCE

Example 7.2

Autocorrelation

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Dpt. Applied Economics III (Econometrics and Statistics)

Example 7.2. Autocorrelation.

Questions.

Load the file `chicken.gdt`.

a. Estimate a regression model of consumption of chicken on disposable income, the prices of chicken, pork and beef and the avian flu epidemic. Write down the Sample Regression Function.

b. Is the effect of the avian flu epidemic statistically significant?

c. Under the assumption that the error term is autocorrelated, use a statistic robust to autocorrelation to test whether the effect of the avian flu epidemic is statistically significant.

d. Comment the time series plot of the OLS residuals.

Questions.

e. Perform the Durbin-Watson test to analyse whether the error term follows a first order autoregressive process.

f. Perform the Breusch-Godfrey test to analyse whether the error term follows a first order autoregressive process.

g. Given the results obtained in these two autocorrelation tests, how would you test the individual significance of the variable avian flu epidemic? Justify your answer in terms of the properties of the estimators.

h. Interpret the results.

Example 7.2. Autocorrelation.

The results of estimating the model proposed in item a. by OLS are shown below.

gretl: model 2

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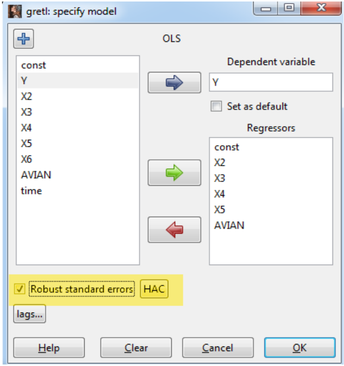
```
Model 2: OLS, using observations 1990-2012 (T = 23)
Dependent variable: Y

              coefficient    std. error    t-ratio    p-value
  ----------------------------------------------------------
  const        32.9461       3.41690        9.642     2.63e-08 ***
  X2            0.00645340   0.00407790     1.583     0.1320
  X3          -10.6778       3.71893       -2.871     0.0106   **
  X4            3.91841      1.36528        2.870     0.0106   **
  X5            1.16019      1.07269        1.082     0.2945
  AVIAN         2.74647      0.927321       2.962     0.0087   ***

Mean dependent var   39.66957   S.D. dependent var   7.372950
Sum squared resid    44.76912   S.E. of regression   1.622799
R-squared            0.962565   Adjusted R-squared   0.951555
F(5, 17)             87.42504   P-value(F)           1.62e-11
Log-likelihood      -40.29487   Akaike criterion     92.58974
Schwarz criterion    99.40270   Hannan-Quinn         94.30318
rho                  0.370274   Durbin-Watson        1.169501
```

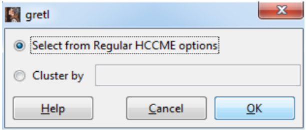
It is possible to test the individual significance of the avian flu epidemic effect using these results, if all the assumptions of the Multiple Regression Model are satisfied.

To estimate the covariance matrix of the OLS estimators robust to autocorrelation, select the option <u>Robust standard errors</u> in the **specify model** dialog box.

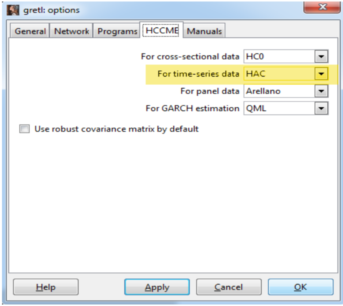


Example 7.2. Autocorrelation.

Mark <u>Select from regular HCCME options</u> in the dialog box.



And select HAC to use the Newey-West estimator for the covariance matrix of the OLS estimators.



Notice that the only difference in the OLS estimation results comes from the new standard errors robust to autocorrelation. As a consequence the t-ratios and the p-values are different as well.

gretl: model 3

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Model 3: OLS, using observations 1990-2012 (T = 23)
Dependent variable: Y
HAC standard errors, bandwidth 2 (Bartlett kernel)

	coefficient	std. error	t-ratio	p-value	
const	32.9461	3.65126	9.023	6.83e-08	***
X2	0.00645340	0.00372001	1.735	0.1009	
X3	-10.6778	3.21690	-3.319	0.0041	***
X4	3.91841	1.13074	3.465	0.0030	***
X5	1.16019	0.789811	1.469	0.1601	
AVIAN	2.74647	0.515741	5.325	5.58e-05	***

Mean dependent var	39.66957	S.D. dependent var	7.372950
Sum squared resid	44.76912	S.E. of regression	1.622799
R-squared	0.962565	Adjusted R-squared	0.951555
F(5, 17)	78.13359	P-value(F)	4.04e-11
Log-likelihood	-40.29487	Akaike criterion	92.58974
Schwarz criterion	99.40270	Hannan-Quinn	94.30318
rho	0.370274	Durbin-Watson	1.169501

Example 7.2. Autocorrelation.

Plot of the OLS residuals against time.

