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Universidad del País Vasco | Euskal Herriko Unibertsitatea

CAMPUS OF INTERNATIONAL EXCELLENCE

Example 4.1

Functional form

Pilar González and Susan Orbe

Dpt. Applied Economics III (Econometrics and Statistics)

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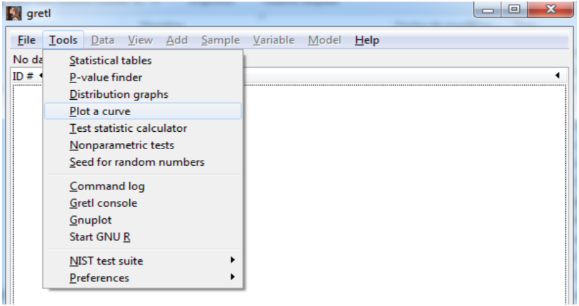
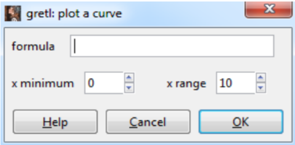
Example 4.1.1. Plotting functions.

To plot a function, go up to the Menu Bar and click

`Tools -> Plot a curve`

Write the function in the dialog box using the operators described in Example 3.2.3. Beware of the range of feasible values for X.

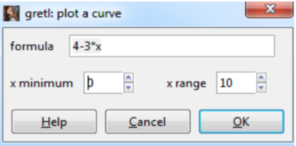


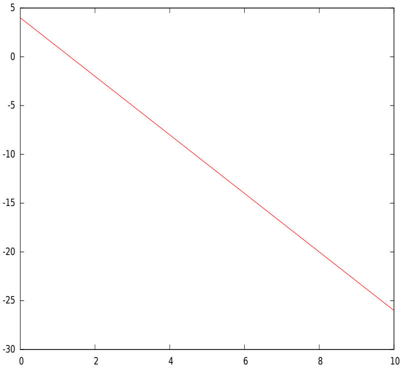


Example 4.1.1. Plotting functions.

Linear function.

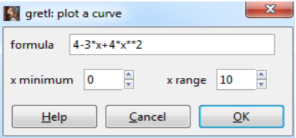
$Y = 4 - 3X$

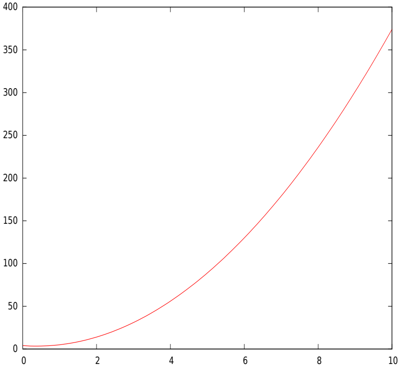




Quadratic function.

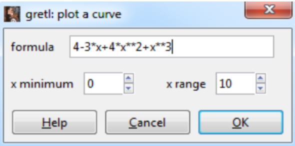
$$Y = 4 - 3X + 4X^2$$

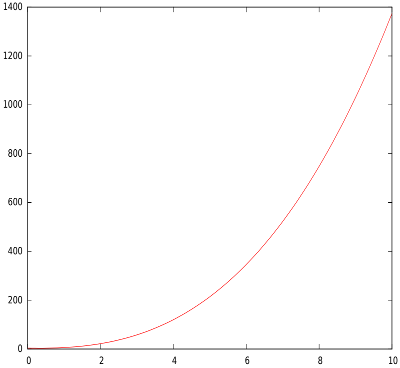




Cubic function.

$$Y = 4 - 3X + 4X^2 + X^3$$

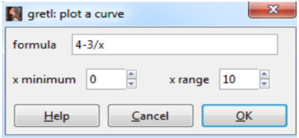


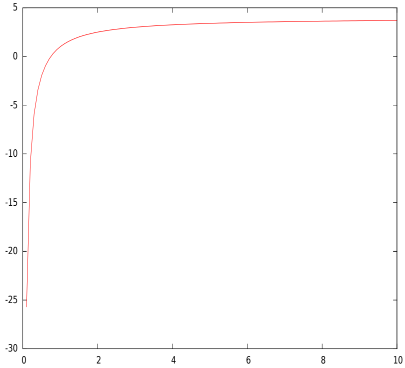


Example 4.1.1. Plotting functions.

