MATHEMATICA 6.0

EJERCICIO PROPUESTO 1 a

In[1]:=
```
Limit[((1 + Tan[x])/(1 + Sin[x]))^(1/Sin[x]), x → 0, Direction → 1]
```

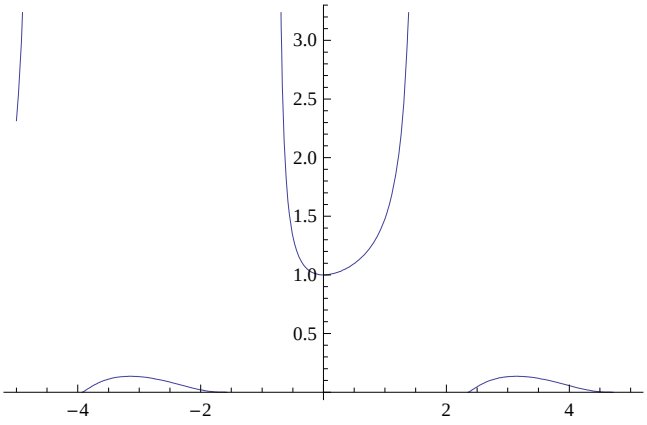
Out[1]= 1

In[2]:=
```
Limit[((1 + Tan[x])/(1 + Sin[x]))^(1/Sin[x]), x → 0, Direction → -1]
```

Out[2]= 1

In[3]:=
```
Plot[((1 + Tan[x])/(1 + Sin[x]))^(1/Sin[x]), {x, -5, 5}]
```

Out[3]=



$$\lim_{x \to 0} \left(\frac{1 + \tan[x]}{1 + \sin[x]} \right)^{\frac{1}{\sin[x]}} = 1$$

