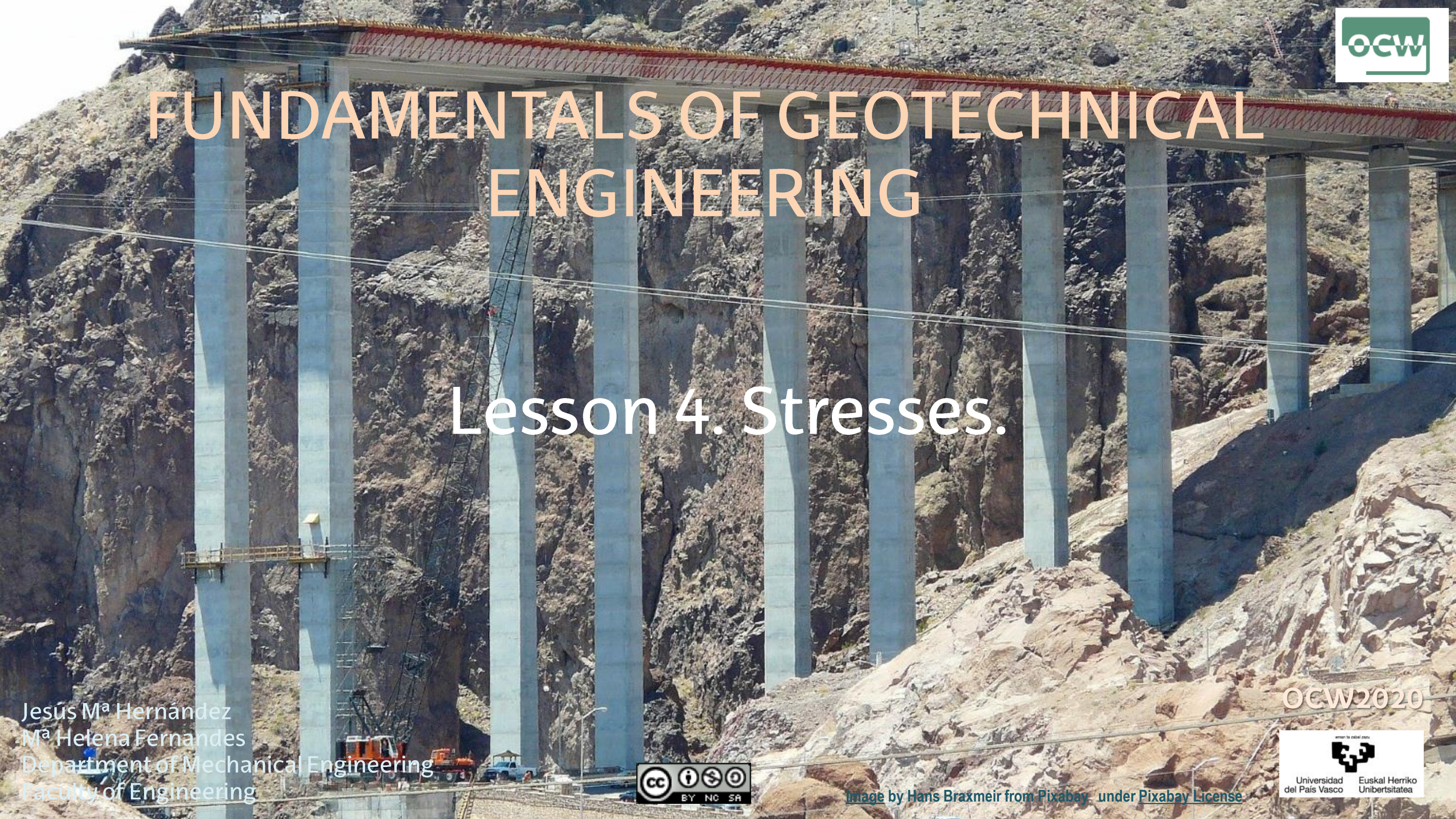


FUNDAMENTALS OF GEOTECHNICAL ENGINEERING

Lesson 4. Stresses.

Jesús Mª Hernández
Mª Helena Fernandes
Department of Mechanical Engineering
Faculty of Engineering

OCW2020

4. STRESSES.

LESSON OVERVIEW

In this lesson, an introduction to stresses in geotechnical engineering is presented. First, geostatic stresses and effective stresses due to the presence of water are defined and explained. Then, induced stresses by external loads are presented following the theory of elasticity. Finally, basic concepts about permeability are introduced; these concepts are necessary to understand, at the end of the lesson, the distribution of stresses in soils having low permeability, and the consolidation process.

LEARNING OUTCOMES

On completion this lesson, the student will be able to:

- ✓ Apply the Terzaghi's law and the coefficient of lateral earth pressure to obtain relationships between horizontal and vertical stresses and pore water pressure at the in-situ state.
- ✓ Determine vertical and horizontal stresses and pore water pressure at any point in the ground, plot their variation and plot Mohr's circles at the in-situ state.
- ✓ Determine induced stresses at any point in the soil due to external loads (basic cases).
- ✓ Know and understand why the flow of water in soils occurs.
- ✓ Know and understand how the distribution of stresses between the different phases of a saturated soil occurs, the consolidation process and its implications in relation to the distribution of stresses in soils along the lifespan of a structure.

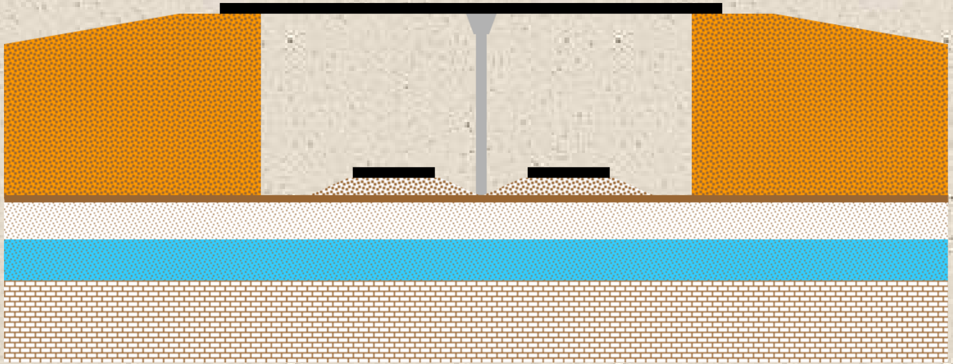
CONTENTS

1. Effects on soils due to applied surface loads.
2. Vertical stress due to the self weight of soil. Geostatic stress.
3. The principle of effective stress.
4. Horizontal stress. Mohr circle.
5. Stress distribution due to applied surface loads.
6. Flow of water in soil. Permeability.
7. Stress distribution between the phases of a soil.

