MATHS BASIC COURSE FOR UNDERGRADUATES

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SOLUTIONS: 1st SUBJECT. SET THEORY

SOLUTION EXERCISE 1: It is clear that the relations \(\{1, 2\} \subseteq A\) and \(\{1, 4\} \not\subseteq A\) fulfill. These are the subsets of \(A\):

\[\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \text{ and } \{1, 2, 3\}.\]

Thus, the set \(A\) has 8 subsets.

SOLUTION EXERCISE 2:

\(A \times B = \{(1, x), (2, x), (3, x), (1, y), (2, y), (3, y)\}\).

SOLUTION EXERCISE 3:

\[(A \cup B) \cup (A \cap (C \cup B)) = A \cup B\]

\[(A \cap B) \cup (C \cap A) \cup (A^c \cap B^c)^c = A \cup B\]

SOLUTION EXERCISE 4: To prove that \(A \not\subseteq B\), it is enough to find a counterexample; for instance, \(16 \in A\) but \(16 \notin B\). On the other hand, in an analogous way, since \(14 \in B\) and \(14 \notin A\), it follows that \(B \not\subseteq A\).

SOLUTION EXERCISE 5: First of all, let us observe that \(\mathbb{R}\) satisfies the following three properties: reflexive, symmetric and transitive.

- Reflexive: for any \(n \in \mathbb{Z}\), \(n \mathbb{R} n\), since \(n - n = 0\) and the number 0 can be considered an even number.

- Symmetric: for any \(m, n \in \mathbb{Z}\), if \(m \mathbb{R} n\) then \(m - n\) is even, and also \(n - m = -(m - n)\) is even. Thus \(n \mathbb{R} m\).

- Transitive: for any \(m, n, t \in \mathbb{Z}\) such that \(m \mathbb{R} n\) and \(n \mathbb{R} t\) we have that \(m - n = 2t_1\) and \(n - t = 2t_2\), for some \(t_1, t_2 \in \mathbb{Z}\). Thus, \((m - n) + (n - t) = m - t = 2t_1 + 2t_2 = 2(t_1 + t_2)\), and in particular, it is an even number, i.e \(m \mathbb{R} t\).

On the other hand, the integer numbers that are related through \(\mathbb{R}\) to 2 are

\[\mathcal{Z} = \{x \in \mathbb{Z} : x \mathbb{R} 2\} = \{x \in \mathbb{Z} : x - 2 = 2t, t \in \mathbb{Z}\} = \{x \in \mathbb{Z} : x = 2 + 2t, t \in \mathbb{Z}\},\]

which coincides with the set formed by all the multiples of 2. The equivalence class of 2008 corresponds to \(\overline{2008} = \{x \in \mathbb{Z} : x \mathbb{R} 2008\} = \{x \in \mathbb{Z} : x - 2008 = 2t, t \in \mathbb{Z}\} = \{x \in \mathbb{Z} : x = 2008 + 2t, t \in \mathbb{Z}\},\) which corresponds to the set formed by all the multiples of 2. Finally, the equivalence class of \(-11\) corresponds to the set formed for all the odd integer numbers.
**SOLUTION EXERCISE 6**: The proof follows in an analogous way as in Exercise 5.

**SOLUTION EXERCISE 7**: \( (a, b) = \{(c, d) \in \mathbb{Z} \times \mathbb{Z}^* \mid \frac{c}{d} = \frac{\alpha}{\beta} \} \), and the quotient set \((\mathbb{Z} \times \mathbb{Z}^*)/\mathbb{R} = \mathbb{Q} \).