Números Complejos – Operaciones

$$Z = |Z|_{\theta} = \rho_{\theta} \equiv x + y\,i \quad ; \quad Z_1 = |Z_1|_{\theta_1} = (\rho_1)_{\theta_1} \equiv x_1 + y_1\,i \quad ; \quad Z_2 = |Z_2|_{\theta_2} = (\rho_2)_{\theta_2} \equiv x_2 + y_2\,i$$

	Forma Polar	**Forma Binómica** *(Cartesiana)*
Suma $Z_1 + Z_2$		$(x_1 + x_2) + i\,(y_1 + y_2)$
Diferencia $Z_1 - Z_2$		$(x_1 - x_2) + i\,(y_1 - y_2)$
Producto $Z_1 \,.\, Z_2$	$(\rho_1 \cdot \rho_2)_{\theta_1+\theta_2}$	$i^2 = -1$ *(producto algebraico)* $(x_1 x_2 - y_1 y_2) + i\,(x_1 y_2 + x_2 y_1)$
Cociente Z_1 / Z_2	$\left(\frac{\rho_1}{\rho_2}\right)_{\theta_1-\theta_2}$	$\frac{(x_1 + y_1\,i)(x_2 - y_2\,i)}{(x_2 + y_2\,i)(x_2 - y_2\,i)} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\,\frac{-x_1 y_2 + y_1 x_2}{x_2^2 + y_2^2}$
Potencia $Z^n \quad (n \in \mathbb{N})$	$(\rho_\theta)^n = (\rho^n)_{n\theta}$	**Newton: n** *pequeño* $(x + y\,i)^n = \sum_{p=0}^{p=n} \binom{n}{p} x^{n-p} (yi)^p$ $\binom{n}{p} = \frac{n!}{p!(n-p)!}$
Raiz $\sqrt[n]{Z} \quad (n \in \mathbb{N})$	$(\sqrt[n]{\rho})_{\frac{\theta+2k\pi}{n}}$; $k = 0, 1, 2, ..., n-1$	*n=2:* $\sqrt{a+bi} = x + yi$ $a + bi = (x + yi)^2 = (x^2 - y^2) + 2xyi$ $\begin{cases} x^2 - y^2 = a \\ 2xy = b \end{cases} \rightarrow (x_1, y_1) \,;\, (x_2, y_2)$

ln z **Logaritmo Neperiano**	**ln** ρ **+ i (** θ **+2k** π **)** **ln** ρ **+ i** θ (*determinación principal: k=0*) **(** θ : *en radianes* **)**	
Logaritmo base Z $\log_z \mathbf{z_1}$	$\mathbf{\log_z z_1} = \frac{\ln Z_1}{\ln Z}$ **= . . . .**	
Potencia compleja $z_1^{z_2} = Z$	**Ln z = z₂ . (lₙ z₁) = ... = a + bi** (*) $z = e^{a+bi} = e^a \cdot e^{ib} = (e^a)_b = (e^a \cdot \cos b) + i(e^a \cdot \text{sen } b)$	

(*) **Forma exponencial:** ($\forall$ z $\in$ $\mathbb{Z}$, θ: *en radianes*) $Z = |z|_\theta = \rho_\theta \equiv \rho_{\theta+2k\pi} \equiv \rho \cdot e^{i(\theta+2k\pi)}$

$Z = |z|_\theta = \rho_\theta \equiv \rho \cdot e^{i\theta}$ (*determinación principal*)

Forma trigonométrica: **(P→R)** $\Rightarrow$ $Z = |Z|_\theta = \rho_\theta \equiv (\rho \cos\theta) + i(\rho \,\text{sen}\theta) = \rho\,[\cos\theta + i(\text{sen}\theta)]$

Euler:
$$\begin{cases} 1_\theta \equiv e^{i\theta} = \cos\theta + i(\text{sen}\theta) \\ 1_{-\theta} \equiv e^{-i\theta} = \cos\theta - i(\text{sen}\theta) \end{cases} \rightarrow \begin{cases} \cos\theta = (e^{i\theta} + e^{-i\theta})/2 = \text{Ch}(i\theta) \\ \text{sen}\theta = (e^{i\theta} + e^{-i\theta})/2i = -i\,\text{Sh}(i\theta) \end{cases}$$
(θ: *en radianes* **)**

PLANO POLAR – Líneas coordenadas

$$x^2 + y^2 = R^2 \iff \rho = R \quad \textbf{circunferencias}$$

$$y = mx \iff \theta_0 = \text{Arc tg } m : \; 2 \text{ semirrectas} : \begin{cases} \theta = \theta_0 \\ \theta = \theta_0 + \pi \end{cases}$$