4. LESSON: SOLVED EXERCISES

1. The time takes a student to get from home to college varies uniformly between 35 and 45 minutes. What time should he leave home with a minimum probability of 0.8 to get to class on time if classes start at 8 a.m.?

$X =$ "Minutes from home to college"

The variable follows an uniform distribution.

$$P(X \leq x) \geq 0.8 \Rightarrow \frac{x-35}{45-35} \geq 0.8 \Rightarrow x \geq 43$$

So that, if the student leave home at 7:17a.m. or earlier, will get to class on time with a probability of 0.8 or higher.

2. In a store, the time we wait from the entry of a customer to the entry of the next customer is distributed exponentially with a mean of 5 minutes. Calculate the probability that we will have to wait between 8 and 10 minutes until the entry of the next customer.

X : 'Minutes we have to wait until the entry of the next customer'

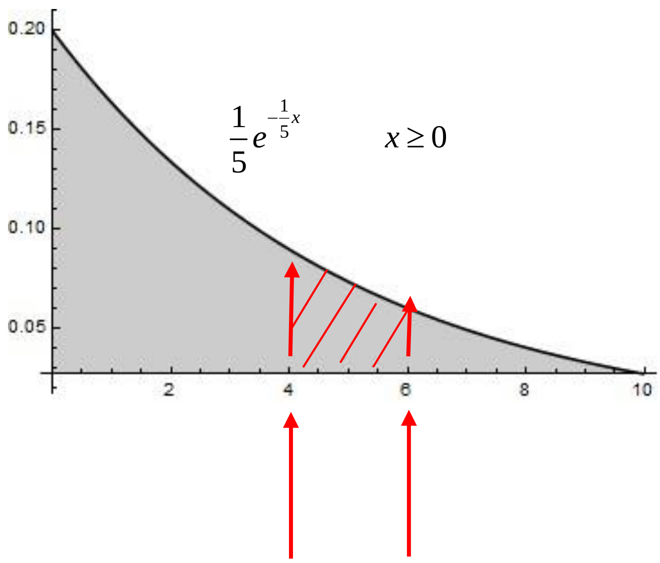
$$E(X) = 5 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{5}$$

$$f(x) = \begin{cases} \frac{1}{5} e^{-\frac{1}{5}x} & x \geq 0 \quad \wedge \quad \lambda > 0 \\ 0 & \text{other cases} \end{cases}$$

$$F(x) = P(X \leq x) = \begin{cases} 1 - e^{-\frac{1}{5}x} & x \geq 0 \\ 0 & \text{other cases} \end{cases}$$

$$P(4 \leq X \leq 6) = P(X \leq 6) - P(X \leq 4)$$

The exercise requires to calculate the probability of an interval. For calculating the surface that appears in the graph this is what we should do.



$$P(4 \leq X \leq 6) = \left(1 - e^{-\frac{1}{5} \cdot 6}\right) - \left(1 - e^{-\frac{1}{5} \cdot 4}\right) = 0.148$$

3. A pot of jam is classified as 'syrup' if the amount of sugar is between 420 and 520 g. The manufacturer when analysing pots observes that the average weight is 465 g, with a standard deviation of 30 g. Knowing that the weight of the sugar follows a normal distribution,
   a) What percentage of the manufacturer's production cannot be labeled as 'syrup'?
   b) What are the two central values that we have to set so that among them there are 50% of the pots?

X : 'Sugar amount in g inside the pot'

$420 \le X \le 520 \rightarrow$ Pot that could be consider as 'Syrup'

X<420 eta X>520 $\rightarrow$ Pot that could not be consider as 'Syrup

$E(X) = 465$ eta $\sigma = 30$ X ~ N(465;30)

$a)$

Two ways:

1)

$$P(420 \le X \le 520) = P(X \le 520) - P(X \le 420)$$

$$Z = \frac{x - 465}{30}$$

$$P(420 \le X \le 520) = P(Z \le \frac{520 - 465}{30}) - P(Z \le \frac{420 - 465}{30}) = P(Z \le 1.83) - P(Z \le -1.5)$$

$P(Z \le 1.83) = 0.9664$ This value was consulted in the table

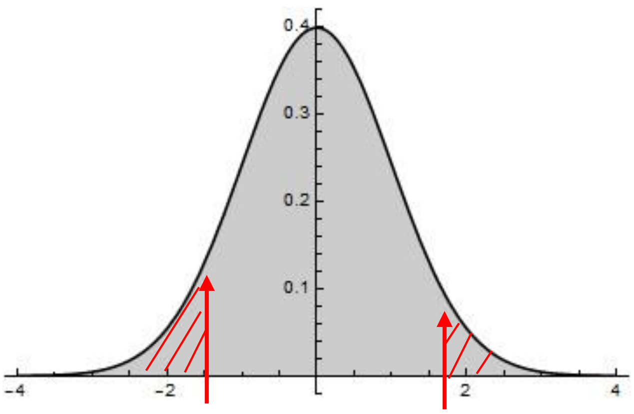
$P(Z \le -1.5)$ In our table we have no negative values, so we need to apply symmetry

$$P(Z \le -1.5) = P(Z \ge 1.5) = 1 - P(Z \le 1.5) = 1 - 0.9332 = 0.0668$$

$$P(420 \le X \le 520) = P(Z \le 1.83) - P(Z \le -1.5) = 0.9664 - 0.0668 = 0.8996$$

$$1 - P(420 \le X \le 520) = 0.1004$$

So that, the 10.04% could not be labelled as 'syrup'



Standardized normal distribution

The probability we have to calculate is represented by the red surface. As the value of the entire surface below the curve is 1, we must subtract the calculated intermediate probability value to obtain the red surface.

