CÁLCULO DE PRIMITIVAS

[6.1] $\int \sqrt[3]{x}\ \sqrt[4]{2+\sqrt[3]{x^2}}\ dx$

Solución

$$\int \sqrt[3]{x}\ \sqrt[4]{2+\sqrt[3]{x^2}}\ dx = \int x^{1/3}(2+x^{2/3})^{1/4}\,dx = \begin{bmatrix} x^{2/3}=t & \Rightarrow x = t^{3/2} \\ dx=(3/2)\,t^{1/2}\,dt & \end{bmatrix} = \frac{3}{2}\int t(2+t)^{1/4}\,dt =$$

$$= \begin{bmatrix} 2+t=z^4 \\ dt = 4z^3\,dz \end{bmatrix} = 6\int (z^8-2z^4)\,dz = 6\left(\frac{z^9}{9}-\frac{2z^5}{5}\right)+cte =$$

$$=\frac{2}{3}(2+x^{2/3})^{9/4} - \frac{12}{5}(2+x^{2/3})^{5/4} + cte =$$

$$=\frac{2}{15}(2+x^{2/3})^{5/4}\,(5x^{2/3}-8)+cte$$

Indefinite integral: Show steps

$$\int \sqrt[3]{x}\ \sqrt[4]{2+x^{2/3}}\ dx = \frac{2}{15}\left(x^{2/3}+2\right)^{5/4}\left(5\,x^{2/3}-8\right) + \text{constant}$$

[6.2] $\int \frac{x}{\cos^2 x}\,dx$

Solución

Integrando por partes se obtiene:

$$\int \frac{x}{\cos^2 x}\,dx = \begin{bmatrix} x=u & \xrightarrow{d} & dx=du \\ \dfrac{dx}{\cos^2 x}=dv & \xrightarrow{\int} & \operatorname{tg} x = v \end{bmatrix} = x\operatorname{tg} x - \int \operatorname{tg} x\,dx = x\operatorname{tg} x + \ln|\cos x| + cte$$

Indefinite integral: Show steps

$$\int \frac{x}{\cos^2(x)}\,dx = x\tan(x) + \log(\cos(x)) + \text{constant}$$

$\log(x)$ is the natural logarithm »

[6.3] $\displaystyle\int \frac{(3x-7)\,dx}{x^3+x^2+4x+4}$

Solución

Descomponiendo en fracciones simples:

$$I=\int \frac{(3x-7)\,dx}{x^3+x^2+4x+4}=\int \frac{(3x-7)\,dx}{(x+1)(x^2+4)}=\int\left(\frac{A}{x+1}+\frac{Bx+C}{x^2+4}\right)dx$$

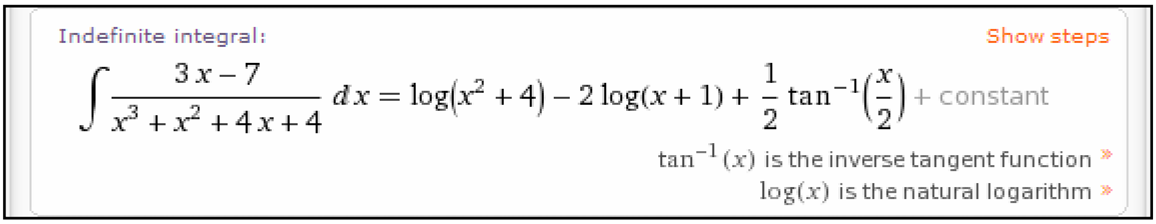
Se obtiene el sistema:

$$3x-7=A(x^2+4)+(Bx+C)(x+1)$$

$$\begin{cases} x=0 & \Rightarrow & -7=4A+C \\ x=-1 & \Rightarrow & -10=5A \\ x=1 & \Rightarrow & -4=5A+2B+2C \end{cases} \quad \Rightarrow \quad \begin{cases} C=1 \\ A=-2 \\ B=2 \end{cases}$$

$$I=\int\left(\frac{-2}{x+1}+\frac{2x+1}{x^2+4}\right)dx=-2\ln\left|x+1\right|+\ln\left|x^2+4\right|+\frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right)+cte=$$

$$=\ln\frac{x^2+4}{(x+1)^2}+\frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right)+cte$$