Example 7.2. Autocorrelation.

The Durbin-Watson statistic is always displayed in the last row of the estimation output window when dealing with time series data.

gretl: model 2

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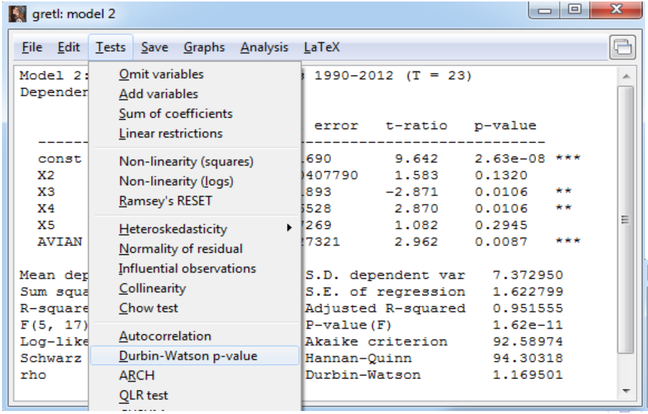
```
Model 2: OLS, using observations 1990-2012 (T = 23)
Dependent variable: Y

             coefficient    std. error    t-ratio    p-value
  ---------------------------------------------------------
  const       32.9461       3.41690        9.642     2.63e-08 ***
  X2           0.00645340   0.00407790     1.583     0.1320
  X3         -10.6778       3.71893       -2.871     0.0106   **
  X4           3.91841      1.36528        2.870     0.0106   **
  X5           1.16019      1.07269        1.082     0.2945
  AVIAN        2.74647      0.927321       2.962     0.0087   ***

Mean dependent var   39.66957   S.D. dependent var   7.372950
Sum squared resid    44.76912   S.E. of regression   1.622799
R-squared            0.962565   Adjusted R-squared   0.951555
F(5, 17)             87.42504   P-value(F)           1.62e-11
Log-likelihood      -40.29487   Akaike criterion     92.58974
Schwarz criterion    99.40270   Hannan-Quinn         94.30318
rho                  0.370274   Durbin-Watson        1.169501
```

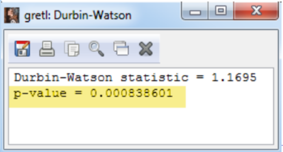
Example 7.2. Autocorrelation.

To perform the Durbin-Watson test we need its p-value. Select

`Tests -> Durbin-Watson p-value`

The null hypothesis of no autocorrelation is rejected if the p-value is smaller than the level of significance chosen.



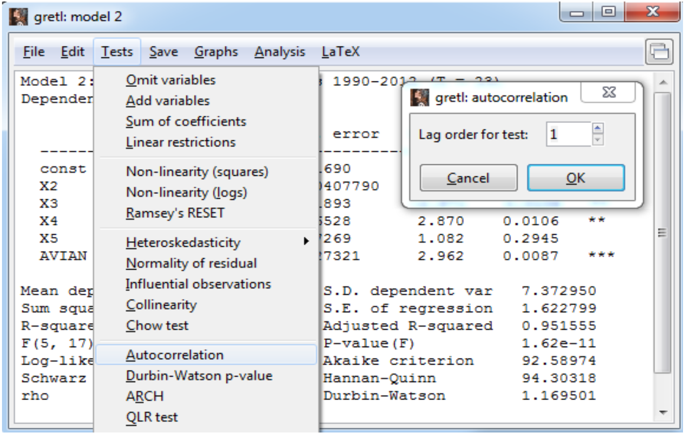


Example 7.2. Autocorrelation.

To perform the Breusch-Godfrey autocorrelation test, use the **Tests** pulldown menu.

`Tests -> Autocorrelation`

Write down the lag order for the test, in this case 1.



Example 7.2. Autocorrelation.

The results of estimating the Breusch-Godfrey auxiliary regression are shown below.

gretl: autocorrelation

```
Breusch-Godfrey test for first-order autocorrelation
OLS, using observations 1990-2012 (T = 23)
Dependent variable: uhat

             coefficient   std. error   t-ratio   p-value
  --------------------------------------------------------
  const       0.0780902    3.20167       0.02439  0.9808
  X2          0.00296443   0.00414833    0.7146   0.4851
  X3          2.23397      3.69102       0.6052   0.5535
  X4         -0.543116     1.31298      -0.4137   0.6846
  X5         -1.11627      1.17486      -0.9501   0.3562
  AVIAN       0.0230535    0.868924      0.02653  0.9792
  uhat_1      0.596444     0.325105      1.835    0.0852  *

  Unadjusted R-squared = 0.173802

Test statistic: LMF = 3.365825,
with p-value = P(F(1,16) > 3.36582) = 0.0852

Alternative statistic: TR^2 = 3.997453,
with p-value = P(Chi-square(1) > 3.99745) = 0.0456

Ljung-Box Q' = 2.2193,
with p-value = P(Chi-square(1) > 2.2193) = 0.136
```

Example 7.2. Autocorrelation.

