



[2.2] Determinar  $x \in \mathbb{R}$  tal que:  $\text{sh}^4 x - 2\text{ch}^2 x - 1 = 0$

*Solución:*

Teniendo presente la igualdad fundamental de la trigonometría hiperbólica:

$$\text{ch}^2 x - \text{sh}^2 x = 1$$

$$\text{sh}^4 x - 2\text{ch}^2 x - 1 = \text{sh}^4 x - 2\text{sh}^2 x - 3 = 0 \Rightarrow$$

$$\text{sh}^2 x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \text{ (absurdo)} \end{cases}$$

$$\begin{cases} \text{sh} x = \sqrt{3} & \Rightarrow x = \arg \text{sh}(\sqrt{3}) = \ln(\sqrt{3} + 2) \\ \text{sh} x = -\sqrt{3} & \Rightarrow x = \arg \text{sh}(-\sqrt{3}) = \ln(-\sqrt{3} + 2) \end{cases}$$

**Input:** *Mathematica form*

$\sinh^4(x) - 2 \cosh^2(x) - 1 = 0$

cosh(x) is the hyperbolic cosine function >  
sinh(x) is the hyperbolic sine function >

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**Root plot:**

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**Alternate form:**

$\cosh(2x) + 2 = \sinh^4(x)$

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**Solutions:** *Approximate forms*

$x = i\pi n - \frac{i\pi}{2}, \quad n \in \mathbb{Z}$

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$x = \log(2 - \sqrt{3}) + i\pi n, \quad n \in \mathbb{Z}$

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$x = \log(2 + \sqrt{3}) + i\pi n, \quad n \in \mathbb{Z}$

$\mathbb{Z}$  is the set of integers >  
log(x) is the natural logarithm >

[2.3] Usando las definiciones de las funciones hiperbólicas, simplificar la expresión:

$$\frac{\operatorname{ch}(\ln x) + \operatorname{sh}(\ln x)}{\operatorname{ch}(\ln x) - \operatorname{sh}(\ln x)}$$

Solución:

$$\begin{cases} \operatorname{ch}(\ln x) = \frac{e^{\ln x} + e^{-\ln x}}{2} = \frac{x + \frac{1}{x}}{2} = \frac{x^2 + 1}{2x} \\ \operatorname{sh}(\ln x) = \frac{e^{\ln x} - e^{-\ln x}}{2} = \frac{x - \frac{1}{x}}{2} = \frac{x^2 - 1}{2x} \end{cases}$$

Por lo tanto: 
$$\frac{\operatorname{ch}(\ln x) + \operatorname{sh}(\ln x)}{\operatorname{ch}(\ln x) - \operatorname{sh}(\ln x)} = \frac{\frac{x^2 + 1}{2x} + \frac{x^2 - 1}{2x}}{\frac{x^2 + 1}{2x} - \frac{x^2 - 1}{2x}} = \frac{2x^2}{2} = x^2$$

**Input:** Mathematica form

$$\frac{\cosh(\log(x)) + \sinh(\log(x))}{\cosh(\log(x)) - \sinh(\log(x))}$$

cosh(x) is the hyperbolic cosine function >  
 log(x) is the natural logarithm >  
 sinh(x) is the hyperbolic sine function >

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**Exact result:**

$$\frac{\frac{x^2 - 1}{2x} + \frac{x^2 + 1}{2x}}{\frac{x^2 + 1}{2x} - \frac{x^2 - 1}{2x}}$$


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**Plots:**

(x from -1 to 1)

(x from -6 to 6)

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**Alternate forms:**

$$x^2$$

$$\frac{\frac{x^2 - 1}{2x} + \frac{x^2 + 1}{2x}}{\frac{x^2 + 1}{2x} - \frac{x^2 - 1}{2x}}$$

[2.4] Resolver en  $\mathbb{R}$  la ecuación:  $2^{\ln \operatorname{sh} x + \ln \operatorname{ch} x} = 4^{\ln \sqrt{e}}$

Solución:

$$2^{\ln \operatorname{sh} x + \ln \operatorname{ch} x} = 4^{\ln \sqrt{e}} \Rightarrow 2^{\ln(\operatorname{sh} x \cdot \operatorname{ch} x)} = 2^{2 \ln \sqrt{e}} \Rightarrow \ln(\operatorname{sh} x \cdot \operatorname{ch} x) = \ln e \Rightarrow$$

$$\Rightarrow \operatorname{sh} x \cdot \operatorname{ch} x = e \Rightarrow \frac{1}{2} \operatorname{sh} 2x = e \Rightarrow \operatorname{sh} 2x = 2e \Rightarrow$$

$$\Rightarrow 2x = \operatorname{argsh}(2e) = \ln\left(2e + \sqrt{1 + 4e^2}\right) \Rightarrow x = \frac{1}{2} \ln\left(2e + \sqrt{1 + 4e^2}\right)$$

