Integral Indefinida

$$\int f(x)\,dx = F(x) + C \quad (\forall C \in \mathbb{R}) :\Leftrightarrow \quad F'(x) = f(x)$$

F(x): función Primitiva de f(x)

Operador Lineal

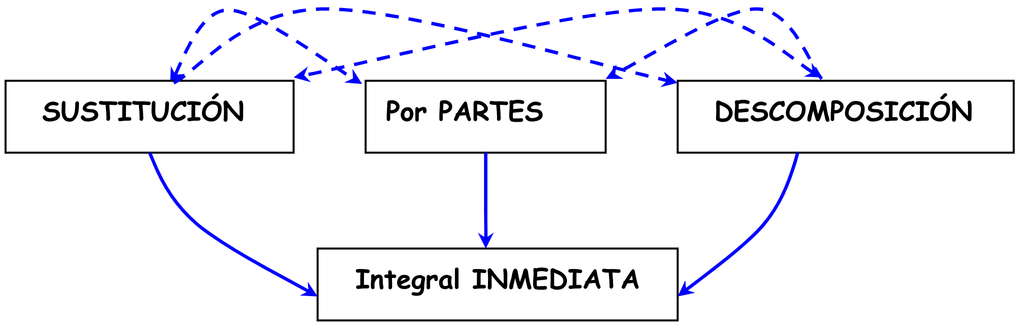
$$\int k\,f(x)\,dx = k\int f(x)\,dx \qquad \forall k \in \mathbb{R}$$

$$\int [f(x) + g(x) + \ldots]\,dx = \int f(x)\,dx + \int g(x)\,dx + \ldots.$$

$$\int [k\,f(x) + m\,g(x) + \ldots]\,dx = k\int f(x)\,dx + m\int g(x)\,dx + \ldots.$$

MÉTODOS de Integración

Transformación de Funciones



Por PARTES

Primitivas de las funciones Inversas: Arc sen , Arc cos , Arg Sh , ...
Logaritmos. Producto de funciones distintas......

Modelos: $\int u(x)\cdot v'(x)\,dx$; $\int u(x)\cdot \frac{v'(x)}{k}\,dx$

Ejemplos : 1) $I = \int P(x)\cdot e^{ax}\,dx ; \int P(x)\cdot \sin x\,dx ; \int P(x)\cdot \text{sh}x\,dx \rightarrow \left| \begin{matrix} u(x) = P(x) \rightarrow du = u'(x)\,dx \\ dv = g\,dx \rightarrow v = \int g\,dx \end{matrix} \right| \Rightarrow \boxed{I = u\cdot v - \int v\,du = ...}$

2) $I = \int \ln x\,dx \rightarrow \left| \begin{matrix} u(x) = \ln x \rightarrow du = \\ dv = dx \quad \rightarrow v = \int dx \end{matrix} \right| \Rightarrow I = u\cdot v - \int v\,du =$

SUSTITUCIÓN

Funciones Irracionales

Ejemplos : 3) $I = \int \frac{x^2\,dx}{\sqrt[3]{4+x^3}} \rightarrow \left| \begin{matrix} 4 + x^3 = t \\ 3x^2 dx = d\,t \end{matrix} \right| \Rightarrow I = \frac{1}{3}\int \frac{dt}{\sqrt[3]{t}} =$

4) $I = \int \frac{4\,dx}{\sqrt{x^2+6x+25}} = \frac{4\,dx}{\sqrt{(x+3)^2+5^2}} \rightarrow \left| \begin{matrix} x+3 = 5t \\ dx = 5d\,t \end{matrix} \right| \rightarrow I = \int \frac{dt}{\sqrt{t^2+1}} =$

5) $I = \int \frac{\text{sh}x\,dx}{3\text{ch}^2 x + 6} \rightarrow \left| \begin{matrix} \text{ch}x = t \\ \text{sh}x\,dx = d\,t \end{matrix} \right| \rightarrow I = \int \frac{dt}{3t^2+6} =$

Tabla de Integrales INMEDIATAS

$$\int dx = x + C$$

$$(n \neq -1): \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$\int \sqrt[n]{x^m}\, dx = \int x^{m/n} dx = \frac{x^{\frac{m}{n}+1}}{\frac{m}{n}+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{1}{x-a} dx = \ln|x-a| + C$$

$$\int \frac{1}{x^2} dx = \frac{-1}{x} + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int e^x dx = e^x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \mathrm{Sh}\, x\, dx = \mathrm{Ch}\, x + C$$

$$\int \mathrm{Ch}\, x\, dx = \mathrm{Sh}\, x + C$$

$$\int \mathrm{tg}\, x\, dx = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + C$$

$$\int \mathrm{ctg}\, x\, dx = \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + C$$

$$\int \mathrm{Th}x\, dx = \int \frac{\mathrm{Sh}x}{\mathrm{Ch}x} dx = \ln(\mathrm{Ch}x) + C$$

$$\int \mathrm{Coth}x\, dx = \int \frac{\mathrm{Ch}x}{\mathrm{Sh}x} dx = \ln|\mathrm{Sh}x| + C$$