Inverse function.

$Y = 4 - \frac{3}{X}$

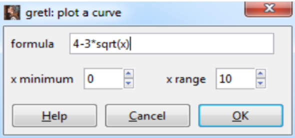


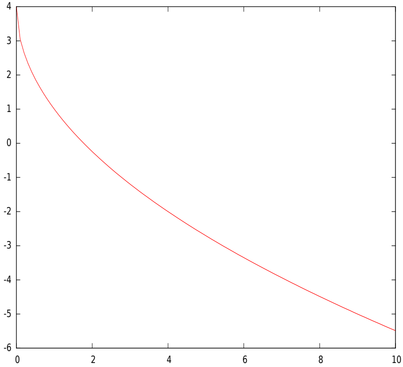


Example 4.1.1. Plotting functions.

Squared root function.

$Y = 4 - 3\sqrt{X}$

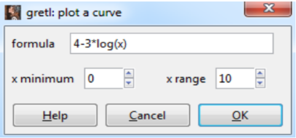


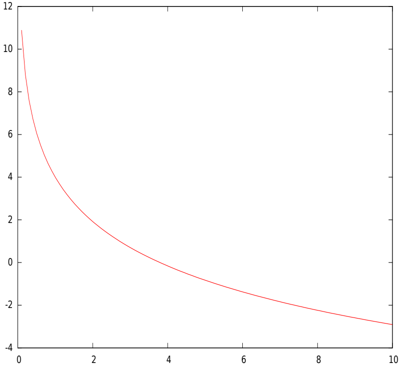


Example 4.1.1. Plotting functions.

Logarithmic function.

$Y = 4 - 3 \ln X$





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Example 4.1.2. Functional form.

Game performance.

Load the data file `golf.gdt` that you may find in the sample folder POE 4th ed (Hill et al. (2008)).

This file contains data from a golf player on variables:

age: age in decades

$score =$ actual score - par

actual score = total number of hits taken during a round of golf, plus any penalty strokes

par for the course = the number of hits allotted for any given course

$score < 0$: under par

$score > 0$: over par

Example 4.1.2. Functional form.

Questions.

a. Plot the relationship between the variables score and age. Delete the fitted line.

b. What is the appropriate functional form?

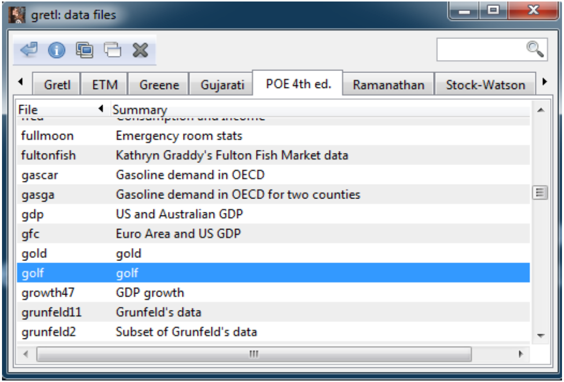
c. Add variables age^2 and age^3 to the file.

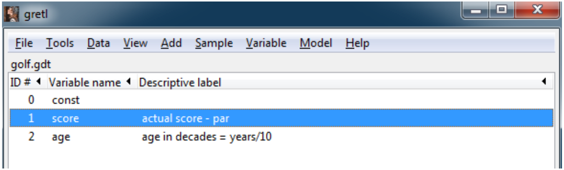
d. Specify an econometric model to determine the score of the golf player as a linear function of his age. What is the effect of age on score?

e. Specify an econometric model to determine the score of the golf player as a quadratic function of his age. What is the effect of age on score?

f. Specify an econometric model to determine the score of the golf player as a cubic function of his age. What is the effect of age on score?

Example 4.1.2. Functional form.