Input: Mathematica form

$$2^{\log(\sinh(x)) + \log(\cosh(x))} = 4^{\log(\sqrt{e})}$$

$\cosh(x)$  is the hyperbolic cosine function »  
 $\log(x)$  is the natural logarithm »  
 $\sinh(x)$  is the hyperbolic sine function »

Result:

$$2^{\log(\sinh(x)) + \log(\cosh(x))} = 2$$

Plot:

Solutions: Exact forms

$$x \approx 1.19732$$

$$x \approx -1.19732 + 1.5708 i$$

$$x \approx -1.19732 - 1.5708 i$$

Input: Mathematica form

$$\frac{1}{2} \log\left(2e + \sqrt{1 + 4e^2}\right)$$

$\log(x)$  is the natural logarithm »

Decimal approximation: More digits

$$1.1973237358068877168262600915442339151706112948783834331129\dots$$

Alternate form:

$$\frac{1}{2} \sinh^{-1}(2e)$$

$\sinh^{-1}(x)$  is the inverse hyperbolic sine function »

[2.5] Resolver en  $\mathbb{R}$  el sistema: 
$$\begin{cases} \text{sh}(x) + \text{ch}(y) = 1 \\ \text{ch}(x) + \text{sh}(y) = 1 \end{cases}$$

*Solución:*

Elevando al cuadrado las dos ecuaciones se obtiene:

$$\begin{cases} \text{sh}^2(x) + 2\text{sh}(x)\text{ch}(y) + \text{ch}^2(y) = 1 \\ \text{ch}^2(x) + 2\text{ch}(x)\text{sh}(y) + \text{sh}^2(y) = 1 \end{cases}$$

Restando miembro a miembro:

$$-1 + 2[\text{sh}(x)\text{ch}(y) - \text{ch}(x)\text{sh}(y)] + 1 = 0 \Rightarrow \text{sh}(x - y) = 0 \Rightarrow x - y = 0 \Rightarrow x = y$$

Sustituyendo en la primera ecuación:

$$\text{sh}(x) + \text{ch}(x) = 1 \Rightarrow e^x = 1 \Rightarrow \boxed{x = y = 0}$$

The screenshot shows the Wolfram Mathematica interface. The input is: `solve {sinh(x) + cosh(y) = 1, cosh(x) + sinh(y) = 1} for x, y`. Below the input, it says: `cosh(x) is the hyperbolic cosine function` and `sinh(x) is the hyperbolic sine function`. The result is: `y = 2 i pi c1 and x = 2 i pi c2 and (c1 | c2) in Z`. At the bottom, it says: `Computed by: Wolfram Mathematica` and `Download as: PDF | Live Mathematica`.

[2.6] Demostrar que  $\text{sh}(a + bi) = \text{sh} a \cos b + i \text{ch} a \text{sen} b$ . A partir de este resultado, calcular:

$$\text{sh}\left(1 + \frac{\pi}{2}i\right)$$

*Solución:*

Recuérdese que, en virtud de las fórmulas de Euler, se tiene:

$$\begin{cases} \text{sh}(ib) = \frac{e^{ib} - e^{-ib}}{2} = \frac{(\cos b + i \text{sen} b) - (\cos b - i \text{sen} b)}{2} = i \text{sen} b \\ \text{ch}(ib) = \frac{e^{ib} + e^{-ib}}{2} = \frac{(\cos b + i \text{sen} b) + (\cos b - i \text{sen} b)}{2} = \cos b \end{cases}$$

$$\operatorname{sh}(a + bi) = \operatorname{sh} a \operatorname{ch}(bi) + \operatorname{ch} a \operatorname{sh}(bi) = \operatorname{sh} a \cos b + i \operatorname{ch} a \operatorname{sen} b$$

$$\operatorname{sh}\left(1 + \frac{\pi}{2}i\right) = \operatorname{sh} 1 \cos \frac{\pi}{2} + i \operatorname{ch} 1 \operatorname{sen} \frac{\pi}{2} = i \operatorname{ch} 1 = \frac{e + e^{-1}}{2} i = \left(\frac{e^2 + 1}{2e}\right) i$$

Input: Mathematica form

$\operatorname{sinh}(a + b i)$

*i* is the imaginary unit »  
sinh(x) is the hyperbolic sine function »

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Alternate form assuming all variables are real:

$\operatorname{sinh}(a) \cos(b) + i \operatorname{cosh}(a) \sin(b)$

cosh(x) is the hyperbolic cosine function »

Input: Mathematica form

$\operatorname{sinh}\left(1 + \frac{\pi i}{2}\right)$

*i* is the imaginary unit »  
sinh(x) is the hyperbolic sine function »

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Exact result:

$i \operatorname{cosh}(1)$

cosh(x) is the hyperbolic cosine function »

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Decimal approximation: More digits

1.543080634815243778477905620757061682601529112365863704737... *i*