FUNCIONES HIPERBÓLICAS

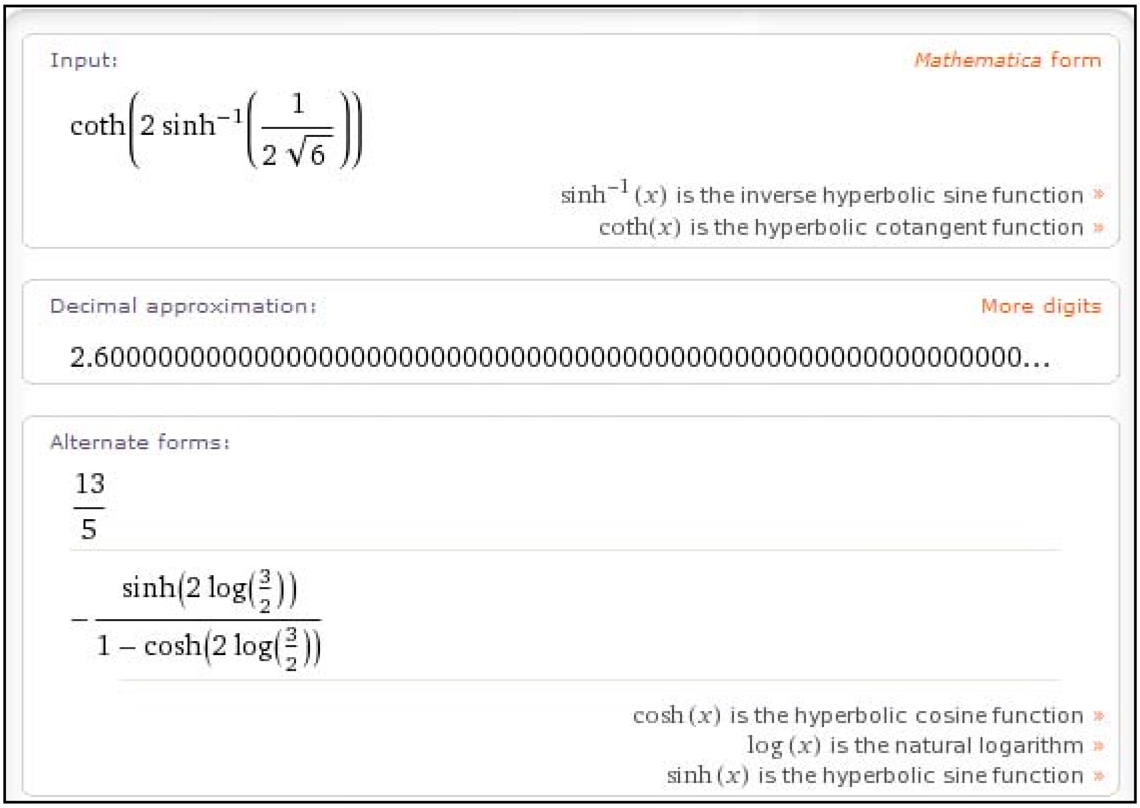
[2.1] Obtener, **sin utilizar calculadora** , $\coth 2x$, siendo $\operatorname{sh} x = \dfrac{1}{2\sqrt{6}}$

Solución:

$$x = \operatorname{arg\,sh}\frac{1}{2\sqrt{6}} = \ln\left(\frac{1}{2\sqrt{6}} + \sqrt{\frac{1}{24}+1}\right) = \ln\left(\frac{1}{2\sqrt{6}} + \frac{5}{2\sqrt{6}}\right) = \ln\frac{6}{2\sqrt{6}} = \ln\frac{\sqrt{6}}{2}$$

$$\coth 2x = \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}} = \frac{e^{2\ln(\sqrt{6}/2)}+e^{-2\ln(\sqrt{6}/2)}}{e^{2\ln(\sqrt{6}/2)}-e^{-2\ln(\sqrt{6}/2)}} = \frac{(6/4)+(4/6)}{(6/4)-(4/6)} =$$

$$= \frac{(3/2)+(2/3)}{(3/2)-(2/3)} = \frac{(9+4)/6}{(9-4)/6} = \frac{13}{5}$$



[2.2] Determinar $x \in \mathbb{R}$ tal que: $\operatorname{sh}^4 x - 2\operatorname{ch}^2 x - 1 = 0$