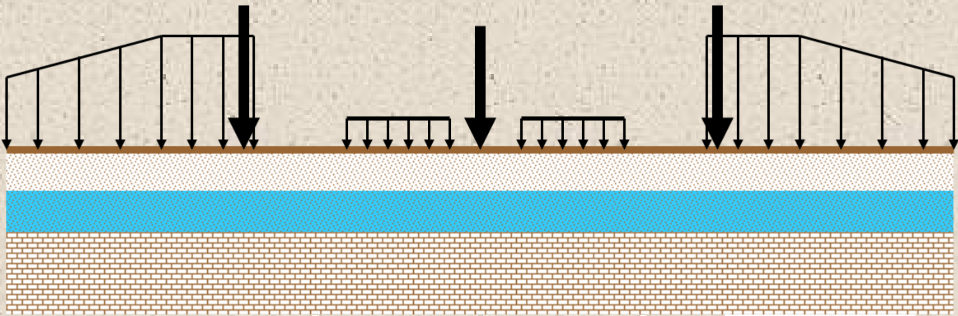
EFFECTS ON SOILS DUE TO APPLIED SURFACE LOADS (I)

At a point within a soil mass, stresses will be developed as a result of the soil lying above the point and by any structural or other loading imposed onto that soil mass.

Loads from civil engineering projects are modelled as distributed loads (uniform or not) and, in some special cases, as point loads.

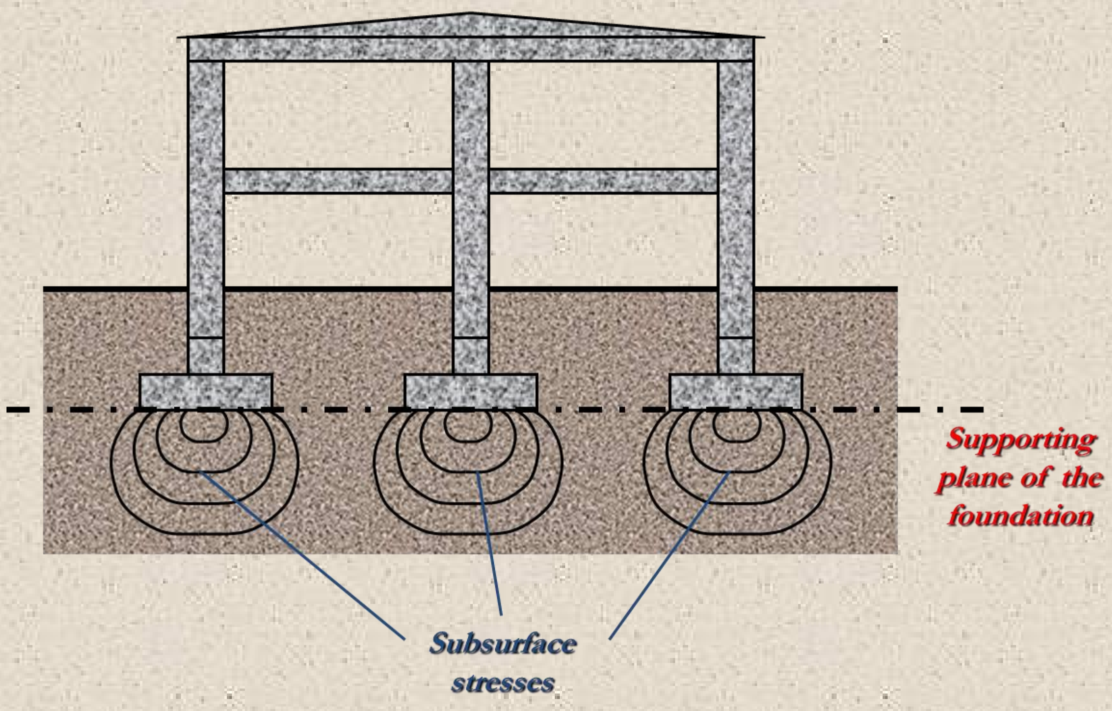


(Real)



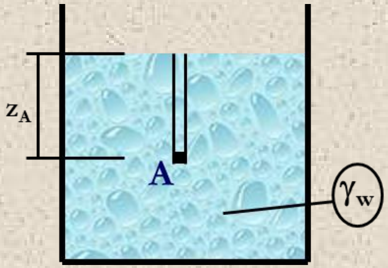
(Model)

EFFECTS ON SOILS DUE TO APPLIED SURFACE LOADS (II)



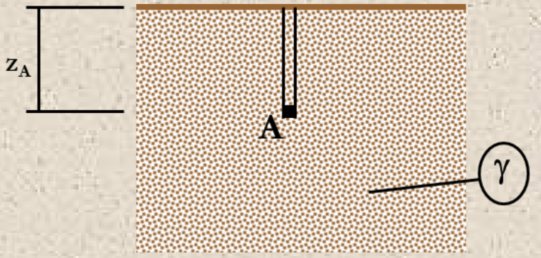
VERTICAL STRESS DUE TO THE SELF WEIGHT OF SOIL – GEOSTATIC STRESS (I)

When the ground surface is horizontal, for example in sedimentary soils, an analogy between geostatic stress and water pressure can be established.



"Water pressure at point A (at depth z_A) is equal to the weight per unit area of the column of water above that depth"

$$u_A = \frac{\gamma_w \cdot (S \cdot z_A)}{S} = \gamma_w \cdot z_A \quad (kN/m^2)$$



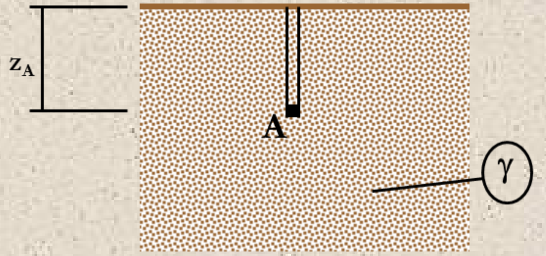
"The total vertical stress at a point A (at depth z_A) is equal to the weight per unit area of all material above that depth"

$$\sigma_v = \frac{\gamma \cdot (S \cdot z_A)}{S} = \gamma \cdot z_A \quad (kN/m^2)$$

Grows linearly as depth increases.