2)

$P(X \geq 520) + P(X \leq 420)$

$Z = \dfrac{x-465}{30}$

$P(X \geq 520) + P(X \leq 420) = P(Z \geq \dfrac{520-465}{30}) + P(Z \leq \dfrac{420-465}{30}) = P(Z \geq 1.83) + P(Z \leq -1.5)$

$P(Z \geq 1.83)$ The values we have in the table are $P(Z \leq z)$.

So that we have to apply symmetry and find the value in the table.

$P(Z \geq 1.83) = 1 - P(Z \leq 1.83) = 1 - 0.9664 = 0.0336$

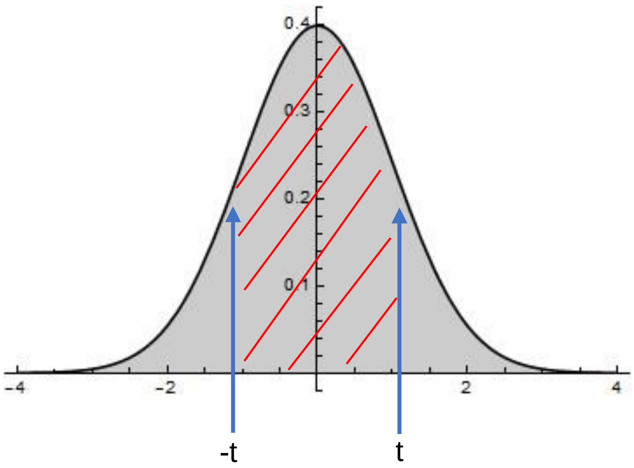
$P(Z \leq -1.5)$ In our table there are not negative values, so we have to apply symmetry.

$P(Z \leq -1.5) = P(Z \geq 1.5) = 1 - P(Z \leq 1.5) = 1 - 0.9332 = 0.0668$

$P(X \geq 520) + P(X \leq 420) = P(Z \geq 1.83) + P(Z \leq -1.5) = 0.0336 - 0.0668 = 0.1004$

So that 10.04% could not labbeled as 'syrup'.

b)



Standardized normal distribution

This section requires the calculation of the central values *b* and *a* (to use the standardized normal distribution after the change of variable -t and t). Each of them is at the same distance from distribution mean. The information they give us is that the red surface between them must be 50%.

We will use standardized normal distribution:

$$P(b \le X \le a) = P(X \le a) - P(X \le b)$$

$$Z = \frac{x-465}{30}$$

For solving this section we will apply symmetry many times.

$$P(b \le \mathrm{X} \le a) = P(-t \le Z \le t) = P(Z \le \frac{a-465}{30}) - P(Z \le \frac{b-465}{30}) = P(Z \le t) - P(Z \le -t)$$

$$P(b \le \mathrm{X} \le a) = P(Z \le t) - P(Z \le -t) = P(Z \le t) - P(Z \ge t) =$$

$$P(Z \le t) - \left(1 - P(Z \le t)\right) = -1 + 2P(Z \le t) = 0.5 \Rightarrow P(Z \le t) = 0.75$$

In the table we could checked for what value of t the probability is 0.75.

$$P(Z \le t) = 0.75 \Rightarrow t = 0.68$$

$$t = \frac{a-465}{30} \Rightarrow a = 485.4 \quad \text{and}$$

$$-t = \frac{b-465}{30} \Rightarrow b = 444.6$$

So that a and b central values are 485.4 and 444.6.

4. In a factory, two types of pieces are produced, one of them made with renewable materials and another with raw materials derived from oil.
   a) If 2% is defective, what is the probability that when selecting 100 pieces there will be no more than 3 defective?
   b) 40% are pieces based on renewable materials. What is the probability that when selecting 600 pieces, 352 or more pieces are made of raw materials derived from oil?

a)

X : 'Amount of defective pieces'

n=100; p=0.02 X ~ $B(100;0.02)$

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$P(X \leq 3) = \binom{100}{0} 0.02^0 (1-0.02)^{100-0} + \binom{100}{1} 0.02^1 (1-0.02)^{100-1} + \binom{100}{2} 0.02^2 (1-0.02)^{100-2} + \binom{100}{3} 0.02^3 (1-0.02)^{100-3} = 0.8589$$

$P(X \leq 3) = 0.8589$

b)

X : 'Amount of defective pieces made with raw materals derived from oil'

n=600; p=0.6 X ~ $B(600;0.6)$

$P(X \geq 352)$ for the calculus, the binomial distribution will be approximated with the normal distribution. Otherwise, we should add many terms. We will check if the conditions for the approximation are satisfied:

n=600>30

np=360 $\geq$ 5

nq=240 $\geq$ 5

Conditions are satisfied. We could do the approximation.

$$X \sim B(600;0.6) \cong X \sim N(\mu=np;\sigma^2=npq)=N(\mu=600\cdot 0.6;\sigma^2=600\cdot 0.6\cdot 0.4)=N(\mu=360;\sigma^2=144)$$

$$P(X_{discrete} \geq 352)=P(X_{continuous} \geq 351.5)$$

$$P(X_{discrete} \geq 352)=P(X_{continuous} \geq 351.5) = P(Z \geq \frac{351.5-360}{12})=P(Z \geq -0.71)$$

$P(Z \geq -0.71)=P(Z \leq 0.71)$ this value is taken from the standardized normal table

$P(Z \leq 0.71) = 0.7611$

$$P(X_{discrete} \geq 352)=P(X_{continuous} \geq 351.5) = 0.7611$$