Solución:

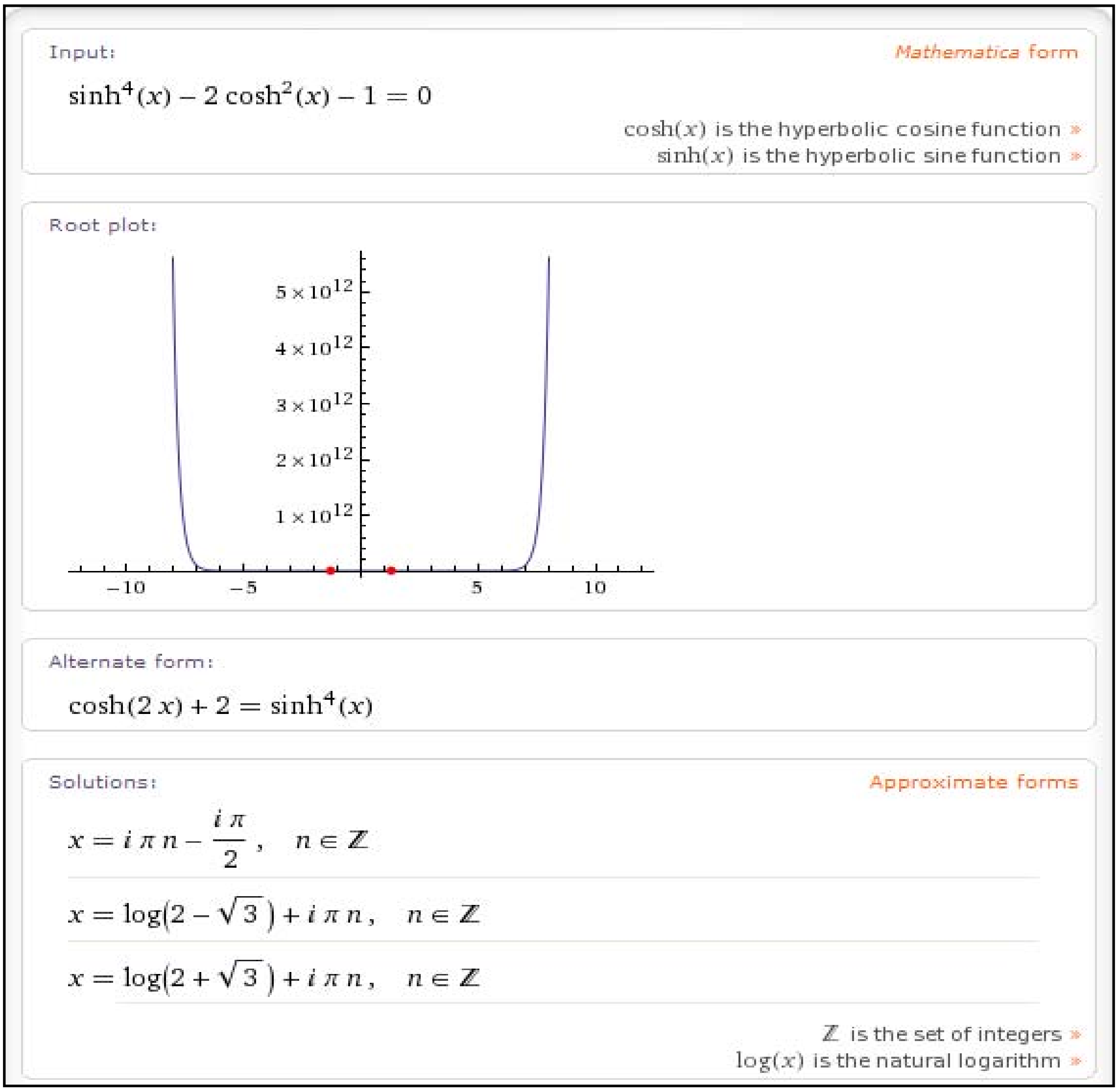
Teniendo presente la igualdad fundamental de la trigonometría hiperbólica:

$$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$$

$$\operatorname{sh}^4 x - 2\operatorname{ch}^2 x - 1 = \operatorname{sh}^4 x - 2\operatorname{sh}^2 x - 3 = 0 \quad \Rightarrow$$

$$\operatorname{sh}^2 x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \\ -1 \quad \text{(absurdo)} \end{cases}$$

$$\begin{cases} \operatorname{sh} x = \sqrt{3} & \Rightarrow \quad x = \operatorname{arg\,sh}(\sqrt{3}) = \ln(\sqrt{3}+2) \\ \operatorname{sh} x = -\sqrt{3} & \Rightarrow \quad x = \operatorname{arg\,sh}(-\sqrt{3}) = \ln(-\sqrt{3}+2) \end{cases}$$



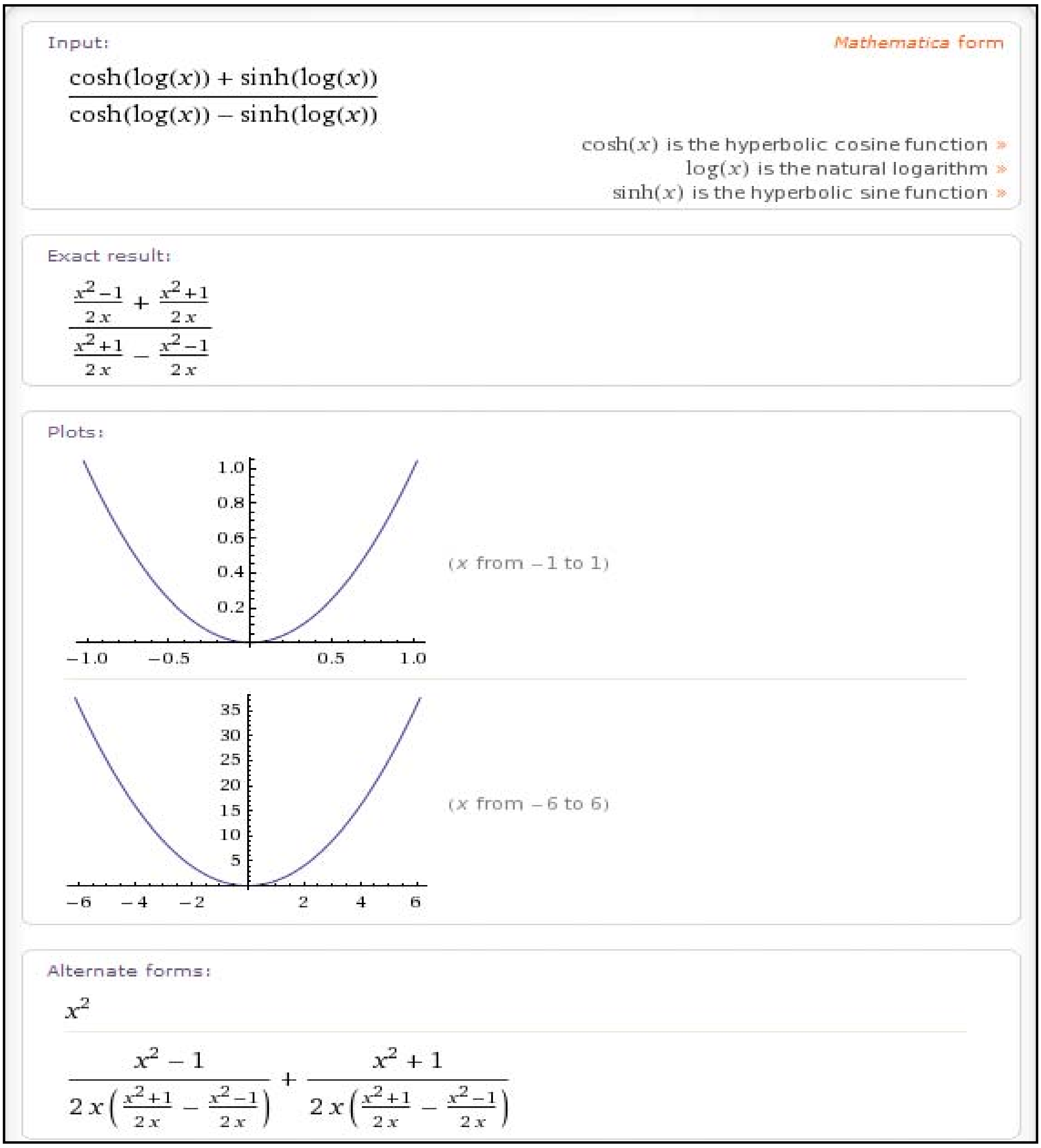
[2.3] Usando las definiciones de las funciones hiperbólicas, simplificar la expresión:

$$\frac{\mathrm{ch}(\ln x)+\mathrm{sh}(\ln x)}{\mathrm{ch}(\ln x)-\mathrm{sh}(\ln x)}$$

Solución:

$$\begin{cases} \mathrm{ch}(\ln x)=\dfrac{e^{\ln x}+e^{-\ln x}}{2}=\dfrac{x+\dfrac{1}{x}}{2}=\dfrac{x^2+1}{2x} \\ \mathrm{sh}(\ln x)=\dfrac{e^{\ln x}-e^{-\ln x}}{2}=\dfrac{x-\dfrac{1}{x}}{2}=\dfrac{x^2-1}{2x} \end{cases}$$

Por lo tanto: $$\frac{\mathrm{ch}(\ln x)+\mathrm{sh}(\ln x)}{\mathrm{ch}(\ln x)-\mathrm{sh}(\ln x)}=\frac{\dfrac{x^2+1}{2x}+\dfrac{x^2-1}{2x}}{\dfrac{x^2+1}{2x}-\dfrac{x^2-1}{2x}}=\frac{2x^2}{2}=x^2$$



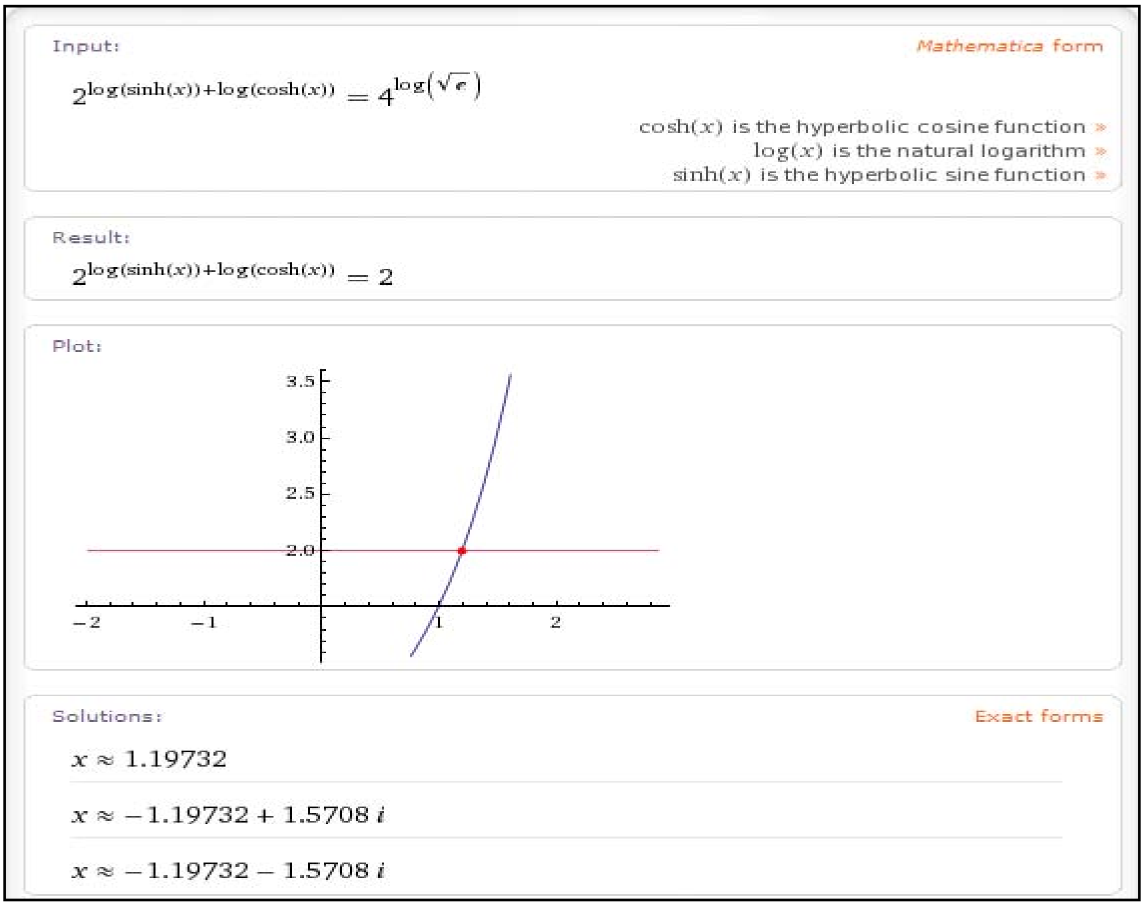
[2.4] Resolver en $\mathbb{R}$ la ecuación: $2^{\ln \operatorname{sh} x + \ln \operatorname{ch} x} = 4^{\ln \sqrt{e}}$