Indefinite integral: Show steps

$$\int \frac{3x-7}{x^3+x^2+4x+4}\,dx=\log(x^2+4)-2\log(x+1)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function »

$\log(x)$ is the natural logarithm »

[6.4] $\displaystyle\int \frac{3x+2}{(x^2-4x+8)^2}\,dx$

Solución

Integrando por Hermite:

$$I=\int \frac{3x+2}{(x^2-4x+8)^2}\,dx=\frac{Ax+B}{x^2-4x+8}+\int \frac{Cx+D}{x^2-4x+8}\,dx$$

Derivando: $\dfrac{3x+2}{(x^2-4x+8)^2}=\dfrac{A(x^2-4x+8)-(Ax+B)(2x-4)}{(x^2-4x+8)^2}+\dfrac{Cx+D}{x^2-4x+8}$

$$3x+2=8A-Ax^2-2Bx+4B+Cx^3-4Cx^2+8Cx+Dx^2-4Dx+8D$$

Por coeficientes indeterminados se obtiene el sistema:

$$\begin{cases} x^3 & \Rightarrow & 0=C \\ x^2 & \Rightarrow & 0=-A-4C+D \\ x & \Rightarrow & 3=-2B+8C-4D \\ x^0 & \Rightarrow & 2=8A+4B+8D \end{cases} \Rightarrow \begin{cases} C=0 \\ D=A \\ 3=-2B-4A \\ 2=4B+16A \end{cases} \Rightarrow \begin{cases} C=0 \\ D=1 \\ 8=8A \quad \Rightarrow \quad A=1 \\ B=-7/2 \end{cases}$$

$$I=\int\frac{3x+2}{(x^2-4x+8)^2}dx=\frac{x-7/2}{x^2-4x+8}+\int\frac{dx}{x^2-4x+8}=\frac{x-7/2}{x^2-4x+8}+\int\frac{dx}{(x-2)^2+4}=$$

$$=\frac{x-7/2}{x^2-4x+8}+\frac{1}{2}\int\frac{\frac{1}{2}dx}{\left(\frac{x}{2}-1\right)^2+1}=\frac{x-7/2}{x^2-4x+8}+\frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}-1\right)+cte$$

Indefinite integral: Show steps

$$\int\frac{3x+2}{(x^2-4x+8)^2}\,dx=\frac{1}{2}\left(\frac{2x-7}{x^2-4x+8}+\tan^{-1}\left(\frac{x-2}{2}\right)\right)+\text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function »

[6.5] Resolver las integrales:

a) $\displaystyle\int\frac{x^3-x-1}{\sqrt{x^2+2x+2}}dx$ b) $\displaystyle\int(3x^2+6x+5)\operatorname{arctg}x\,dx$

Solución

a) $\displaystyle\int\frac{x^3-x-1}{\sqrt{x^2+2x+2}}dx=(Ax^2+Bx+C)\sqrt{x^2+2x+2}+k\int\frac{dx}{\sqrt{x^2+2x+2}}$ (método alemán)

$$\frac{x^3-x-1}{\sqrt{x^2+2x+2}}=(2Ax+B)\sqrt{x^2+2x+2}+\frac{(Ax^2+Bx+C)(2x+2)}{2\cdot\sqrt{x^2+2x+2}}+\frac{k}{\sqrt{x^2+2x+2}}$$