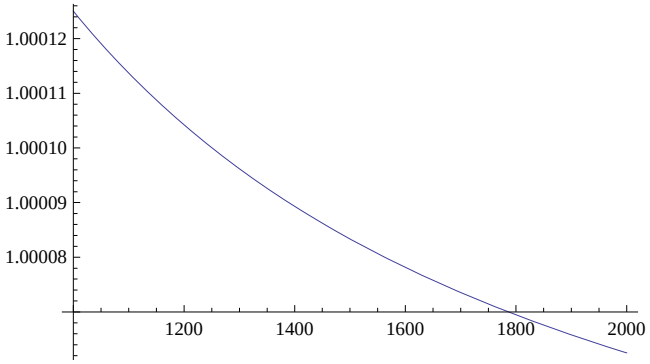
EJERCICIO PROPUESTO 1 b

In[4]:=
```
Limit[(Sqrt[x + Sqrt[x]] - Sqrt[x - Sqrt[x]]), x → ∞]
```

Out[4]= 1

In[7]:=
```
Plot[(Sqrt[x + Sqrt[x]] - Sqrt[x - Sqrt[x]]), {x, 1000, 2000}]
```

Out[7]=



$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}\right)=1$$

EJERCICIO PROPUESTO 2

In[8]:=
```
Limit[1/(2 - 2^(1/x)), x → 0, Direction → 1]
```

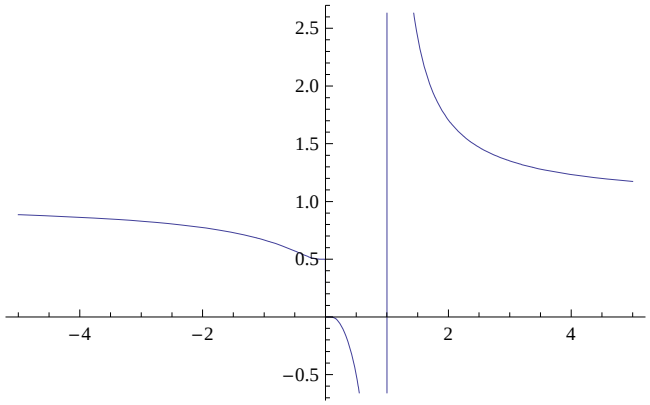
Out[8]=
$$\frac{1}{2}$$

In[9]:=
```
Limit[1/(2 - 2^(1/x)), x → 0, Direction → -1]
```

Out[9]=
```
0
```

In[10]:=
```
Plot[1/(2 - 2^(1/x)), {x, -5, 5}]
```

Out[10]=



EJERCICIO PROPUESTO 3

In[11]:=
$$D\left[\text{Log}\left[\frac{x-1}{x+1}\right], x\right]$$

Out[11]=
$$\frac{(1+x)\left(-\frac{-1+x}{(1+x)^2}+\frac{1}{1+x}\right)}{-1+x}$$

In[12]:=
```
Simplify[%]
```

Out[12]=
$$\frac{2}{-1+x^2}$$

In[13]:=
$$D\left[\text{Sin}[x]^{\text{Log}[\text{Cos}[x]]}, x\right]$$

Out[13]=
$$\text{Sin}[x]^{\text{Log}[\text{Cos}[x]]}\,(\text{Cot}[x]\,\text{Log}[\text{Cos}[x]]-\text{Log}[\text{Sin}[x]]\,\text{Tan}[x])$$

In[14]:=
```
Simplify[%]
```

Out[14]=
$$\text{Sin}[x]^{\text{Log}[\text{Cos}[x]]}\,(\text{Cot}[x]\,\text{Log}[\text{Cos}[x]]-\text{Log}[\text{Sin}[x]]\,\text{Tan}[x])$$

EJERCICIO PROPUESTO 4

In[15]:=
$$D\left[\frac{x-y}{3xy}, \{x, 2\}, \{y, 4\}\right]$$

Out[15]=
$$0$$

In[16]:=
$$\partial_{\{x,2\},\{y,4\}}\frac{x-y}{3xy}$$

Out[16]=
$$0$$

In[20]:=
$$D\left[\frac{\text{Sin}[xy]}{3x+y}, \{x, 2\}, \{y, 1\}\right]\ //\ \text{Simplify}$$

Out[20]=
$$-\frac{1}{(3x+y)^4}\left(y\left(-6y+27x^4y+27x^3y^2+9x^2y^3+x\left(-18+y^4\right)\right)\text{Cos}[xy]+\left(54+9x^2y^2+6xy^3+y^4\right)\text{Sin}[xy]\right)$$

In[21]:=
$$\partial_{\{x,2\},\{y,1\}}\frac{\text{Sin}[xy]}{3x+y}$$

Out[21]=
$$\frac{18x\,\text{Cos}[xy]}{(3x+y)^3}+\frac{12y\,\text{Cos}[xy]}{(3x+y)^3}-\frac{6\,\text{Cos}[xy]}{(3x+y)^2}-\frac{xy^2\,\text{Cos}[xy]}{3x+y}-\frac{54\,\text{Sin}[xy]}{(3x+y)^4}+\frac{6xy\,\text{Sin}[xy]}{(3x+y)^2}+\frac{y^2\,\text{Sin}[xy]}{(3x+y)^2}-\frac{2y\,\text{Sin}[xy]}{3x+y}$$

In[22]:= **Simplify[%]**

Out[22]=
$$-\frac{1}{(3x+y)^4}\left(y\left(-6y+27x^4y+27x^3y^2+9x^2y^3+x\left(-18+y^4\right)\right)\text{Cos}[xy]+\left(54+9x^2y^2+6xy^3+y^4\right)\text{Sin}[xy]\right)$$

EJERCICIO PROPUESTO 5

In[23]:= **Dt[x^2 (x + y) == Log[Sqrt[x^2 + y^2]], x]**

Out[23]=
$$2x(x+y)+x^2(1+\text{Dt}[y,x])==\frac{2x+2y\,\text{Dt}[y,x]}{2\left(x^2+y^2\right)}$$

In[24]:= **Solve[%, Dt[y, x]]**

Out[24]=
$$\left\{\left\{\text{Dt}[y,x]\to\frac{x-3x^4-2x^3y-3x^2y^2-2xy^3}{x^4-y+x^2y^2}\right\}\right\}$$

