3. Gaia: Programen egiaztapena

2. Ariketa-orria: Asignazioak eta konposaketa sekuentziala

1. Ondoko baieztapenetan post-baldintzak ({ ___ }) bete:

1.1. $\{\ \exists i\ (\ txiki \le i \le txiki + 5\ \wedge\ A(i) > 0\ )\ \}$
```
txiki := txiki+2;
```
$\{\ \underline{\exists i\ (\ txiki - 2 \le i \le txiki + 3\ \wedge\ A(i) > 0\ )}\ \}$

1.2. $\{\ 1 \le erdia \le n\ \}$
```
A(erdia) := z;
```
$\{\ \underline{\exists i\ (\ 1 \le i \le n\ \wedge\ A(i) = z\ )}\ \}$

1.3. $\{\ bikoitia(k)\ \wedge\ y \times z^k = p\ \}$
```
k := k/2;
z := z*z;
```
$\{\ \underline{y \times z^k = p}\ \}$

2. Idatzi post-baldintza bete dadin exekutatu beharreko agindua (____;):

2.1. $\{\ 1 \le i \le n\ \wedge\ z = \sum_{k=1}^{i-1} A(k)\ \}$
```
z := z+A(i) ;
```
$\{\ 1 \le i \le n\ \wedge\ z = \sum_{k=1}^{i} A(k)\ \}$

2.2. $\{\ \forall i\ (\ 1 \le i < muga\ \rightarrow\ A(i) = i^2\ )\ \wedge\ 1 \le muga \le n\ \}$
```
A(muga) := muga*muga ;
```
$\{\ \forall i\ (\ 1 \le i \le muga\ \rightarrow\ A(i) = i^2\ )\ \wedge\ 1 \le muga \le n\ \}$

2.3. $\{\ m = max(A(1..i))\ \wedge\ 1 \le i < n\ \wedge\ A(i+1) > m\ \}$
```
m := A(i+1) ;
```
$\{\ m = max(A(1..i+1))\ \wedge\ 1 \le i < n\ \}$

3. Markatu zuzena den aukera:

3.1. Zein da post-baldintza zuzena?

$\{\ a = i\ \}$
```
i := i+1;
```
a) $\{\ a = i + 1\ \}$ [ ]

b) $\{\ a = i - 1\ \}$ [X]

c) $\{\ i = i + 1\ \wedge\ a = i\ \}$ [ ]

3.2. Zein da baieztapen zuzena?

a) $\{\ i-1>0\ \}$ [*X*]

```
i := i-1;
```

$\{\ i>0\ \}$

b) $\{\ i>0\ \}$ [ ]

```
i := i-1;
```

$\{\ i-1>0\ \}$

4. Hurrengo frogapenetan hutsuneak bete (___):

4.1. $\{\ bikoitia(k)\ \wedge\ y\times z^k=p\ \}$

```
k := k/2;
```

$\{\ y\times z^{2\times k}=p\ \}$

Frogapena:

1. $(\ bikoitia(k)\ \wedge\ y\times z^k=p\ )\ \rightarrow\ (\ \underline{y\times z^{\frac{k}{2}\times 2}=p}\ )$
2. $\{\ \underline{y\times z^{\frac{k}{2}\times 2}=p}\ \}$
   ```
   k := k/2;
   ```
   $\{\ y\times z^{k\times 2}=p\ \}$ **(AA)**
3. $\{\ bikoitia(k)\ \wedge\ y\times z^k=p\ \}$
   ```
   k := k/2;
   ```
   $\{\ y\times z^{k\times 2}=p\ \}$ 1, 2 eta **(ODE)**

4.2. $\{\ batura=g\ \}$

```
g := g+1;
batura := batura+g;
```

$\{\ batura=2\times g-1\ \}$

Frogapena:

1. $(\ batura=g\ )\ \rightarrow\ (\ \underline{batura=g+1-1}\ )$
2. $\{\ \underline{batura=g+1-1}\ \}$
   ```
   g := g+1;
   ```
   $\{\ batura=g-1\ \}$ **(AA)**
3. $\{\ batura=g\ \}$
   ```
   g := g + 1;
   ```
   $\{\ batura=g-1\ \}$ 1, 2 eta **(ODE)**
4. $(\ batura=g-1\ )\ \rightarrow\ (\ batura+g-g=g-1\ )\ \rightarrow$
   $(\ \underline{batura+g=2\times g-1}\ )$
5. $\{\ \underline{batura+g=2\times g-1}\ \}$
   ```
   batura := batura+g;
   ```
   $\{\ batura=2\times g-1\ \}$ **(AA)**
6. $\{\ batura=g-1\ \}$

```
   batura := batura+g;
```
$\{\ batura = 2 \times g - 1\ \}$ 4, 5 eta **(ODE)**

7. $\{\ batura = g\ \}$
```
   g := g+1;
   batura := batura+g;
```
$\{\ batura = 2 \times g - 1\ \}$ 3, 6 eta **(KPE)**

4.3.
$\{\ z = p^k\ \}$
```
k := k+1;
z := z*p;
```
$\{\ z = p^k\ \}$

Frogapena:

1. $(\ z = p^k\ ) \rightarrow (\ z = p^{k+1-1}\ )$
2. $\{\ z = p^{k+1-1}\ \}$
```
   k := k+1;
```
$\{\ \underline{z = p^{k-1}}\ \}$ **(AA)**
3. $\{\ z = p^k\ \}$
```
   k := k+1;
```
$\{\ \underline{z = p^{k-1}}\ \}$ 1, 2 eta **(ODE)**
4. $(\ \underline{z = p^{k-1}}\ ) \rightarrow (\ z \times p = p^{k-1} \times p\ ) \rightarrow (\ \underline{z \times p = p^k}\ )$
5. $\{\ \underline{z \times p = p^k}\ \}$
```
   z := z*p;
```
$\{\ z = p^k\ \}$ **(AA)**
6. $\{\ \underline{z = p^{k-1}}\ \}$
```
   z := z*p;
```
$\{\ z = p^k\ \}$ 4, 5 eta **(ODE)**
7. $\{\ z = p^k\ \}$
```
   k := k+1;
   z := z*p;
```
$\{\ z = p^k\ \}$ 3, 6 eta **(KPE)**

5. Ondokoen artean egiaztatu zuzena den baieztapena eta justifikatu kontradibide baten bidez zuzena ez dena:

5.1. (A) $\{\ 4 \times x = 5^{k+1} - 1\ \}$
```
   k := k+1;
   x := x+5^k;
```
$\{\ 4 \times x = 5^{k+1} - 1\ \}$

(B) $\{\ x = 5^{k+1}\ \}$
```
   k := k+1;
   x := x+5^k;
```
$\{\ x = 5^{k+1}\ \}$

(A) baieztapena zuzena da eta (B) ez da zuzena [X]

(B) baieztapena zuzena da eta (A) ez da zuzena [ ]

Frogapena:

1. $\{\ 4 \times x = 5^{k+1} - 1\ \}$
   `k := k+1;`
   $\{\ 4 \times x = 5^{k} - 1\ \}$ **(AA)**
2. $(\ 4 \times x = 5^{k} - 1\ ) \rightarrow (\ 4 \times x + 4 \times 5^{k} = 5^{k} - 1 + 4 \times 5^{k}\ ) \rightarrow$
   $(\ 4 \times (x + 5^{k}) = 5^{k+1} - 1\ )$
3. $\{\ 4 \times (x + 5^{k}) = 5^{k+1} - 1\ \}$
   `x := x + 5^k;`
   $\{\ 4 \times x = 5^{k+1} - 1\ \}$ **(AA)**
4. $\{\ 4 \times x = 5^{k+1} - 1\ \}$
   `k := k+1;`
   `x := x + 5^k;`
   $\{\ 4 \times x = 5^{k+1} - 1\ \}$ **1, 2, 3, (KPE) eta (ODE)**

Kontradibidea: $\{\ k = 1\ \wedge\ x = 25\ \}$

25 eta 5^2 baliokideak dira, baina $25 + 5^2 \neq 5^3$.