$$x^3 - x - 1 \equiv (2Ax + B)(x^2 + 2x + 2) + (Ax^2 + Bx + C)(x + 1) + k =$$

$$= 3Ax^3 + (2B + 5A)x^2 + (4A + 3B + C)x + (2B + C + k)$$

$$\Rightarrow \quad \begin{cases} 3A & = & 1 \\ 2B + 5A & = & 0 \\ 4A + 3B + C & = & -1 \\ 2B + C + k & = & -1 \end{cases} \quad \Rightarrow \quad \begin{cases} A & = & 1/3 \\ B & = & -5/6 \\ C & = & 1/6 \\ k & = & 1/2 \end{cases}$$

$$I = \int \frac{x^3 - x - 1}{\sqrt{x^2 + 2x + 2}}\, dx = \left(\frac{x^2}{3} - \frac{5x}{6} + \frac{1}{6}\right)\sqrt{x^2 + 2x + 2} + \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + 2x + 2}} =$$

$$= \left(\frac{x^2}{3} - \frac{5x}{6} + \frac{1}{6}\right)\sqrt{x^2 + 2x + 2} + \frac{1}{2} J$$

$$J = \int \frac{dx}{\sqrt{x^2 + 2x + 2}} = \int \frac{dx}{\sqrt{(x+1)^2 + 1}} = \left\{ \begin{matrix} x + 1 & = & t \\ dx & = & dt \end{matrix} \right\} = \int \frac{dt}{\sqrt{t^2 + 1}} =$$

$$= \ln\left|t + \sqrt{t^2 + 1}\right| + cte = \ln\left|x + 1 + \sqrt{x^2 + 2x + 2}\right| + cte$$

Por consiguiente:

$$\int \frac{x^3 - x - 1}{\sqrt{x^2 + 2x + 2}}\, dx = \left(\frac{x^2}{3} - \frac{5x}{6} + \frac{1}{6}\right)\sqrt{x^2 + 2x + 2} + \frac{1}{2}\ln\left|x + 1 + \sqrt{x^2 + 2x + 2}\right| + C$$

Indefinite integral: Show steps

$$\int \frac{x^3 - x - 1}{\sqrt{x^2 + 2x + 2}}\, dx =$$

$$\frac{1}{6}\left(\sqrt{x^2 + 2x + 2}\,(2x^2 - 5x + 1) + 3\sinh^{-1}(x + 1)\right) + \text{constant}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

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b) $\displaystyle\int (3x^2 + 6x + 5)\,\text{arctg}\,x\, dx = \left\{ \begin{matrix} \text{arctg}\,x = u & \xrightarrow{d} & du = \dfrac{dx}{1 + x^2} \\ (3x^2 + 6x + 5)\,dx = dv & \xrightarrow{\int} & v = x^3 + 3x^2 + 5x \end{matrix} \right\} =$

$$= (x^3+3x^2+5x)\operatorname{arctg} x - \int \frac{x^3+3x^2+5x}{x^2+1}\,dx = (x^3+3x^2+5x)\operatorname{arctg} x - J$$

$$J = \int \frac{x^3+3x^2+5x}{x^2+1}\,dx = \int\left(x+3+\frac{4x-3}{x^2+1}\right)dx = \frac{x^2}{2}+3x+2\int\frac{2x}{x^2+1}\,dx-3\int\frac{dx}{x^2+1} =$$

$$= \frac{x^2}{2}+3x+2\ln(x^2+1)-3\operatorname{arctg} x + cte$$

Por lo tanto:

$$\int (3x^2+6x+5)\operatorname{arctg} x\,dx = (x^3+3x^2+5x)\operatorname{arctg} x - \frac{x^2}{2} - 3x - 2\ln(x^2+1) + 3\operatorname{arctg} x + C$$

$$\int (3x^2+6x+5)\operatorname{arctg} x\,dx = (x^3+3x^2+5x+3)\operatorname{arctg} x - \frac{x^2}{2} - 3x - 2\ln(x^2+1) + C$$

