MATHEMATICA 6.0

EJERCICIO PROPUESTO 1

In[1]:=
```
pe[x_, y_] :=
 x[[1]] y[[1]] + x[[1]] y[[2]] + x[[2]] y[[1]] + 2 x[[2]] y[[2]] + x[[3]] y[[3]] + x[[4]] y[[4]]
```

In[4]:=
```
b = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

Base canónica de $\mathbb{R}^4$

In[5]:=
```
Orthogonalize[b, pe]
```

Out[5]=
```
{{1, 0, 0, 0}, {-1, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
```

Base ortonormal de $\mathbb{R}^4$.
Con esta versión de Mathematica se obtiene bases ortonormales

EJERCICIO PROPUESTO 2

Comprobamos que se trata de un sistema incompatible

In[13]:= $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \end{pmatrix}; B = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix};$

In[14]:=
```
LinearSolve[A, B]
```

LinearSolve::nosol : Linear equation encountered that has no solution. ≫

Out[14]=
```
LinearSolve[{{1, 0, 1}, {0, 1, 2}, {1, 2, 2}, {1, 1, 3}}, {{1}, {1}, {1}, {1}}]
```

El sistema es incompatible. Vamos a calcular la solución aproximada

In[15]:=
```
u = {{1, 0, 1, 1}, {0, 1, 2, 1}, {1, 2, 2, 3}};
```

In[16]:=
```
RowReduce[u]
```

Out[16]=
```
{{1, 0, 0, 1}, {0, 1, 0, 1}, {0, 0, 1, 0}}
```

u es un sistema libre

In[17]:=
```
w = Orthogonalize[u]
```

Out[17]= $\left\{\left\{\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0\right\}, \left\{0, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}\right\}$

w es un sistema ortonormal

In[18]:=
```
b = {1, 1, 1, 1};
```

In[19]:=
```
bp = Sum[Projection[b, w[[i]]], {i, 3}]
```

Out[19]= $\left\{\frac{2}{3}, \frac{2}{3}, 1, \frac{4}{3}\right\}$

Mejor aproximación de b en $\mathcal{L}$ (u)

In[20]:= `s = Solve[x u[[1]] + y u[[2]] + z u[[3]] == bp, {x, y, z}]`

Out[20]= $\left\{\left\{x \to \frac{1}{3},\ y \to 0,\ z \to \frac{1}{3}\right\}\right\}$

La solución aproximada es : $x = \frac{1}{3},\ y = 0,\ z = \frac{1}{3}$

EJERCICIO PROPUESTO 3

In[21]:= `Solve[x1 - x4 == x2 - x4 == x3]`

Solve::svars : Equations may not give solutions for all "solve" variables. ≫

Out[21]= $\{\{x1 \to x2,\ x3 \to x2 - x4\}\}$

S = {(x2, x2, x2 - x4, x4)} = {x2 (1, 1, 1, 0) + x4 (0, 0, -1, 1)} = ℒ ({(1, 1, 1, 0), (0, 0, -1, 1)})

B_S = {(1, 1, 1, 0), (0, 0, -1, 1)}, dim (S) = 2

In[22]:= `x = {1, 1, 1, 1};`

In[23]:= `b = {{1, 1, 1, 0}, {0, 0, -1, 1}};`

In[32]:= `w = Orthogonalize[b]`

Out[32]= $\left\{\left\{\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ 0\right\},\ \left\{\frac{1}{\sqrt{15}},\ \frac{1}{\sqrt{15}},\ -\frac{2}{\sqrt{15}},\ \sqrt{\frac{3}{5}}\right\}\right\}$

w base ortonormal de S

In[33]:= `xp = Sum[Projection[x, w[[i]]], {i, 2}]`

Out[33]= $\left\{\frac{6}{5},\ \frac{6}{5},\ \frac{3}{5},\ \frac{3}{5}\right\}$

xp es la mejor aproximación de x en S

In[34]:= `e = xp - x`

Out[34]= $\left\{\frac{1}{5},\ \frac{1}{5},\ -\frac{2}{5},\ -\frac{2}{5}\right\}$

In[35]:= `Norm[e]`

Out[35]= $\sqrt{\frac{2}{5}}$

Norma del vector de error

In[36]:= `y = {2, 4, 2, 2};`

In[37]:= `yp = Sum[Projection[y, w[[i]]], {i, 2}]`

Out[37]= $\left\{\frac{8}{3} + \frac{2\sqrt{\frac{3}{5}} + \frac{2}{\sqrt{15}}}{\sqrt{15}},\ \frac{8}{3} + \frac{2\sqrt{\frac{3}{5}} + \frac{2}{\sqrt{15}}}{\sqrt{15}},\ \frac{8}{3} - \frac{2\left(2\sqrt{\frac{3}{5}} + \frac{2}{\sqrt{15}}\right)}{\sqrt{15}},\ \sqrt{\frac{3}{5}}\left(2\sqrt{\frac{3}{5}} + \frac{2}{\sqrt{15}}\right)\right\}$

In[38]:= Simplify[%]

Out[38]= $\left\{\frac{16}{5}, \frac{16}{5}, \frac{8}{5}, \frac{8}{5}\right\}$

yp es la mejor aproximación de x en S

In[41]:= Norm[yp - y] // Simplify

Out[41]= $2\sqrt{\frac{3}{5}}$

Norma del vector de error

EJERCICIO PROPUESTO 4

In[42]:= $A = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix};$

In[43]:= Eigenvalues[A]

Out[43]= {3, 2, 0}

Valores propios de A :
$\lambda_1 = 3$, k1 = 1
$\lambda_2 = 2$, k2 = 1
$\lambda_3 = 0$, k3 = 1

In[44]:= Eigenvectors[A]

Out[44]= {{3, 2, 1}, {0, 1, 1}, {0, -1, 1}}

Subespacios propios de A :
V (3) = $\mathcal{L}$ ({(3, 2, 1)}), d1 = 1
V (2) = $\mathcal{L}$ ({(0, 1, 1)}), d2 = 1
V (0) = $\mathcal{L}$ ({(0, -1, 1)}) d3 = 1

A es diagonalizable ya que k1 + k2 + k3 = 3
d1 = k1, d2 = k2, d3 = k3

In[46]:= $P = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix};$

$D = P^{-1} AP$

In[47]:= D = MatrixPower[P, -1].A.P // MatrixForm

Out[47]//MatrixForm=
$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

D matriz diagonal formada por los valores propios de A

La matriz real A no es diagonalizable ortogonalmente ya que no es simétrica