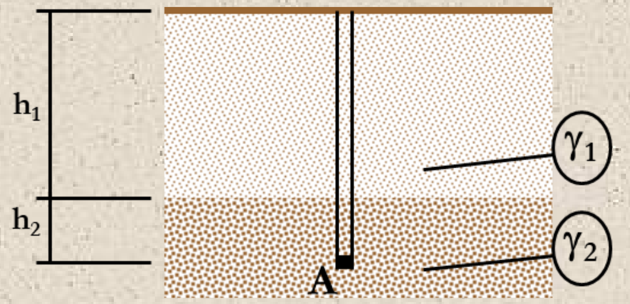
4. STRESSES.

VERTICAL STRESS DUE TO THE SELF WEIGHT OF SOIL – GEOSTATIC STRESS (II)



$$\sigma_v = \frac{\gamma \cdot (S \cdot z_A)}{S} = \gamma \cdot z_A \quad (kN/m^2)$$

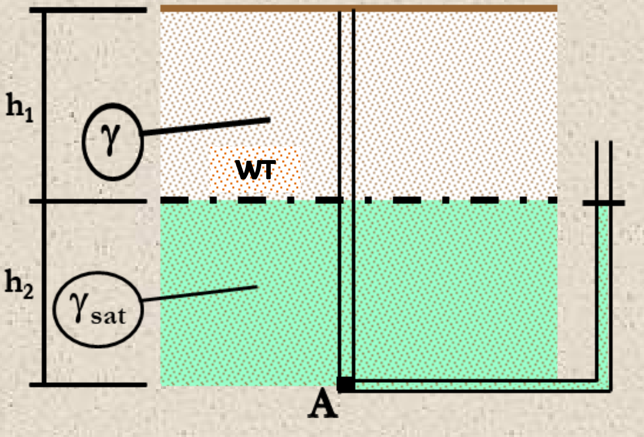
Grows linearly as depth increases.

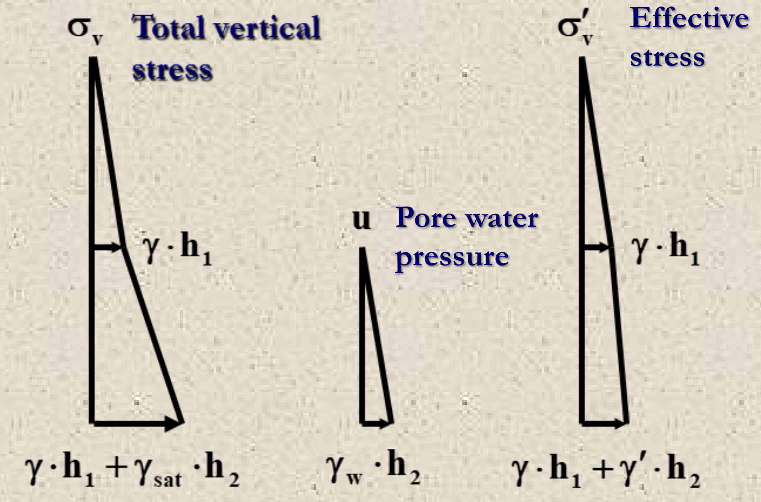


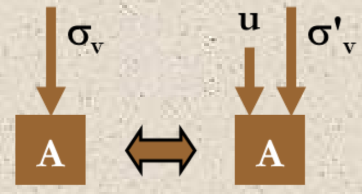
$$\sigma_v = \frac{\gamma_1 \cdot (S \cdot h_1)}{S} + \frac{\gamma_2 \cdot (S \cdot h_2)}{S} =$$

$$= \gamma_1 \cdot h_1 + \gamma_2 \cdot h_2 = \sum \gamma_i \cdot h_i$$

THE PRINCIPLE OF EFFECTIVE STRESS







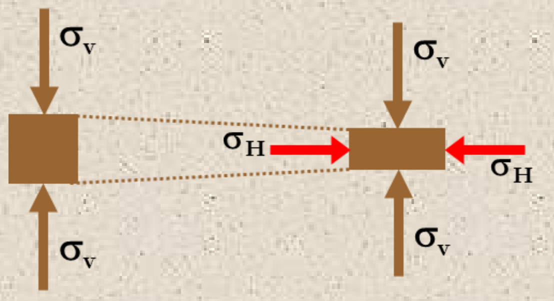
u: pore water pressure

σ'_v: effective stress

$$\sigma_v = \sigma'_v + u$$

Terzaghi's law

HORIZONTAL STRESS. MOHR CIRCLE.



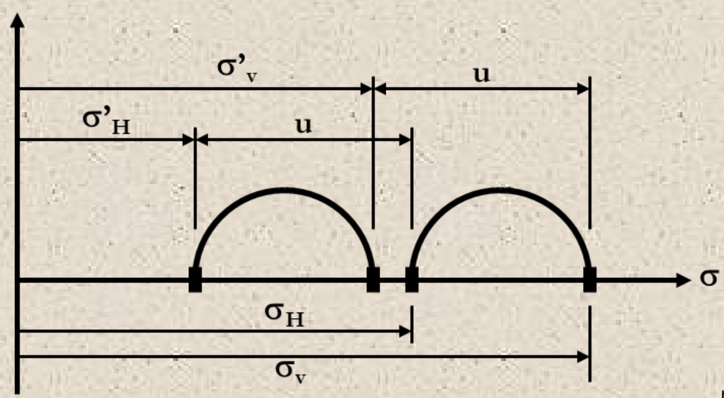
Horizontal stress and vertical stress are related