Indefinite integral: Show steps

$$\int (3x^2+6x+5)\tan^{-1}(x)\,dx = x^3\tan^{-1}(x) - \frac{x^2}{2} - 2\log(x^2+1) + 3x^2\tan^{-1}(x) - 3x + 5x\tan^{-1}(x) + 3\tan^{-1}(x) + \text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function »
$\log(x)$ is the natural logarithm »

[6.6] $\displaystyle\int \frac{dx}{1+\cos^2 x}$

Solución

$$\int\frac{dx}{1+\cos^2 x} = \left\{\begin{array}{l} \operatorname{tg} x = t \quad\Rightarrow\quad dx = \dfrac{dt}{1+t^2} \\ \cos^2 x = \dfrac{1}{1+t^2} \end{array}\right\} = \int \frac{\dfrac{dt}{1+t^2}}{1+\dfrac{1}{1+t^2}} = \int\frac{dt}{t^2+2} =$$

$$= \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{t}{\sqrt{2}}\right) + cte = \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{\operatorname{tg} x}{\sqrt{2}}\right) + cte$$

Indefinite integral: Show steps

$$\int \frac{1}{1+\cos^2(x)}\,dx = \frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}} + \text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function »

[6.7] $\int \frac{\operatorname{ch} 3x}{5+2\operatorname{sh} 3x}dx$

Solución

$$\int \frac{\operatorname{ch} 3x}{5+2\operatorname{sh} 3x}dx = \left\{ \begin{array}{c} 5+2\operatorname{sh} 3x = t \\ 6\operatorname{ch} 3x\, dx = dt \end{array} \right\} = \frac{1}{6}\int \frac{dt}{t} = \frac{1}{6}\ln|t| + cte = \frac{1}{6}\ln|5+2\operatorname{sh} 3x| + cte$$

Indefinite integral: Show steps

$$\int \frac{\cosh(3x)}{5+2\sinh(3x)}\,dx = \frac{1}{6}\log(2\sinh(3x)+5) + \text{constant}$$

$\cosh(x)$ is the hyperbolic cosine function »
$\log(x)$ is the natural logarithm »
$\sinh(x)$ is the hyperbolic sine function »

[6.8] Calcular las siguientes integrales:

a) $\int \frac{dx}{\sqrt{(a^2-x^2)^3}}$ b) $\int \frac{dx}{(1+x^2)^{3/2}}$

Solución

a) $$\int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \left[\begin{array}{c} x = a\operatorname{sen} t \\ dx = a\cos t\, dt \end{array} \right] = \frac{1}{a^2}\int \frac{\cos t\, dt}{\cos^3 t} = \frac{1}{a^2}\int \frac{dt}{\cos^2 t} = \frac{1}{a^2}\operatorname{tg} t + cte =$$

$$= \frac{1}{a^2}\operatorname{tg}\arcsin\frac{x}{a} + cte = \frac{1}{a^2}\frac{x}{\sqrt{a^2-x^2}} + cte$$

Indefinite integral: Show steps

$$\int \frac{1}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{a^2\sqrt{a^2-x^2}} + \text{constant}$$

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b) $$\int \frac{dx}{(1+x^2)^{3/2}} = \int (1+x^2)^{-3/2}dx = \left[\begin{array}{c} x^2 = t \\ dx = (1/2)t^{-1/2}dt \end{array} \right] = (1/2)\int t^{-1/2}(1+t)^{-3/2}dt =$$

$$= (1/2)\int t^{-1/2}\cdot t^{-3/2}\left(\frac{1+t}{t}\right)^{-3/2}dt = (1/2)\int t^{-2}\left(\frac{1+t}{t}\right)^{-3/2}dt = \left[\begin{array}{c} \frac{1+t}{t} = z \\ dt = \frac{-dz}{(z-1)^2} \end{array} \right] =$$

