2. LESSON: PROPOSED EXERCISES

To solve the following proposed exercises, the theory corresponding to lesson 2 as well as the examples shown in the solved exercises of the same lesson will be very helpful.

1. A machine produces plastic bottles of four different colours. The probability of producing bottles per colour is shown in the following table:

Colour	1	2	3	4
Probability	4/25	6/25	17/50	13/50

Let be X the discrete random discrete variable that defines the colour of plastic bottles. Find the characteristic function of this random variable and calculate the first two moments with respect to the origin using the characteristic function.

Solution: $\Psi(t)=\frac{1}{50}\left(8e^{it}+12e^{2it}+17e^{3it}+13e^{4it}\right)$; $\alpha_1=\frac{27}{10}$; $\alpha_2=\frac{417}{50}$

2. Let be X the discrete random variable with the following distribution function:

$$F(x)=\begin{cases} 0 & x<0 \\ 1/5 & 0\le x<1 \\ 1/3 & 1\le x<2 \\ 7/15 & 2\le x<3 \\ 1 & x\ge 3 \end{cases}$$

Calculate the probability function, the moments generating function and using this last one, calculate the variance of the variable.

Solution:

$$p(x)=\begin{cases} 1/5 & x=0 \\ 2/15 & x=1 \\ 2/15 & x=2 \\ 8/15 & x=3 \\ 0 & \text{other cases} \end{cases} \quad ;\ \alpha(w)=\frac{2}{15}\left(\frac{3}{2}+e^{w}+e^{2w}+4e^{3w}\right);\ \sigma^2=\frac{4}{5}$$

3. The gasoline consumption (l/100 km) of a new model of a well-known motorcycle factory manufactured in the north of Italy is defined by a continuous random variable. This variable has the following density function:

$$f(x)=\begin{cases} mxe^{-x^2} & x>0 \\ 0 & x\le 0 \end{cases}$$

a. Which is the value of the constant m for $f(x)$ to be a density function?

b. Get the distribution function of the continuous random variable.

c. Calculate the probability that the gasoline consumption of the new model is higher than 2.5 litre/100 km and the probability that it is between $0-1.5$ litre/100 km.

Solution: a) $m=2$; b) $F(x)=\begin{cases} 1-e^{-x^2} & x>0 \\ 0 & x\le 0 \end{cases}$; c) 0,8946 eta 0,0190

4. A high-level technician in an English industry measures the concentration of lead in some paints for aeronautical use. Let be X = "lead concentration" continuous random variable. The mean concentration of lead is 3 ppm and the moments generating function of this random variable is $\alpha(w)=\dfrac{\left(1+e^{aw}\right)^2}{4}$. Calculate:

a. The value of the constant a.

b. The first three moments with respect to the origin of the variable X.

c. Standard deviation.

Solution: a) $a=3$; b) $\alpha_1=3,\ \alpha_2=\dfrac{27}{2},\ \alpha_3=\dfrac{135}{2}$; c) $\dfrac{3\sqrt{2}}{2}$

5. Let be $\Psi_X(t)=kt^2+1$ and $\Psi_Y(t)=2kt$ the characteristic functions of the independent random variables X and Y respectively. If $Z=3X+Y$ and the mean of the variable Z is 2. Calculate:

a. The value of the constant k.

b. The means of the variables X and Y.

Solution: a) $k=i$; b) $\alpha_{1x}=0$; $\alpha_{1y}=2$