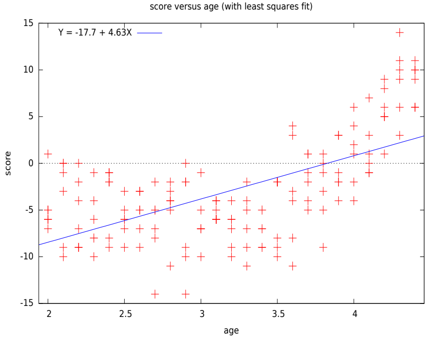




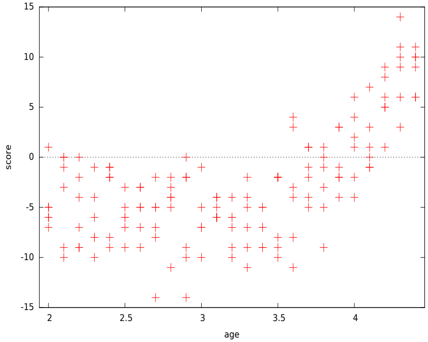
Example 4.1.2. Functional form.

Plot the relationship between score and age and edit the plot to delete the OLS fitted line.

```
View -> Graph specified vars    -> X-Y scatter ...
```



Default plot



Modified plot

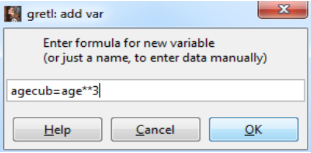
Example 4.1.2. Functional form.

The X-Y scatter shows that the relationship between score and age is not linear. Therefore a polynomial function (quadratic or cubic) seems to be more appropriate. To estimate these functional forms (see Lesson 5) it is necessary to generate the variables age^2 and age^3.

- To add age^2: highlight the variable age with the cursor, go up to the Menu Bar and click

  `Add -> Squares of selected variables`

- To add age^3: highlight the variable age, right-click, select the option *Define new variable...* from the pulldown menu and write the formula.



Example 4.1.2. Functional form.

Linear relationship between variables

Econometric model:

$$score_t = \beta_1 + \beta_2 \, age_t + u_t$$

$$\text{PRF}: \quad E(score_t) = \beta_1 + \beta_2 \, age_t$$

Effect of age on score:

$$\frac{\Delta score}{\Delta \, age} = \beta_2$$

The effect of age on score is β_2, constant throughout the sample.

Quadratic relationship between variables

Econometric model:

$$score_t = \beta_1 + \beta_2\, age_t + \beta_3\, age_t^2 + u_t$$

$$\text{PRF}: \quad E(score_t) = \beta_1 + \beta_2\, age_t + \beta_3\, age_t^2$$

Effect of age on score:

$$\frac{\Delta score}{\Delta\, age} \approx \beta_2 \; + 2\, \beta_3\, age$$

The effect of age on score is NOT constant throughout the sample because it depends on age.

Example 4.1.2. Functional form.

Cubic relationship between variables

Econometric model:

$$score_t = \beta_1 + \beta_2\, age_t + \beta_3\, age_t^2 + \beta_4\, age_t^3 + u_t$$

$$\text{PRF :} \quad E(score_t) = \beta_1 + \beta_2\, age_t + \beta_3\, age_t^2 + \beta_4\, age_t^3$$

Effect of age on score:

$$\frac{\Delta score}{\Delta\, age} \approx \beta_2 + 2\,\beta_3\, age + 3\,\beta_4\, age^2$$

The effect of age on score is NOT constant throughout the sample because it depends on age.

Example 4.1.2. Functional form.

Production function.

Load the data file `cobb.gdt` that you may find in the folder denoted by POE 4th ed (Hill et al. (2008)).

This file contains data from 33 firms on variables:

k: capital
l: labour
q: output

It is assumed that the appropriate production function for these firms is the Cobb-Douglas function:

$$q = A\, k^{\beta_2}\, l^{\beta_3} \quad (1)$$

Example 4.1.2. Functional form.