K_0, coefficient of earth pressure at rest

$$K_0 = \frac{\sigma'_H}{\sigma'_V}$$

Soil type	K0
Granular, loose	0.5 – 0.6
Granular, dense	0.3 – 0.5
Cohesive, soft	0.9 – 1.1
Cohesive, hard	0.8 – 0.9

$$\sigma_H = \sigma'_H + u$$

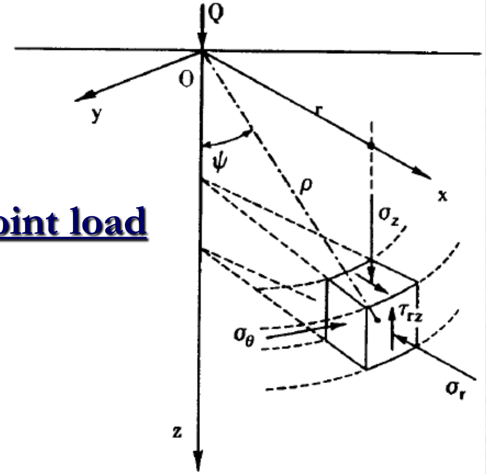


Mohr's circles

STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (I)

- Solutions based on Theory of Elasticity. Assumptions:
  - Soil is a semi-infinite, homogeneous and isotropic material.
  - Also, it is elastic and linear.
- NOT TRUE ⇒ Approximate solutions.
- Stresses and strains are proportional to applied loads.

Point load



$$\sigma_z = \frac{3\cdot Q}{2\cdot\pi\cdot z^2}\cdot\cos^5\psi$$

$$\sigma_r = \frac{Q}{2\cdot\pi\cdot z^2}\cdot\left[3\cdot\cos^3\psi\cdot\text{sen}^2\psi-(1-2\cdot\nu)\cdot\frac{\cos^2\psi}{(1+\cos\psi)}\right]$$

$$\sigma_\theta = -(1-2\cdot\nu)\cdot\frac{Q}{2\cdot\pi\cdot z^2}\cdot\left[\cos^3\psi-\frac{\cos^2\psi}{(1+\cos\psi)}\right]$$

$$\tau_{rz} = \frac{3\cdot Q}{2\cdot\pi\cdot z^2}\cdot\cos^4\psi\cdot\text{sen}\,\psi$$

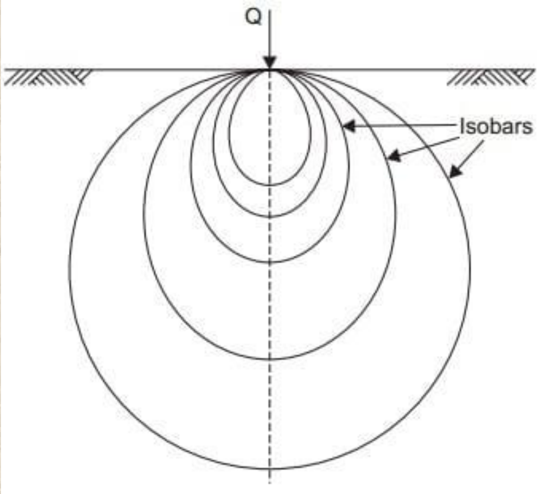
STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (II).

Rock Type	Poisson's Ratio
Igneous	0.10 - 0.40
Granite	0.17
Diorite	0.10 - 0.20
Gabbro	0.20 - 0.35
Rhyolite	0.20 - 0.40
Andesite	0.20
Basalt	0.10 - 0.20
Sedimentary	0.10 - 0.30
Conglomerate	0.10 - 0.15
Sandstone	0.14
Shale	0.10
Mudstone	0.15
Dolomite	0.15
Limestone	0.30
Metamorphic	0.15 - 0.30
Gneiss	0.24
Schist	0.15 - 0.25
Phyllite	0.26
Slate	0.20 - 0.30
Marble	0.15 - 0.30
Quartzite	0.17

Type of soil	Poisson's ratio
Normally consolidated soft clay	0.40
Medium clay	0.30
Overconsolidated hard clay	0.15
Sand and granular soils	0.30

Table D.24. Indicative values of the Poisson's ratio.

Technical Building Code. Foundations.

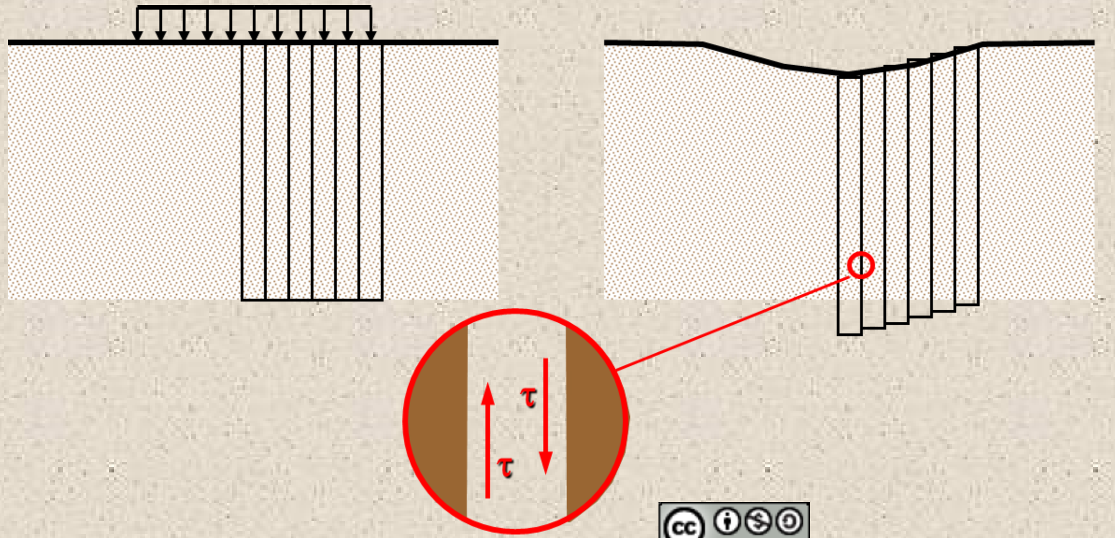


Pressure bulb for a point load. Connects all points below the ground surface at which the vertical pressure is the same.

STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (III).

What is that greek letter τ?

- It is called "shear stress".
- It is a kind of "friction" which arises between soil particles when deformation (distortion) occurs.



STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (IV).