Results (I).

Sample regression function:

$$\begin{aligned}\widehat{Y}_t &= 32.9461 + 0.006345340\,X2_t - 10.6778\,X3_t + 3.91841\,X4_t + \\ &+ 1.16019\,X5_t + 2.74647\,AVIAN_t \qquad t = 1990, \ldots, 2012\end{aligned}$$

Individual significance test for the avian flu epidemic explanatory variable.

$$H_0 : \beta_6 = 0 \qquad t = \frac{\hat{\beta}_6 - 0}{\hat{\sigma}_{\hat{\beta}_6}} \overset{H_0}{\sim} t(N-k)$$
$$H_0 : \beta_6 \neq 0$$

where $\hat{\sigma}_{\hat{\beta}_6}$ is computed using the estimate of the covariance matrix of the OLS estimators give by $\hat{V}(\hat{\beta}) = \hat{\sigma}(X'X)^{-1}$.

$$|t| = |\ 2.962\ | > 2.10982 = t_{0.05/2}(40-7)$$

Therefore, the null hypothesis is rejected at a 5 % significance level and it may be concluded that the avian flu epidemic is a statistically significant variable.

Example 7.2. Autocorrelation.

Results (II).

Individual significance test for avian flu epidemic using Newey-West robust to autocorrelation estimate of the covariance matrix.

$$H_0 : \beta_6 = 0 \qquad t = \frac{\hat{\beta}_6 - 0}{\hat{\sigma}^R_{\hat{\beta}_6}} \overset{H_0,a}{\sim} N(0,1)$$
$$H_0 : \beta_6 \neq 0$$

where $\hat{\sigma}^R_{\hat{\beta}_6}$ is computed using the estimator of the covariance matrix of the OLS estimators robust to autocorrelation proposed by Newey-West.

Since $t =\mid 5.325 \mid > 1.96 = N_{0.05/2}(0,1)$, the null hypothesis is rejected at a 5 % significance level and it may be concluded that the avian flu epidemic is a statistically significant variable.

The **residuals time series plot** shows a group of negative residuals at the beginning of the sample followed by a group of positive residuals and so on. These clusters of residuals, positive and negative, may suggest the presence of a first order autoregressive process in the error term. Therefore, the assumption of no autocorrelation in the error term may not hold. The presence of autocorrelation should be checked using some autocorrelation tests.

Example 7.2. Autocorrelation.

Results (III).

Durbin-Watson test.

$$\begin{cases} H_0 : \rho = 0 \quad \text{(No autocorrelation)} \\ H_a : u_t \; = \; \rho \, u_{t-1} + v_t \quad \rho > 0 \quad \text{(Positive autocorrelation)} \end{cases}$$

Test statistic: $$DW = \frac{\displaystyle\sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2}{\displaystyle\sum_{t=1}^{T} \hat{u}_t^2}$$

Decision rule: $DW = 1.169501 \in (d_L = 0.8949, d_U = 1.9196)$

Therefore, at the 5 % significance level, it is not possible to reach any conclusion using the critical values of the Durbin-Watson tables.
Since the Durbin-Watson p-value is 0.0008, it may concluded that the null hypothesis of no autocorrelation is rejected at the 5 % significance level, that is, the error term follows a first order autoregressive process.

Example 7.2. Autocorrelation.

Results (IV).

Breusch-Godfrey test.

H_0 : No autocorrelation of order 1

H_a : First order autocorrelation ($p = 1$)

Auxiliary regression:

$$\hat{u}_t = \alpha_1 + +\alpha_2\, X2_t + \alpha_3\, X3_t + \alpha_4\, X4_t + \alpha_5\, X5_t + \alpha_6\, AVIAN_t + \alpha_7 \hat{u}_{t-1} + w_t$$

Test statistic: $BG = TR^2 \overset{H_0,a}{\sim} \chi^2(p = 1)$

Decision rule: $BG = 3.997453 > 3.84 = \chi^2_{0.05}(1)$

Therefore, the null hypothesis of no autocorrelation is rejected at the 5 % significance level, that is, the error term follows a first order autoregressive process.

Results (V).

Conclusions.

The autocorrelation tests conclude that the error term is autocorrelated so the assumption of no autocorrelation is not satisfied.

As a consequence, the OLS estimator of the vector of coefficients is, conditional on X, linear and unbiased but it has not the smallest variance in the class of linear and unbiased estimators.

The estimator of the variance of the error term, $\hat{\sigma}^2 = SSR/(N-k)$ and the standard estimator of the covariance matrix of the OLS estimator are biased. Therefore the inference based on the t-statistic constructed using these estimators is not valid.

It is possible to perform significance tests valid for large samples using an estimator of the covariance matrix of the OLS estimators robust to autocorrelation, for instance the Newey-West estimator. Therefore, the second test performed to check the significance of the avian flu epidemic is the appropriate one.