Questions.

a. Is the Cobb-Douglas production function (1) linear in the coefficients?

b. Is it possible to write the Cobb-Douglas production function as a linear regression model?

c. Given the linear regression model specified in item b.:

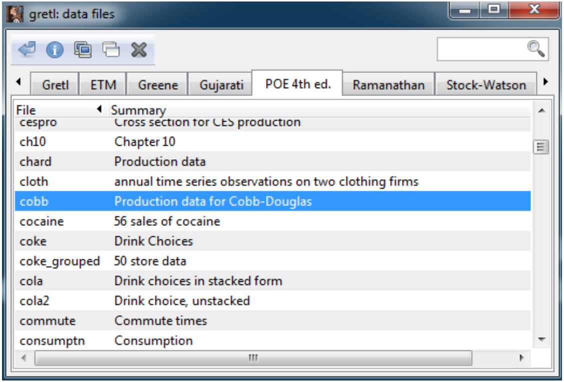
   c.1. What is the partial effect of capital on output? What is the partial effect of labour on output?

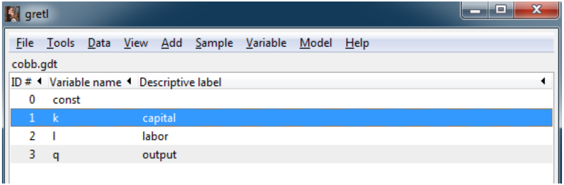
   c.2. What is the elasticity capital-output? What is the elasticity labour-output?

d. Interpret the coefficients of the linear regression model.

e. Transform the data into logarithms using Gretl.

Example 4.1.2. Functional form.





Example 4.1.2. Functional form.

Answers.

a. Is the Cobb-Douglas production function (1) linear in the coefficients?

NO, because the coefficients appear in the exponents of the variables.

b. Is it possible to write the Cobb-Douglas production function as a linear regression model?

YES. Taking logarithms, we get:

$$\ln q_i = \ln A + \beta_2 \ln k_i + \beta_3 \ln l_i + u_i \qquad i = 1, 2, \ldots, 33 \tag{2}$$

The regression model (2) DOES satisfy the linearity assumption because it is linear in the coefficients, even though it is not linear in the variables. It is called a log-log model.

Example 4.1.2. Functional form

Answer.

c.1. **Partial effects**.

Population Regression Function: $E(\ln q_i|X) = \ln A + \beta_2 \ln k_i + \beta_3 \ln l_i$

CAPITAL (holding labour constant).

$$\Delta \ln q = \beta_2 \Delta \ln k \qquad \frac{\Delta q}{q} = \beta_2 \frac{\Delta k}{k}$$

Therefore, the partial effect of capital on output is:

$$\frac{\Delta q}{\Delta k} = \frac{\beta_2}{k} q$$

LABOUR (holding capital constant).

$$\frac{\Delta q}{q} = \beta_2 \frac{\Delta l}{l} \qquad \rightarrow \qquad \frac{\Delta q}{\Delta l} = \frac{\beta_2}{l} q$$

Example 4.1.2. Functional form.

Answers.

c.2. **Elasticities**.

CAPITAL.

$$\text{elasticity}_k = \frac{\Delta q/q}{\Delta k/k} = \frac{\Delta q}{\Delta k}\frac{k}{q} = \frac{\beta_2}{k}\, q\frac{k}{q} = \beta_2$$

LABOUR.

$$\text{elasticity}_l = \frac{\Delta q/q}{\Delta l/l} = \frac{\Delta q}{\Delta l}\frac{l}{q} = \frac{\beta_2}{l}\, q\frac{l}{q} = \beta_2$$

In the log-log model, the marginal effects on output of the explanatory variables (capital and labour) ARE NOT constant while the elasticities capital-output and labour-output ARE constant.

Example 4.1.2. Functional form

Answer.

d. **Interpret the coefficients**.

In the log-log model the coefficients are elasticities.

β_2 is the elasticity capital-output. If capital increases 1 %, holding labour constant, output is expected to increase β_2 %.

β_3 is the elasticity labour-output. If labour increases 1 %, holding capital constant, output is expected to increase β_3 %.

Example 4.1.2. Functional form .

Answer.

e. Transform the data into logarithms using Gretl.

To transform the data into logarithms, highlight variables output, capital and labour, go up to the Menu Bar and click

```
Add -> Logs of selected variables
```