EJERCICIO PROPUESTO 6

In[25]:= $\int \frac{x-2}{\sqrt{x^2+x+1}}\,dx$

Out[25]=
$$\sqrt{1+x+x^2}-\frac{5}{2}\,\text{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]$$

In[26]:= $\int \sqrt{2x^2+3x-1}\,dx$

Out[26]=
$$\frac{1}{32}\left(4(3+4x)\sqrt{-1+3x+2x^2}-17\sqrt{2}\,\text{Log}\left[3+4x+2\sqrt{-2+6x+4x^2}\right]\right)$$

In[27]:= $\int_0^{2\pi} (1-\text{Cos}[\varphi])^2\,d\varphi$

Out[27]= 3π

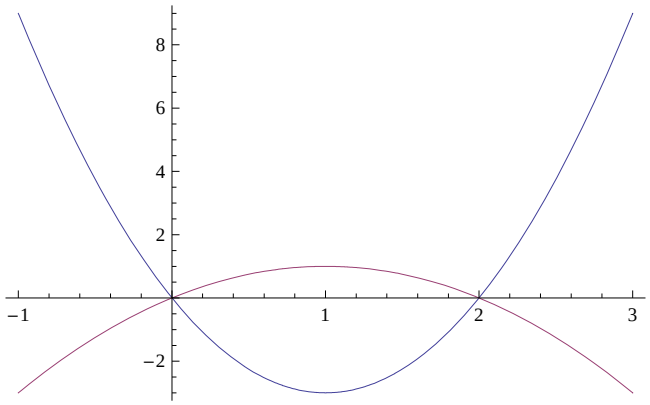
In[28]:= $\int_1^2 \frac{10x^2}{(x^3+1)^2}\,dx$

Out[28]= $\frac{35}{27}$

EJERCICIO PROPUESTO 7

In[29]:=
```
Plot[{3 x^2 - 6 x, 2 x - x^2}, {x, -1, 3}]
```

Out[29]=



Puntos de corte de las gráficas x = 0 y x = 2

In[30]:=
```
area = ∫_0^2 (2 x - x^2 - (3 x^2 - 6 x)) dx
```

Out[30]= $\frac{16}{3}$

El área limitada por las curvas es $\frac{16}{3}$ u²

EJERCICIO PROPUESTO 8

In[31]:=
```
Tgrad3 = Normal[Series[Log[1 + x], {x, 0, 3}]]
```

Out[31]= $x - \frac{x^2}{2} + \frac{x^3}{3}$

In[32]:=
```
Tgrad6 = Normal[Series[Log[1 + x], {x, 0, 6}]]
```

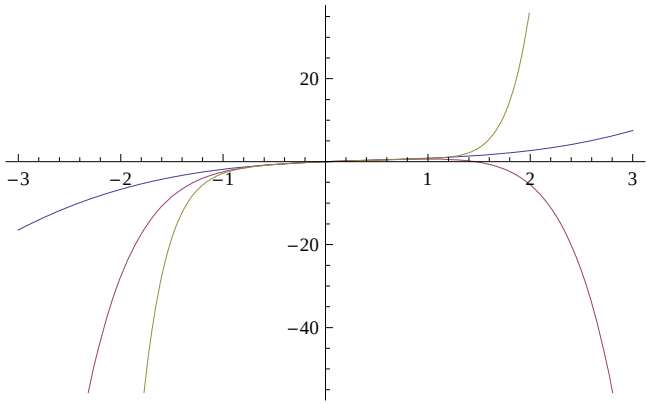
Out[32]= $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6}$

In[33]:=
```
Tgrad9 = Normal[Series[Log[1 + x], {x, 0, 9}]]
```

Out[33]= $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8} + \frac{x^9}{9}$

In[37]:=
```
Plot[{Tgrad3, Tgrad6, Tgrad9}, {x, -3, 3}]
```

Out[37]=



EJERCICIO PROPUESTO 9

In[38]:=
```
ec = y'''[x] - 6 y''[x] + 11 y'[x] - 6 y[x] == Exp[2 x] - 6 x^2 + 4 x - 3;
```