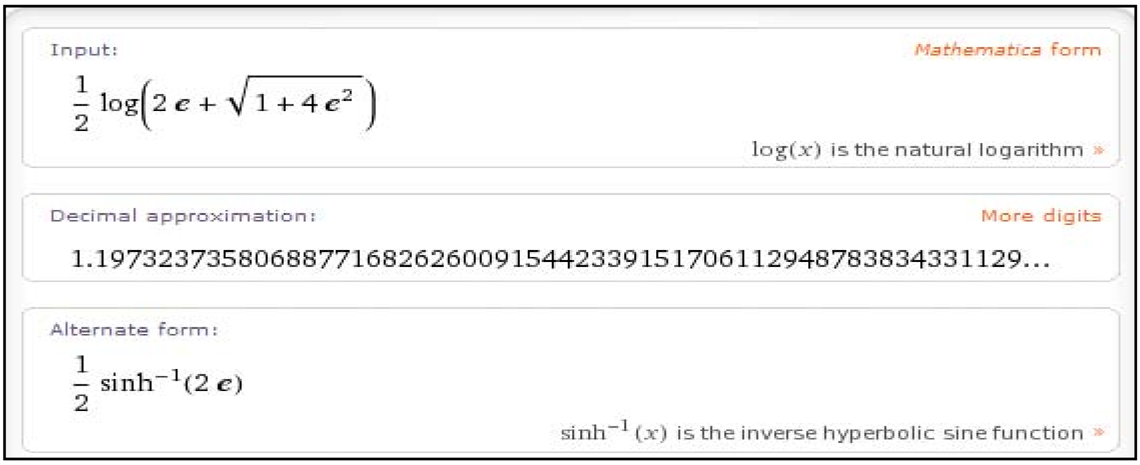
Solución:

$$2^{\ln \operatorname{sh} x + \ln \operatorname{ch} x} = 4^{\ln \sqrt{e}} \Rightarrow 2^{\ln(\operatorname{sh} x \cdot \operatorname{ch} x)} = 2^{2\ln \sqrt{e}} \Rightarrow \ln(\operatorname{sh} x \cdot \operatorname{ch} x) = \ln e \Rightarrow$$

$$\Rightarrow \operatorname{sh} x \cdot \operatorname{ch} x = e \Rightarrow \frac{1}{2}\operatorname{sh} 2x = e \Rightarrow \operatorname{sh} 2x = 2e \Rightarrow$$

$$\Rightarrow 2x = \operatorname{arg\,sh}(2e) = \ln\left(2e + \sqrt{1+4e^2}\right) \Rightarrow x = \frac{1}{2}\ln\left(2e + \sqrt{1+4e^2}\right)$$





[2.5] Resolver en $\mathbb{R}$ el sistema: $\begin{cases} \text{sh}(x)+\text{ch}(y)=1 \\ \text{ch}(x)+\text{sh}(y)=1 \end{cases}$

Solución:

Elevando al cuadrado las dos ecuaciones se obtiene:

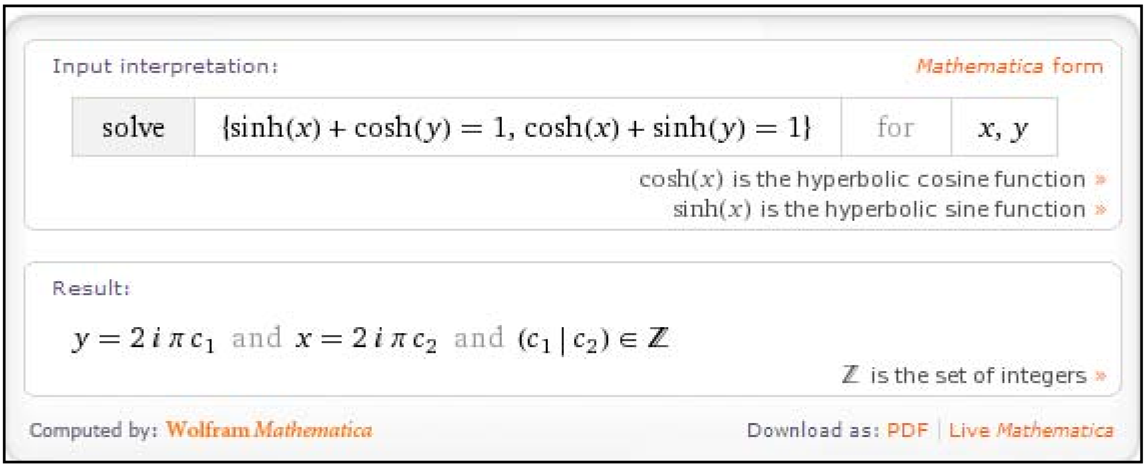
$$\begin{cases} \text{sh}^2(x)+2\,\text{sh}(x)\,\text{ch}(y)+\text{ch}^2(y)=1 \\ \text{ch}^2(x)+2\,\text{ch}(x)\,\text{sh}(y)+\text{sh}^2(y)=1 \end{cases}$$

Restando miembro a miembro:

$$-1+2[\text{sh}(x)\,\text{ch}(y)-\text{ch}(x)\,\text{sh}(y)]+1=0 \quad\Rightarrow\quad \text{sh}(x-y)=0 \quad\Rightarrow\quad x-y=0 \quad\Rightarrow\quad x=y$$

Sustituyendo en la primera ecuación:

$$\text{sh}(x)+\text{ch}(x)=1 \quad\Rightarrow\quad e^x=1 \quad\Rightarrow\quad \boxed{x=y=0}$$



[2.6] Demostrar que $\text{sh}(a+bi)=\text{sh}\,a\cos b+i\,\text{ch}\,a\,\text{sen}\,b$. A partir de este resultado, calcular:

$$\text{sh}\left(1+\frac{\pi}{2}i\right)$$

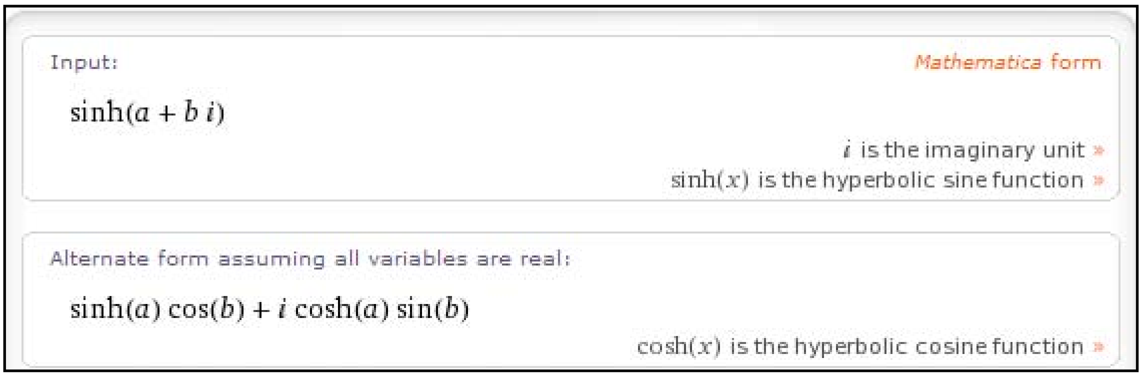
Solución:

Recuérdese que, en virtud de las fórmulas de Euler, se tiene:

$$\begin{cases} \text{sh}(ib)=\dfrac{e^{ib}-e^{-ib}}{2}=\dfrac{(\cos b+i\,\text{sen}\,b)-(\cos b-i\,\text{sen}\,b)}{2}=i\,\text{sen}\,b \\ \text{ch}(ib)=\dfrac{e^{ib}+e^{-ib}}{2}=\dfrac{(\cos b+i\,\text{sen}\,b)+(\cos b-i\,\text{sen}\,b)}{2}=\cos b \end{cases}$$

$$\operatorname{sh}(a+bi)=\operatorname{sh}a\operatorname{ch}(bi)+\operatorname{ch}a\operatorname{sh}(bi)=\operatorname{sh}a\cos b+i\operatorname{ch}a\operatorname{sen}b$$

$$\operatorname{sh}\left(1+\frac{\pi}{2}i\right)=\operatorname{sh}1\cos\frac{\pi}{2}+i\operatorname{ch}1\operatorname{sen}\frac{\pi}{2}=i\operatorname{ch}1=\frac{e+e^{-1}}{2}i=\left(\frac{e^2+1}{2e}\right)i$$


Input: Mathematica form

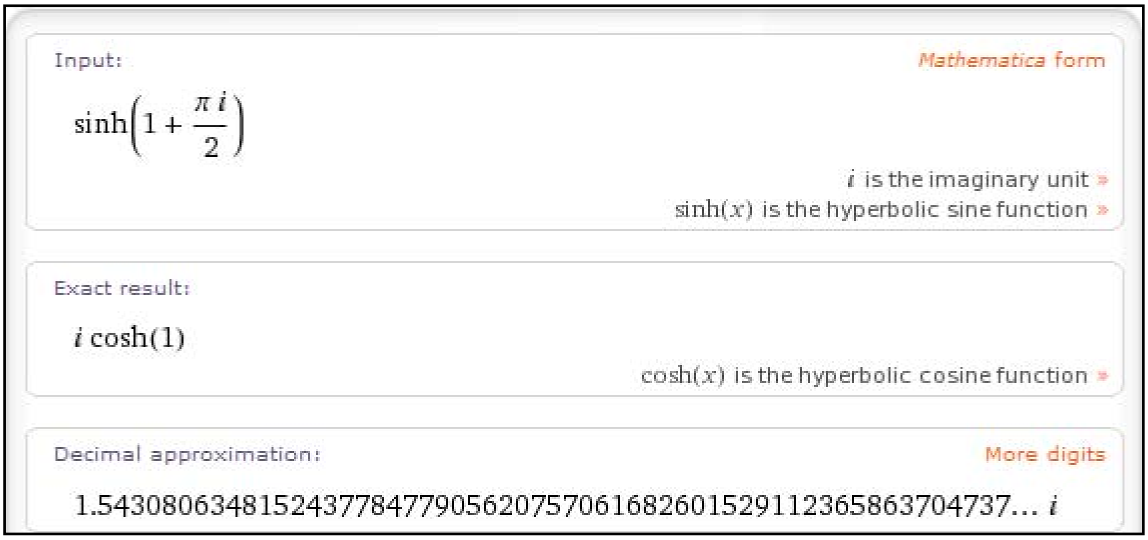
$\sinh(a+b\,i)$

i is the imaginary unit »
$\sinh(x)$ is the hyperbolic sine function »

Alternate form assuming all variables are real:

$\sinh(a)\cos(b)+i\cosh(a)\sin(b)$

$\cosh(x)$ is the hyperbolic cosine function »


Input: Mathematica form

$\sinh\left(1+\frac{\pi\,i}{2}\right)$

i is the imaginary unit »
$\sinh(x)$ is the hyperbolic sine function »

Exact result:

$i\cosh(1)$

$\cosh(x)$ is the hyperbolic cosine function »

Decimal approximation: More digits

$1.5430806348152437784779056207570616826015291123658637047370\ldots\, i$