(I) $$\int \frac{f'(x)}{f(x)} dx = L_n |f(x)| + C$$

(II) $$\int f(x)\,dx = F(x) + C \Rightarrow \int f(ax+b)\,dx = \frac{1}{a} \cdot F(ax+b) + C$$

$$\wedge \int f(x+b)\,dx = F(x+b) + C$$

(III) $$I = \int f(u) \cdot \frac{u'}{k} dx \to \left| \begin{matrix} u(x) = t \\ u'(x)\,dx = dt \end{matrix} \right| \Rightarrow I = \frac{1}{k} \cdot \int f(t)\,dt = \ldots\ldots.$$

$$\int (1+\mathrm{tg}^2 x)\, dx = \int \frac{1}{\cos^2 x} dx = \mathrm{tg}\, x + C$$

$$\int (1+\mathrm{ctg}^2 x)\, dx = \int \frac{1}{\sin^2 x} dx = -\mathrm{ctg}\, x + C$$

$$\int (1+\mathrm{Th}^2 x)\, dx = \int \frac{1}{\mathrm{Ch}^2 x} dx = \mathrm{Th}x + C$$

$$\int (\mathrm{Coth}^2 x - 1)\, dx = \int \frac{1}{\mathrm{Sh}^2 x} dx = -\mathrm{Coth}\, x + C$$

I(x): Irracional

$$\int \frac{1}{\sqrt{1-x^2}} dx = \mathrm{Arcsin}\, x + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \mathrm{ArgCh}\, x + C$$

$$\int \frac{1}{\sqrt{x^2+1}} dx = \mathrm{ArgSh}\, x + C$$

$$\int \frac{x \cdot dx}{\sqrt{1-x^2}} = -\sqrt{1-x^2} + C$$

$$\int \frac{x \cdot dx}{\sqrt{1+x^2}} = \sqrt{1+x^2} + C$$

$$\int \frac{x \cdot dx}{\sqrt{x^2-1}} = \sqrt{x^2-1} + C$$

$1 \dashrightarrow a$ $(a > 0)$

R(x): Racional

$$\mathbf{R}(x) = \int \frac{P(x)}{Q(x)} dx$$

$$\int \frac{1}{x} dx = \ln|x| + C \qquad \int \frac{1}{x-a} dx = \ln|x-a| + C$$

$$\int \frac{B dx}{(x-a)^n} + C = B \frac{(x-a)^{-n+1}}{-n+1} + C$$

$$* \int \frac{1}{1+x^2} dx = Arctgx + C \qquad ** \int \frac{dx}{a+x^2} = \frac{1}{\sqrt{a}} Arc\ tg \frac{x}{\sqrt{a}} + C \quad (a>0)$$

$$\spadesuit \int \frac{1}{1-x^2} dx = ArgThx + C \qquad \spadesuit\spadesuit \int \frac{dx}{a-x^2} = \frac{1}{\sqrt{a}} ArgTh \frac{x}{\sqrt{a}} + C \quad (a>0)$$

$x \dashrightarrow x\text{-}a$

$1 \dashrightarrow b^2$

$$\int \frac{dx}{b-(x-a)^2} = \frac{1}{\sqrt{b}} Arg\ Th \frac{x-a}{\sqrt{b}} + C$$

$$\clubsuit \int \frac{N dx}{(x-a)^2+b^2} = \frac{N}{b} Arc\ tg \frac{x-a}{b} + C$$

$$\int \frac{\mathbf{M(x-a)+N}}{\mathbf{(x-a)^2+b^2}} \mathbf{dx} = \frac{\mathbf{M}^{(I)}}{\mathbf{2}} \int \frac{\mathbf{2(x-a)}}{\mathbf{(x-a)^2+b^2}} \mathbf{dx} + \clubsuit \int \frac{\mathbf{N}}{\mathbf{(x-a)^2+b^2}} dx =$$

$$= \frac{\mathbf{M}}{\mathbf{2}} \mathbf{Ln} \left[\mathbf{(x-a)^2+b^2} \right] + \frac{\mathbf{N}}{\mathbf{b}} \mathbf{Arctg} \frac{\mathbf{x-a}}{\mathbf{b}} + \mathbf{C}$$