Distributed loads

- Point load is not real.
- Foundations transfer the load to the soil.
- Also, fills and embankments are actually loads.
- Real loads can be modelled as distributed loads.
- Subsurface stresses are calculated integrating the point load solution.

STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (V).

Uniform load on a rectangular area (I)

$$\sigma_z = \frac{q}{2\cdot\pi}\cdot\left\{\operatorname{arctg}\frac{a\cdot b}{z\cdot R_3} + \frac{a\cdot b\cdot z}{R_3}\cdot\left[\frac{1}{R_1^2}+\frac{1}{R_2^2}\right]\right\}$$

$$\sigma_x = \frac{q}{2\cdot\pi}\cdot\left\{\operatorname{arctg}\frac{a\cdot b}{z\cdot R_3} - \frac{a\cdot b\cdot z}{R_1^2\cdot R_3}\right\}$$

$$\sigma_y = \frac{q}{2\cdot\pi}\cdot\left\{\operatorname{arctg}\frac{a\cdot b}{z\cdot R_3} - \frac{a\cdot b\cdot z}{R_2^2\cdot R_3}\right\}$$

$$\tau_{xz} = \frac{q}{2\cdot\pi}\cdot\left\{\frac{b}{R_2} - \frac{z^2\cdot b}{R_1^2\cdot R_3}\right\}$$

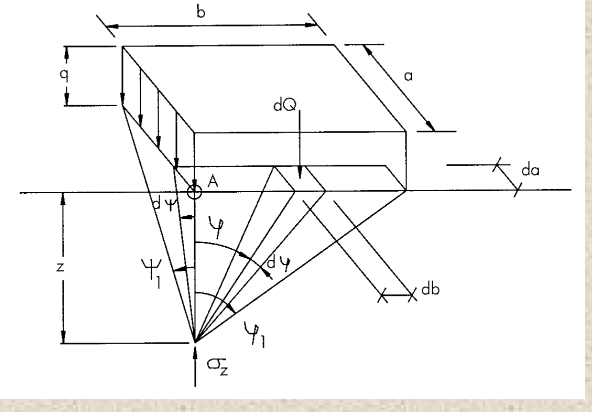
$$\tau_{yz} = \frac{q}{2\cdot\pi}\cdot\left\{\frac{a}{R_1} - \frac{z^2\cdot a}{R_2^2\cdot R_3}\right\}$$

$$\tau_{xy} = \frac{q}{2\cdot\pi}\cdot\left\{1+\frac{z}{R_3} - z\cdot\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right\}$$

$$R_1 = \sqrt{a^2+z^2}$$

$$R_2 = \sqrt{b^2+z^2}$$

$$R_3 = \sqrt{a^2+b^2+z^2}$$



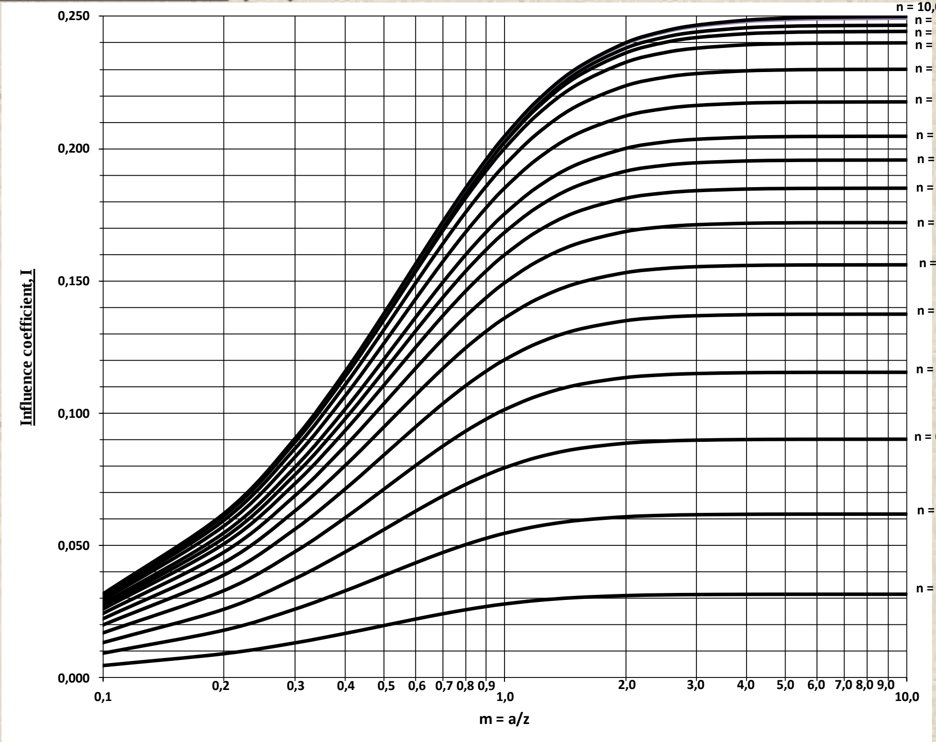
¡¡¡ Stress at a depth z beneath the corner of the rectangle!!!

STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (VI).

Uniform load on a rectangular area (II)

$$\sigma_z = q \cdot I$$

Influence coefficient



n = b

STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (VII).

Uniform load on a rectangular area (II)