$$= (-1/2)\int \frac{(z-1)^2 \cdot z^{-3/2}}{(z-1)^2}dz = (-1/2)\int z^{-3/2}dz = \frac{1}{2}\frac{z^{-1/2}}{(1/2)} + cte = \frac{1}{\sqrt{z}} + cte =$$

$$= \sqrt{\frac{x^2}{1+x^2}} + cte = \frac{x}{\sqrt{1+x^2}} + cte$$

Indefinite integral: Show steps

$$\int \frac{1}{(1+x^2)^{3/2}}\,dx = \frac{x}{\sqrt{x^2+1}} + \text{constant}$$

[6.9] $\int x\sqrt{x^2+2x+2}\,dx$

Solución

$$I = \int x\sqrt{x^2+2x+2}\,dx = \int x\sqrt{(x+1)^2+1}\,dx = \begin{bmatrix} x+1 = \operatorname{sh} t \\ dx = \operatorname{ch} t\,dt \end{bmatrix} =$$

$$= \int (\operatorname{sh} t - 1)\sqrt{\operatorname{sh}^2 t + 1}\,\operatorname{ch} t\,dt = \int (\operatorname{sh} t - 1)\operatorname{ch}^2 t\,dt = \int \operatorname{sh} t\,\operatorname{ch}^2 t\,dt - \int \operatorname{ch}^2 t\,dt =$$

$$= \frac{1}{3}\operatorname{ch}^3 t - \frac{1}{2}\int(\operatorname{ch} 2t + 1)\,dt = \frac{1}{3}\operatorname{ch}^3 t - \frac{1}{4}\operatorname{sh} 2t - \frac{1}{2}t + cte =$$

$$= \frac{1}{3}\operatorname{ch}^3 t - \frac{1}{2}(\operatorname{sh} t\operatorname{ch} t + t) + cte = \operatorname{ch} t\left(\frac{1}{3}\operatorname{sh}^2 t + \frac{1}{3} - \frac{1}{2}\operatorname{sh} t\right) - \frac{1}{2}t + cte =$$

$$= \sqrt{\operatorname{sh}^2 t + 1}\left(\frac{1}{3}\operatorname{sh}^2 t + \frac{1}{3} - \frac{1}{2}\operatorname{sh} t\right) - \frac{1}{2}t + cte$$

Deshaciendo el cambio de variable:

$$I = \sqrt{x^2+2x+2}\left[\frac{1}{3}(x^2+2x+2) - \frac{1}{2}(x+1)\right] - \frac{1}{2}\operatorname{arg\,sh}(x+1) + cte =$$

$$= \sqrt{x^2+2x+2}\left[\frac{1}{3}x^2 + \frac{1}{6}x + \frac{1}{6}\right] - \frac{1}{2}\ln(x+1+\sqrt{x^2+2x+2}) + cte$$

Indefinite integral: Show steps

$$\int x\sqrt{x^2+2x+2}\,dx =$$

$$\frac{1}{6}\left(\sqrt{x^2+2x+2}\,(2x^2+x+1) - 3\sinh^{-1}(x+1)\right) + \text{constant}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function »

[6.10] $\int x^8 \cdot \sqrt[3]{1+x^3}\, dx$

Solución

$$\int x^8 \cdot \sqrt[3]{1+x^3}\, dx = \int x^8 \cdot (1+x^3)^{1/3}\, dx = \begin{bmatrix} m=8 & n=3 \\ p=\dfrac{1}{3} \notin \mathbb{Z} & \dfrac{m+1}{n}=3\in\mathbb{Z} \end{bmatrix} = \begin{bmatrix} \text{binomia} \\ 2^{\text{o}}\ \text{caso} \end{bmatrix} =$$

$$= \left\{ \begin{array}{l} x^3 = z \\ x = z^{1/3} \quad \Rightarrow \quad dx = \dfrac{1}{3} z^{-2/3} dz \end{array} \right\} = \int z^{8/3}(1+z)^{1/3}\frac{1}{3}z^{-2/3}dz = \frac{1}{3}\int z^2(1+z)^{1/3}dz =$$

