

Duality using Lindo

The aim of this laboratory session is to study the entering vector rule and the leaving vector rule used in the dual simplex algorithm, using the software tool called Lindo. Lindo gives you the possibility to select the pivot by yourself, if you want to.

The software tool Lindo is widely used to solve optimization models. You can download a trial version free of charge from: <http://www.lindo.com>, go to "downloads", and there to "Download Classic LINDO". It is available for windows operating system.

1. Consider the two linear models that follow. For each of them, compute the initial tableau using lingo and analyze whether it is primal feasible or dual feasible.

$$1.1 \quad \max z = 2x_1 + 2x_2$$

subject to

$$x_1 - x_2 \leq 2$$

$$2x_1 + 2x_2 \geq 6$$

$$x_1 + 2x_2 \geq 5$$

$$x_1, x_2 \geq 0$$

$$1.2 \quad \min z = 2x_1 + 2x_2$$

subject to

$$x_1 + 3x_2 \geq 3$$

$$x_1 - 3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

2. The dual simplex algorithm is used when the initial tableau is dual feasible. If there are negative x_{B_i} values, the solution to the dual may be improved. The entering vector rule and the leaving vector rule are used to change the basis. The process goes on until the tableau is also primal feasible.

In this exercise you are asked not to follow one of the rules mentioned. You have to show that you understand the consequences that such decision has.

Consider the following linear model:

$$\max z = -4x_1 - 3x_2 - 2x_3$$

subject to

$$x_1 - 2x_2 - 2x_3 \leq -30$$

$$2x_1 - x_2 + x_3 \leq 12$$

$$-2x_1 + 3x_2 - x_3 \leq -18$$

$$x_1, x_2, x_3 \geq 0$$

- 2.1 Compute the initial tableau using lingo. Is it dual feasible? Write the dual model associated to the given model. Extract the solution to the dual model from the initial tableau and prove that it is a feasible solution to the dual model.

2.2 Starting at the initial tableau computed in 2.1, select the entering vector and the leaving vector (go to the "Solve" menu and select the "Pivot" option) according to the following criteria:

- (a) The leaving vector \mathbf{a}_r is selected so that $x_{Br} > 0$, and the entering vector \mathbf{a}_k is selected so that the following holds:

$$\frac{z_k - c_k}{y_{rk}} = \max \left\{ \frac{z_j - c_j}{y_{rj}} / y_{rj} < 0 \right\}$$

Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

- (b) The leaving vector \mathbf{a}_r is selected so that $x_{Br} < 0$ and the following holds:

$$x_{Br} \neq \min \{ x_{Bi} / x_{Bi} < 0 \}$$

and the entering vector \mathbf{a}_k is selected so that the following holds:

$$\frac{z_k - c_k}{y_{rk}} = \max \left\{ \frac{z_j - c_j}{y_{rj}} / y_{rj} < 0 \right\}$$

Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

- (c) The leaving vector \mathbf{a}_r is selected so that the following holds:

$$x_{Br} = \min \{ x_{Bi} / x_{Bi} < 0 \}$$

and the entering vector \mathbf{a}_k is selected so that $y_{rk} > 0$.

Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

- (d) The leaving vector \mathbf{a}_r is selected so that the following holds:

$$x_{Br} = \min \{ x_{Bi} / x_{Bi} < 0 \}$$

and the entering vector \mathbf{a}_k is selected so that $y_{rk} < 0$ and the following holds:

$$\frac{z_k - c_k}{y_{rk}} \neq \max \left\{ \frac{z_j - c_j}{y_{rj}} / y_{rj} < 0 \right\}$$

Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

3. Solve the linear model given in the previous exercise. You can compute the optimal solution directly (go to the "Solve" menu and select the "Solve" option). Lindo will select the entering vector and the leaving vector at each iteration of the algorithm. Extract the optimal solution to the model given and the optimal solution to the dual from the optimal tableau.