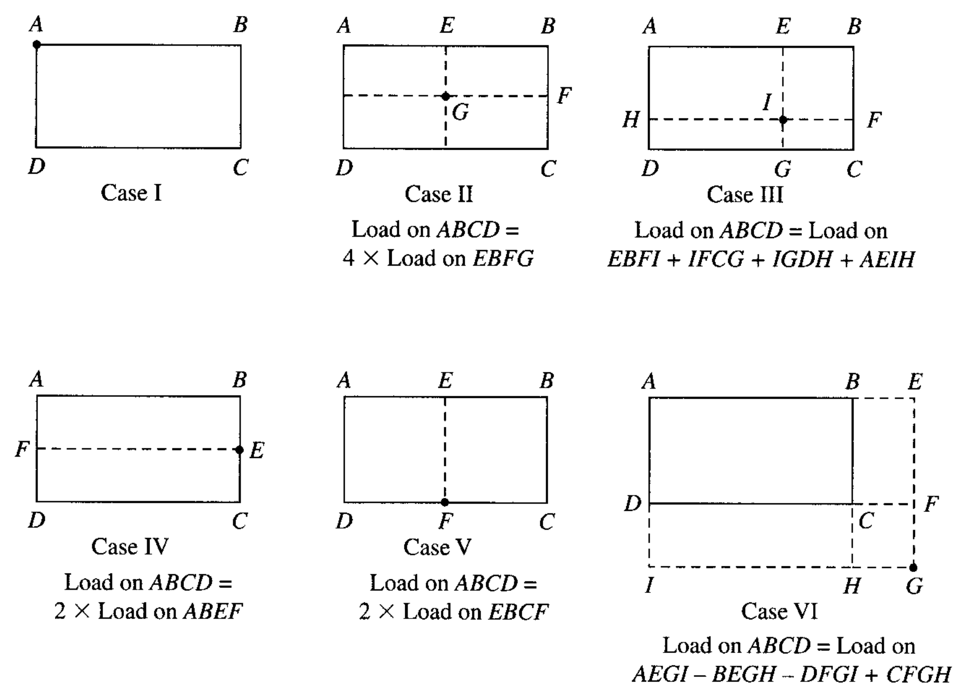
	m = a/z																		
n = b/z	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,2	1,5	2,0	2,5	3,0	4,0	5,0	6,0	10,0
0,1	0,005	0,009	0,013	0,017	0,020	0,022	0,024	0,026	0,027	0,028	0,029	0,030	0,031	0,031	0,031	0,032	0,032	0,032	0,032
0,2	0,009	0,018	0,026	0,033	0,039	0,043	0,047	0,050	0,053	0,055	0,057	0,059	0,061	0,062	0,062	0,062	0,062	0,062	0,062
0,3	0,013	0,026	0,037	0,047	0,056	0,063	0,069	0,073	0,077	0,079	0,083	0,086	0,089	0,089	0,090	0,090	0,090	0,090	0,090
0,4	0,017	0,033	0,047	0,060	0,071	0,080	0,087	0,093	0,098	0,101	0,106	0,110	0,113	0,114	0,115	0,115	0,115	0,115	0,115
0,5	0,020	0,039	0,056	0,071	0,084	0,095	0,103	0,110	0,116	0,120	0,126	0,131	0,135	0,136	0,137	0,137	0,137	0,137	0,137
0,6	0,022	0,043	0,063	0,080	0,095	0,107	0,117	0,125	0,131	0,136	0,143	0,149	0,153	0,155	0,155	0,156	0,156	0,156	0,156
0,7	0,024	0,047	0,069	0,087	0,103	0,117	0,128	0,137	0,144	0,149	0,157	0,164	0,169	0,170	0,171	0,172	0,172	0,172	0,172
0,8	0,026	0,050	0,073	0,093	0,110	0,125	0,137	0,146	0,154	0,160	0,168	0,176	0,181	0,183	0,184	0,185	0,185	0,185	0,185
0,9	0,027	0,053	0,077	0,098	0,116	0,131	0,144	0,154	0,162	0,168	0,178	0,186	0,192	0,194	0,195	0,195	0,196	0,196	0,196
1,0	0,028	0,055	0,079	0,101	0,120	0,136	0,149	0,160	0,168	0,175	0,185	0,194	0,200	0,202	0,203	0,204	0,204	0,204	0,205
1,2	0,029	0,057	0,083	0,106	0,126	0,143	0,157	0,168	0,178	0,185	0,196	0,205	0,212	0,215	0,216	0,217	0,217	0,218	0,218
1,5	0,030	0,059	0,086	0,110	0,131	0,149	0,164	0,176	0,186	0,194	0,205	0,216	0,224	0,227	0,228	0,229	0,230	0,230	0,230
2,0	0,031	0,061	0,089	0,113	0,135	0,153	0,169	0,181	0,192	0,200	0,212	0,224	0,232	0,236	0,238	0,239	0,240	0,240	0,240
2,5	0,031	0,062	0,089	0,114	0,136	0,155	0,170	0,183	0,194	0,202	0,215	0,227	0,236	0,240	0,242	0,243	0,244	0,244	0,244
3,0	0,031	0,062	0,090	0,115	0,137	0,155	0,171	0,184	0,195	0,203	0,216	0,228	0,238	0,242	0,244	0,246	0,246	0,246	0,247
4,0	0,032	0,062	0,090	0,115	0,137	0,156	0,172	0,185	0,195	0,204	0,217	0,229	0,239	0,243	0,246	0,247	0,248	0,248	0,248
5,0	0,032	0,062	0,090	0,115	0,137	0,156	0,172	0,185	0,196	0,204	0,217	0,230	0,240	0,244	0,246	0,248	0,249	0,249	0,249
6,0	0,032	0,062	0,090	0,115	0,137	0,156	0,172	0,185	0,196	0,204	0,218	0,230	0,240	0,244	0,246	0,248	0,249	0,249	0,249
10,0	0,032	0,062	0,090	0,115	0,137	0,156	0,172	0,185	0,196	0,205	0,218	0,230	0,240	0,244	0,247	0,248	0,249	0,249	0,250

$$\sigma_z = q \cdot I$$

Influence coefficient

STRESS DISTRIBUTION DUE TO APPLIED SURFACE LOADS (VIII).

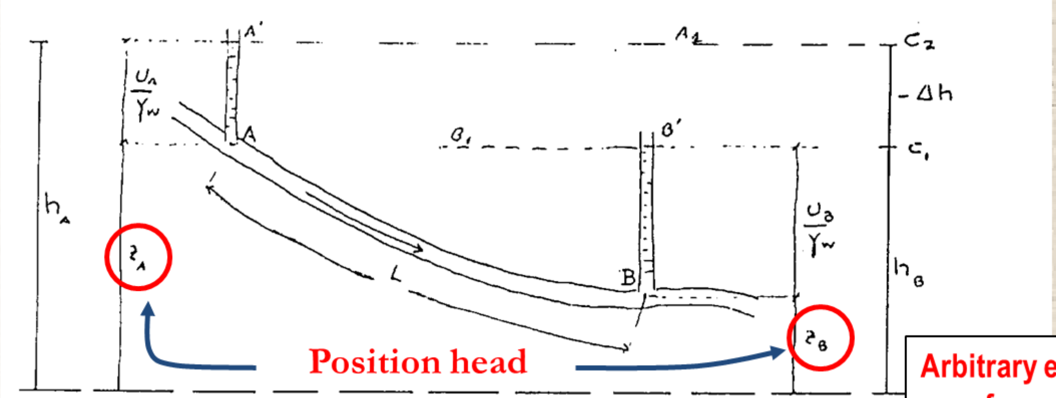
Uniform load on a rectangular area (III)



For stresses beneath points other than the corner of the loaded area, the principle of superposition should be used.