$$= \left\{ \begin{array}{l} 1+z=t^3 \\ dz = 3t^2dt \end{array} \right\} = \frac{1}{3}\int (t^3-1)^2\, t\cdot 3t^2dt = \int (t^6-2t^3+1)t^3\, dt = \int (t^9-2t^6+t^3)\, dt =$$

$$= \frac{t^{10}}{10} - \frac{2t^7}{7} + \frac{t^4}{4} + cte = \frac{(1+z)^{10/3}}{10} - \frac{2(1+z)^{7/3}}{7} + \frac{(1+z)^{4/3}}{4} + cte =$$

$$= \frac{(1+x^3)^{10/3}}{10} - \frac{2(1+x^3)^{7/3}}{7} + \frac{(1+x^3)^{4/3}}{4} + cte$$

Indefinite integral: Show steps

$$\int x^8 \sqrt[3]{1+x^3}\, dx = \frac{1}{140}\left(x^3+1\right)^{4/3}\left(14x^6 - 12x^3 + 9\right) + \text{constant}$$

[6.11] $\int \dfrac{dx}{\sqrt{x^3}\ \sqrt[3]{1+\sqrt[4]{x^3}}}$

Solución

Se trata de una integral binomia de 3ª especie. Escribiéndola en forma estándar:

$$I = \int \frac{dx}{\sqrt{x^3}\ \sqrt[3]{1+\sqrt[4]{x^3}}} = \int x^{-3/2}(1+x^{3/4})^{-1/3}dx$$

Sustituyendo: $x^{3/4} = t \quad \Rightarrow \quad x = t^{4/3} \quad \Rightarrow \quad dx = \dfrac{4}{3}t^{1/3}dt \quad \Rightarrow \quad I = \dfrac{4}{3}\int t^{-5/3}(1+t)^{-1/3}dt$

Multiplicando y dividiendo por $t^{-1/3}$: $I = \dfrac{4}{3}\int t^{-2}\left(\dfrac{1+t}{t}\right)^{-1/3} dt$

Con el nuevo cambio de variable:

$$\frac{1+t}{t}=z \quad \Rightarrow \quad t=\frac{1}{z-1} \quad \Rightarrow \quad dt=-\frac{dz}{(z-1)^2}$$

$$I=-\frac{4}{3}\int (z-1)^2 z^{-1/3}\frac{dz}{(z-1)^2}=-\frac{4}{3}\int z^{-1/3}dz=-2z^{2/3}+cte$$

Finalmente, restituyendo variables: $I=-2\left(\frac{1+x^{3/4}}{x^{3/4}}\right)^{2/3}+cte$

Indefinite integral: Show steps

$$\int \frac{1}{x^{3/2}\sqrt[3]{1+x^{3/4}}}\,dx=-\frac{2\left(x^{3/4}+1\right)^{2/3}}{\sqrt{x}}+\text{constant}$$

[6.12] $\int \operatorname{sen}(\ln x)\,dx$

Solución

$$I=\int \operatorname{sen}(\ln x)\,dx=\left\{\begin{array}{lll}\operatorname{sen}(\ln x)=u & \xrightarrow{d} & du=\frac{1}{x}\cos(\ln x)\,dx\\ dx=dv & \xrightarrow{\int} & v=x\end{array}\right\}=$$

$$=x\operatorname{sen}(\ln x)-\int \cos(\ln x)\,dx=x\operatorname{sen}(\ln x)-J$$

$$J=\int \cos(\ln x)\,dx=\left\{\begin{array}{lll}\cos(\ln x)=u & \xrightarrow{d} & du=-\frac{1}{x}\operatorname{sen}(\ln x)\,dx\\ dx=dv & \xrightarrow{\int} & v=x\end{array}\right\}=$$