5.2. (A) $\{\ 1 \leq v \leq z\ \wedge\ x^z \times y^v = w\ \}$
```
z := z+1;
w := w*x*y;
v := v+1;
```
$\{\ 1 < v \leq z\ \wedge\ x^z \times y^v = w\ \}$

(B) $\{\ 1 < v \leq z\ \wedge\ x^z \times y^v = w\ \}$
```
z := z-1;
w := w*x*y;
v := v-1;
```
$\{\ 1 \leq v \leq z\ \wedge\ x^z \times y^v = w\ \}$

(A) baieztapena zuzena da eta (B) ez da zuzena [X]

(B) baieztapena zuzena da eta (A) ez da zuzena [ ]

Frogapena:

1. $\{\ 1 \leq v \leq z\ \wedge\ x^z \times y^v = w\ \}$
   `z := z+1;`
   $\{\ 1 \leq v \leq z - 1\ \wedge\ x^{z-1} \times y^v = w\ \}$ **(AA)**
2. $\{\ 1 \leq v \leq z - 1\ \wedge\ x^{z-1} \times y^v = w\ \}$
   `w := w*x*y;`
   $\{\ 1 \leq v \leq z - 1\ \wedge\ x^z \times y^{v+1} = w\ \}$ **(AA)**
3. $\{\ 1 \leq v \leq z - 1\ \wedge\ x^z \times y^{v+1} = w\ \}$
   `v := v+1;`
   $\{\ 1 \leq v - 1 \leq z - 1\ \wedge\ x^z \times y^v = w\ \}$ **(AA)**
4. $(\ 1 \leq v - 1 \leq z - 1\ \wedge\ x^z \times y^v = w\ ) \rightarrow$
   $(\ 1 < v \leq z\ \wedge\ x^z \times y^v = w\ )$

5. $\{\ 1 \leq v \leq z\ \wedge\ x^z \times y^v = w\ \}$
```
z := z+1;
w := w*x*y;
v := v+1;
```
1, 2, 3, 4, (KPE)
$\{\ 1 \leq v-1 \leq z-1\ \wedge\ x^z \times y^v = w\ \}$ **eta (ODE)**

Kontradibidea: $\{\ x=2\ \wedge\ y=5\ \wedge\ z=2\ \wedge\ v=2\ \wedge\ w=100\ \}$
$2^2 \times 5^2 = 100$, baina $2^1 \times 5^1 = 10$ eta $10 \neq 1000$.

5.3. (A) $\{\ 1 < k < w\ \wedge\ z = 2^k \times 4^w\ \}$
```
k := k-1;
z := z*8;
w := w+2;
```
$\{\ 1 \leq k < w\ \wedge\ z = 2^k \times 4^w\ \}$

(B) $\{\ 1 \leq k < w\ \wedge\ z = 2^k \times 4^w\ \}$
```
k := k+2;
z := z*8;
w := w+1;
```
$\{\ 1 < k \leq w\ \wedge\ z = 2^k \times 4^w\ \}$

(A) baieztapena zuzena da eta (B) ez da zuzena [X]
(B) baieztapena zuzena da eta (A) ez da zuzena [ ]

Frogapena:

1. $\{\ 1 < k < w\ \wedge\ z = 2^k \times 4^w\ \}$
```
k := k-1;
```
$\{\ 1 < k+1 < w\ \wedge\ z = 2^{k+1} \times 4^w\ \}$ **(AA)**

2. $\{\ 1 < k+1 < w\ \wedge\ z = 2^{k+1} \times 4^w\ \}$
```
z := z*8;
```
$\{\ 1 < k+1 < w\ \wedge\ \frac{z}{8} = 2^{k+1} \times 4^w\ \}$ **(AA)**

3. $\{\ 1 < k+1 < w\ \wedge\ \frac{z}{8} = 2^{k+1} \times 4^w\ \}$
```
w := w+2;
```
$\{\ 1 < k+1 < w-2\ \wedge\ \frac{z}{8} = 2^{k+1} \times 4^{w-2}\ \}$ **(AA)**