FLOW OF WATER IN SOIL. PERMEABILITY (I).

Bernoulli's equation (I)



Arbitrary elevation selected as reference datum

$$u_A = \overline{AA'} \cdot \gamma_w \quad \Rightarrow \quad \overline{AA'} = \frac{u_A}{\gamma_w} \qquad u_B = \overline{BB'} \cdot \gamma_w \quad \Rightarrow \quad \overline{BB'} = \frac{u_B}{\gamma_w}$$

Pressure head

Position head or elevation head is the difference in elevation between the datum and the point (z_A and z_B).

Pressure head is the difference in elevation between the point and the water table, and it is due to the pore water pressure (AA' and BB').

Total head is the sum of pressure head and position head.

$$h = z + \frac{u}{\gamma_w}$$

- A fluid particle flows through void spaces existing between the solid grains of a soil from point A to point B, along a distance L.

FLOW OF WATER IN SOIL. PERMEABILITY (II).

Bernoulli's equation (II)

➢ Flow of water between point A and point B is due to the law of conservation of energy. Applying this law, but dividing energy terms by m·g, leads to:

$$z_A + \frac{v_A^2}{2\cdot g} + \frac{u_A}{\gamma_w} = z_B + \frac{v_B^2}{2\cdot g} + \frac{u_B}{\gamma_w} \quad \text{+ loss energy}$$

Strain "energy"

Kinetic "energy" → Can be ignored because the velocity of flow of water through soil is quite small.

Potential "energy"

$$h_A = h_B + \Delta h$$

Δh ← Head loss

FLOW OF WATER IN SOIL. PERMEABILITY (III).

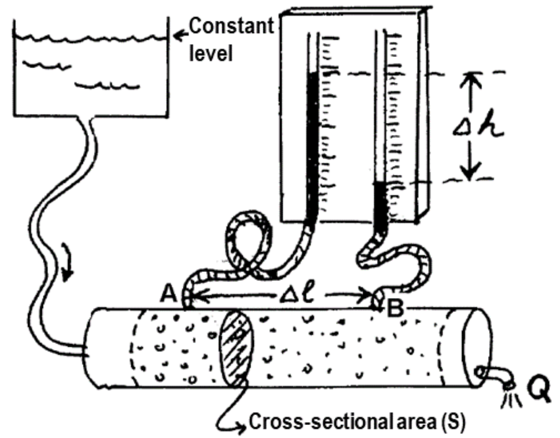
Bernoulli's equation (III)

➔ Hydraulic gradient: $$i_{AB} = \frac{h_A - h_B}{L_{AB}}$$

Flow of water in soil between two point occurs as a result of a total head difference between two points, with the direction of flow being from the higher to the lower head.

FLOW OF WATER IN SOIL. PERMEABILITY (IV).

Darcy's law (1856). Coefficient of permeability.



➢ In 1856 H. Darcy performed experiments to study the flow of water. The figure illustrates Darcy's experiment in which water moves through a soil sample contained in a cylindrical conduit.

➢ Darcy measured the total head at points A and B.

➢ Darcy observed that the flow rate varied directly with both the head difference and the cross-sectional area and inversely proportional to the length of the soil.

$$Q = k \cdot S \cdot i_{AB} = k \cdot S \cdot \frac{h_A - h_B}{\Delta l} = k \cdot S \cdot \frac{\Delta h}{\Delta l}$$

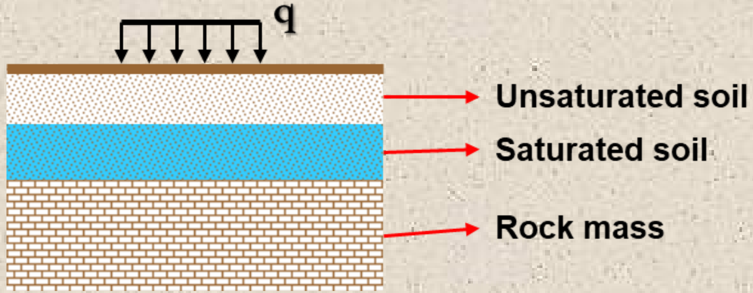
➢ "k" is the *coefficient of permeability* (or hydraulic conductivity). Units of "k" are length per unit of time. In fact, "k" is directly proportional to the velocity of flow - v -, as shown in the expression below, and so the larger "k" is, the quicker the water will flow through soil, and vice versa.

$$Q = v \cdot S = k \cdot i \cdot S \quad \Rightarrow \quad v = k \cdot i$$

Type of soil	k (cm/s)
Poorly graded gravel (GP)	≥ 1
Uniform gravel (GP)	0.2 - 1
Well-graded gravel (GW)	0.05 - 0.3
Uniform sand (SP)	$5\cdot10^{-3}$ - 0.2
Well-graded sand (SW)	10^{-3} - 0.1
Silty sand (SM)	10^{-3} - $5\cdot10^{-3}$
Clayey sand (SC)	10^{-4} - 10^{-3}
Silt (ML)	$5\cdot10^{-5}$ - 10^{-4}
Lean clay (CL)	10^{-8} - 10^{-5}

4. STRESSES.

STRESS DISTRIBUTION BETWEEN THE PHASES OF A SOIL (I)



- Due to structural loads, q, stresses in soil appear. Terzaghi's law is still valid.