In[39]:=
```
transec = LaplaceTransform[ec, x, s] /. {y[0] → 4, y'[0] → 2, y''[0] → 2}
```

Out[39]=
$$-2-2\,s-4\,s^2-6\,\text{LaplaceTransform}[y[x],x,s]+s^3\,\text{LaplaceTransform}[y[x],x,s]+11\,(-4+s\,\text{LaplaceTransform}[y[x],x,s])-6\,\left(-2-4\,s+s^2\,\text{LaplaceTransform}[y[x],x,s]\right)==\frac{1}{-2+s}-\frac{12}{s^3}+\frac{4}{s^2}-\frac{3}{s}$$

In[40]:=
```
sol = Solve[transec, LaplaceTransform[y[x], x, s]]
```

Out[40]=
$$\left\{\left\{\text{LaplaceTransform}[y[x],x,s]\to\frac{2\left(12-10\,s+5\,s^2-35\,s^3+39\,s^4-15\,s^5+2\,s^6\right)}{(-2+s)^2\,s^3\left(3-4\,s+s^2\right)}\right\}\right\}$$

In[41]:=
```
InverseLaplaceTransform[2 (12 - 10 s + 5 s^2 - 35 s^3 + 39 s^4 - 15 s^5 + 2 s^6)/((-2 + s)^2 s^3 (3 - 4 s + s^2)), s, x]
```

Out[41]=
$$2\left(2+e^x-2\,e^{2x}+e^{3x}+\frac{3\,x}{2}-\frac{1}{2}\,e^{2x}\,x+\frac{x^2}{2}\right)$$

In[42]:=
```
Simplify[%]
```

Out[42]=
$$4+2\,e^x+2\,e^{3x}+3\,x+x^2-e^{2x}\,(4+x)$$

La solución de la EDO es $y(x)=4+2\,e^x+2\,e^{3x}+3\,x+x^2-e^{2x}\,(4+x)$

EJERCICIO PROPUESTO 10

In[43]:=
```
ecu = y''''[x] - y[x] == x Exp[x] + Cos[x];
```

In[44]:= **DSolve[ecu, y[x], x]**

Out[44]=
$$\left\{\left\{y[x] \to e^{x} C[1] + e^{-x} C[3] + C[2] \operatorname{Cos}[x] + C[4] \operatorname{Sin}[x] + \frac{1}{16}\left(e^{x} - 2 e^{x} x + 2 e^{x} x^{2} - 4 \operatorname{Cos}[x] + 4 e^{x} \operatorname{Cos}[x]^{2} - 4 e^{x} x \operatorname{Cos}[x]^{2} - 2 \operatorname{Cos}[x] \operatorname{Cos}[2 x] - 4 x \operatorname{Sin}[x] + 4 e^{x} \operatorname{Sin}[x]^{2} - 4 e^{x} x \operatorname{Sin}[x]^{2} - 4 \operatorname{Cos}[x] \operatorname{Sin}[x]^{2}\right)\right\}\right\}$$

In[45]:= **Simplify[%]**

Out[45]=
$$\left\{\left\{y[x] \to \frac{5 e^{x}}{16} - \frac{3 e^{x} x}{8} + \frac{e^{x} x^{2}}{8} + e^{x} C[1] + e^{-x} C[3] + \left(-\frac{3}{8} + C[2]\right) \operatorname{Cos}[x] + \left(-\frac{x}{4} + C[4]\right) \operatorname{Sin}[x]\right\}\right\}$$

La solución de la EDO es

$$\mathbf{y(x) = \frac{5 e^{x}}{16} - \frac{3 e^{x} x}{8} + \frac{e^{x} x^{2}}{8} + e^{x} C[1] + e^{-x} C[3] + \left(-\frac{3}{8} + C[2]\right) \operatorname{Cos}[x] + \left(-\frac{x}{4} + C[4]\right) \operatorname{Sin}[x]}$$