4. $(\ 1 < k+1 < w-2\ \wedge\ \frac{z}{8} = 2^{k+1} \times 4^{w-2}\ ) \rightarrow$
$(\ 1 \leq k < w\ \wedge\ \frac{z}{8} = 2^k \times 4^w \times \frac{2}{16}\ ) \rightarrow$
$(\ 1 \leq k < w\ \wedge\ z = 2^k \times 4^w\ )$

5. $\{\ 1 < k < w\ \wedge\ z = 2^k \times 4^w\ \}$
```
k := k-1;
z := z*8;
w := w+2;
```
$\{\ 1 \leq k < w\ \wedge\ z = 2^k \times 4^w\ \}$ **1, 2, 3, 4, (KPE) eta (ODE)**

Kontradibidea: $\{\ k=1\ \wedge\ w=2\ \wedge\ z=32\ \}$
$2^3 \times 4^3 = 512$, baina $z = 256$ eta $512 \neq 256$.

6. Egiaztatu hurrengo baieztapenak:

6.1. $\{\ n \geq 1\ \wedge\ i = 0\ \}$
`zerorik_ez := true;`
$\{\ 0 \leq i \leq n\ \wedge\ (\ zerorik\_ez \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ \}$

Soluzioa:

1. $(\ n \geq 1\ \wedge\ i = 0\ )\ \rightarrow$
$(\ 0 \leq i \leq n\ \wedge\ (\ true \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ )$
2. $\{\ 0 \leq i \leq n\ \wedge\ (\ true \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ \}$
`zerorik_ez := true;` **(AA)**
$\{\ 0 \leq i \leq n\ \wedge\ (\ zerorik\_ez \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ \}$
3. $\{\ n \geq 1\ \wedge\ i = 0\ \}$
`zerorik_ez := true;` 1, 2 eta **(ODE)**
$\{\ 0 \leq i \leq n\ \wedge\ (\ zerorik\_ez \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ \}$

6.2. $\{\ n \geq 1\ \wedge\ A(1) \neq 0\ \wedge\ zerorik\_ez\ \}$
`i := 1;`
$\{\ 1 \leq i \leq n\ \wedge\ (\ zerorik\_ez \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ \}$

Soluzioa:

1. $\{\ n \geq 1\ \wedge\ A(1) \neq 0\ \wedge\ zerorik\_ez\ \}$
`i := 1;`
$\{\ n \geq i\ \wedge\ A(i) \neq 0\ \wedge\ zerorik\_ez\ \wedge\ i = 1\ \}$ **(AA)**
2. $(\ n \geq i\ \wedge\ A(i) \neq 0\ \wedge\ zerorik\_ez\ \wedge\ i = 1\ )\ \rightarrow\ (\ 1 \leq i \leq n\ \wedge$
$(\ zerorik\_ez \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ )$
3. $\{\ n \geq 1\ \wedge\ A(1) \neq 0\ \wedge\ zerorik\_ez\ \}$
`i := 1;` 1, 2 eta **(ODE)**
$\{\ 1 \leq i \leq n\ \wedge\ (\ zerorik\_ez \leftrightarrow \forall k\ (\ 1 \leq k \leq i\ \rightarrow\ A(k) \neq 0\ )\ )\ \}$

6.3. $\{\ x \geq y\ \}$
`z := x;`
$\{\ z = max(x, y)\ \}$

Soluzioa (goitik beherakoa):

1. $\{\ x \geq y\ \}$
`z := x;`
$\{\ z \geq y\ \wedge\ z = x\ \}$ **(AA)**
2. $(\ z \geq y\ \wedge\ z = x\ )\ \rightarrow\ (\ z = max(x, y)\ )$
3. $\{\ x \geq y\ \}$
`z := x;`
$\{\ z = max(x, y)\ \}$ 1, 2 eta **(ODE)**

Soluzioa (behetik gorakoa):

1. $\{\ x = max(x, y)\ \}$
   `z := x;`
   $\{\ z = max(x, y)\ \}$ **(AA)**
2. $(\ x \geq y\ ) \rightarrow (\ x = max(x, y)\ )$
3. $\{\ x \geq y\ \}$
   `z := x;`
   $\{\ x = max(x, y)\ \}$ 1, 2 eta **(ODE)**