$$=x\cos(\ln x)+\int \operatorname{sen}(\ln x)\,dx=x\cos(\ln x)+I$$

Por lo tanto: $I=x\operatorname{sen}(\ln x)-x\cos(\ln x)-I \quad \Rightarrow \quad 2I=x[\operatorname{sen}(\ln x)-\cos(\ln x)]$

$$I=\frac{x}{2}[\operatorname{sen}(\ln x)-\cos(\ln x)]+cte$$

Indefinite integral: Show steps

$$\int \sin(\log(x))\,dx=\frac{1}{2}\,x\sin(\log(x))-\frac{1}{2}\,x\cos(\log(x))+\text{constant}$$

$\log(x)$ is the natural logarithm »

[6.13] $\displaystyle\int \frac{e^x+1}{e^x-4+4e^{-x}}dx$

Solución

$$\int \frac{e^x+1}{e^x-4+4e^{-x}}dx = \left\{ \begin{aligned} e^x &= t \\ dx &= \frac{dt}{t} \end{aligned} \right\} = \int \frac{t+1}{t-4+\frac{4}{t}}\cdot\frac{dt}{t} = \int \frac{t(t+1)}{(t^2-4t+4)t}dt = \int \frac{t+1}{t^2-4t+4}dt$$

$$\frac{t+1}{t^2-4t+4} = \frac{t+1}{(t-2)^2} = \frac{A}{(t-2)^2}+\frac{B}{t-2}$$

$$t+1 \equiv A+B(t-2) = Bt+(A-2B) \quad \Rightarrow \quad \left\{ \begin{aligned} B &= 1 \\ A-2B &= 1 \end{aligned} \right. \quad \Rightarrow \quad \left\{ \begin{aligned} A &= 3 \\ B &= 1 \end{aligned} \right.$$

$$\int \frac{e^x+1}{e^x-4+4e^{-x}}dx = 3\int \frac{dt}{(t-2)^2}+\int \frac{dt}{t-2} = -\frac{3}{t-2}+\ln|t-2|+cte = \ln\left|e^x-2\right|-\frac{3}{e^x-2}+cte$$

Indefinite integral: Show steps

$$\int \frac{\exp(x)+1}{\exp(x)-4+4\exp(-x)}\,dx = \log(2-e^x)-\frac{3}{e^x-2} + \text{constant}$$

$\log(x)$ is the natural logarithm »

[6.14] $\displaystyle\int \frac{\sqrt{x}+2\cdot\sqrt[6]{x^5}}{\sqrt[6]{x^5}\left(1+\sqrt[3]{x}\right)}dx$

Solución

$$\int \frac{\sqrt{x}+2\cdot\sqrt[6]{x^5}}{\sqrt[6]{x^5}\left(1+\sqrt[3]{x}\right)}dx = \int \frac{x^{1/2}+2\cdot x^{5/6}}{x^{5/6}\left(1+x^{1/3}\right)}dx = \left\{ \begin{aligned} &\mu = m.c.m.(2,3,6) = 6 \\ &x = t^6 \quad \Rightarrow \quad dx = 6t^5dt \end{aligned} \right\} =$$

$$= \int \frac{t^3+2t^5}{t^5(1+t^2)}\cdot 6t^5dt = 6\int \frac{2t^5+t^3}{t^2+1}dt = 6\int\left(2t^3-t+\frac{t}{t^2+1}\right)dt =$$

$$= 6\left[\frac{2t^4}{4}-\frac{t^2}{2}+\frac{1}{2}\ln(1+t^2)\right]+cte = 3\left[t^4-t^2+\ln\left|1+t^2\right|\right]+cte =$$

$$= 3\left[x^{2/3} - x^{1/3} + \ln\left|1 + x^{1/3}\right|\right] + cte = 3\left[\sqrt[3]{x^2} - \sqrt[3]{x} + \ln\left|1 + \sqrt[3]{x}\right|\right] + cte$$