$$\Delta\sigma_v(q) = \Delta\sigma'_V(q) + \Delta u(q)$$

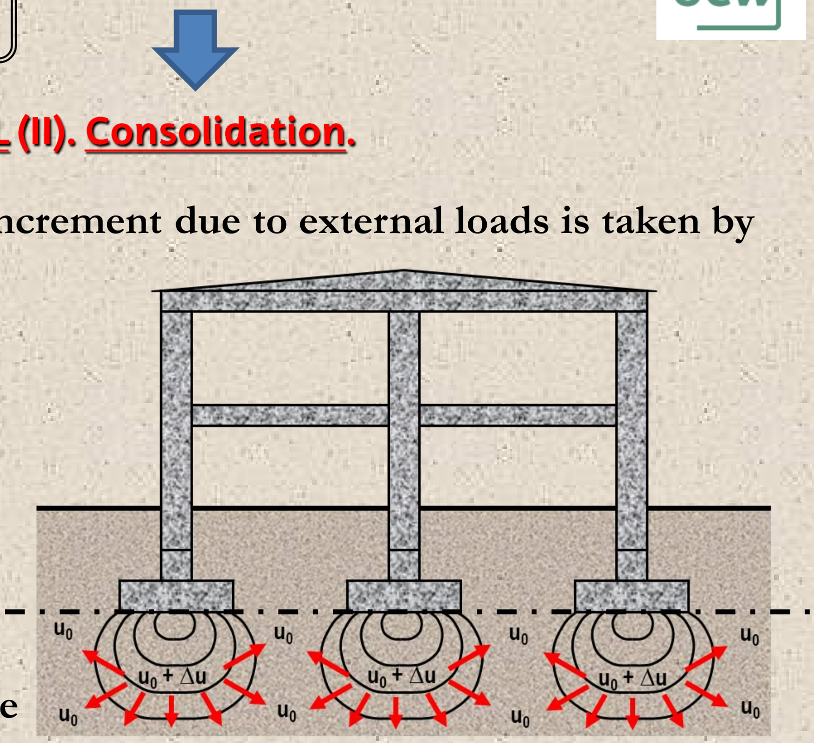
- Also, as soil is under compression, a change in total volume occurs.
- In dry soils, this change in volume is due to a rearrangement of solid particles into a tighter packing. The volume of solid particles itself remains virtually unchanged and only a decrease in the volume of voids occurs. Therefore, all the stress increment is taken by solid particles.
- Also, the behaviour of unsaturated soils where the degree of saturation is low can be considered similar to that of the dry soils.
- In unsaturated soils where the degree of saturation is high, the analysis of stress distribution is very complex and beyond the scope of this course.

4. STRESSES.

STRESS DISTRIBUTION BETWEEN THE PHASES OF A SOIL (II). Consolidation.

➢ In saturated soils, following Terzaghi's law, the stress increment due to external loads is taken by both the solid particles and the water in voids.

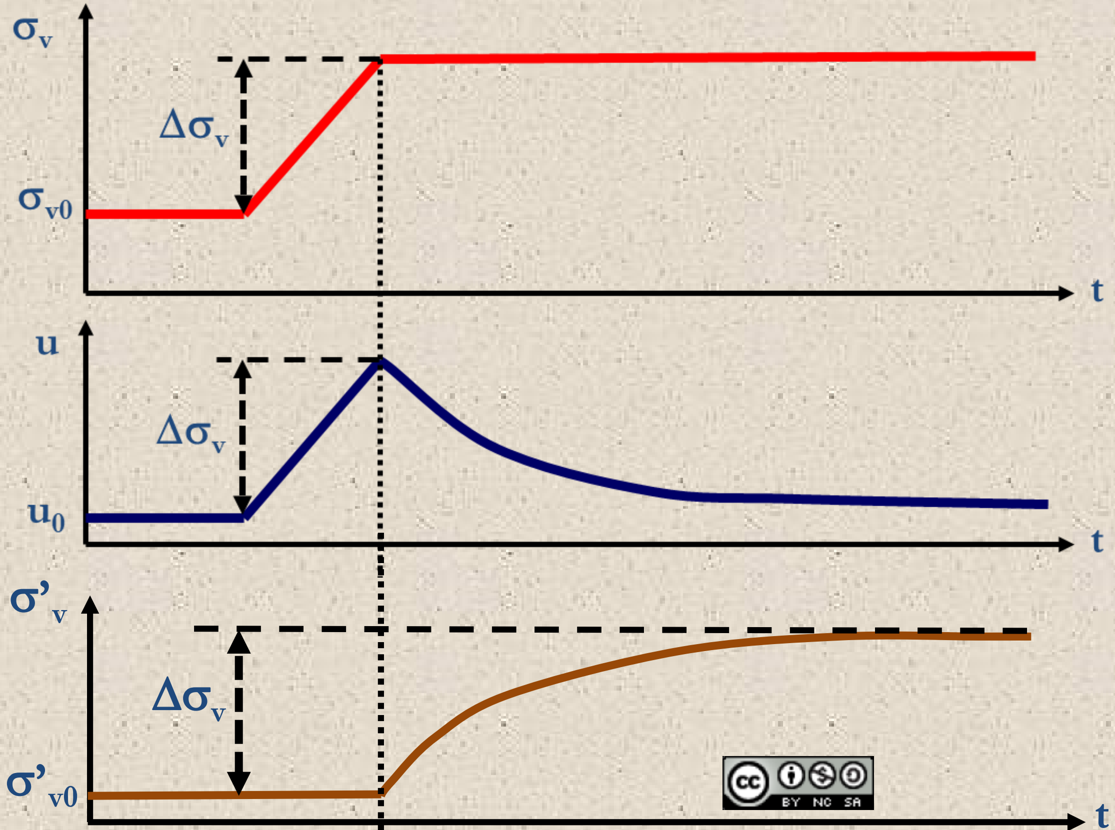
➢ This increase in pore water pressure (Δu) produces a hydraulic gradient in the soil, causing some of the pore water to flow away. As water is discharged, the solid particles consolidate and begin to carry part of the external load. Thus, the stress increment is gradually transferred from the pore water to the soil solids.



➢ Eventually, all the external load is carried by the solids, pore water pressure returns to its initial (hydrostatic) value (Δu = 0) and the flow of pore water ceases.

➢ This process is quite fast (some minutes) in gravel and sand, and very slow (years) in clay, organic soils and silt (at a lesser extent), depending on their coefficients of permeability (high/low respectively).

STRESS DISTRIBUTION BETWEEN THE PHASES OF A SOIL (III). Consolidation.



Consolidation of saturated soils (low permeability)