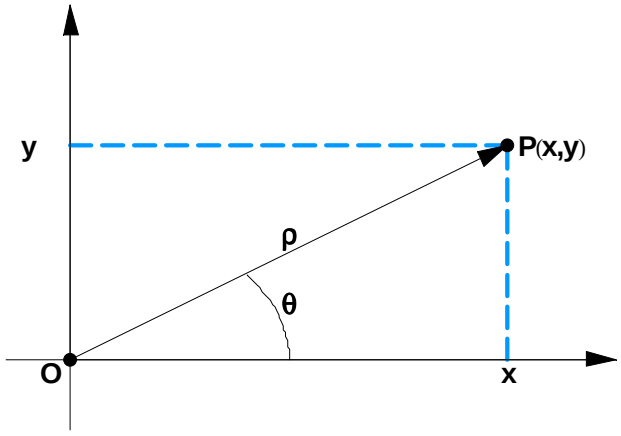
5

REPRESENTACIÓN DE CURVAS EN COORDENADAS POLARES

5.1. Reperesentación gráfica en Coordenadas Polares

En un sistema de ejes coordenados rectangulares OXY, todo punto P queda perfectamente determinado por sus coordenadas cartesianas (x,y), que son las proyecciones sobre los ejes, o por sus coordenadas polares (ρ,ϑ), que es la distancia ρ>0 del punto P al centro de coordenadas y el ángulo ϑ que forma el vector OP con la parte positiva del eje OX

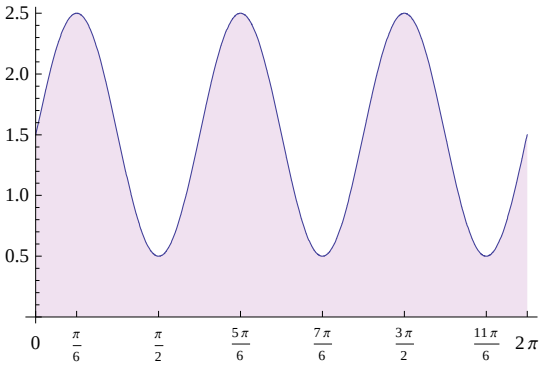


▼ PolarPlot[ ]

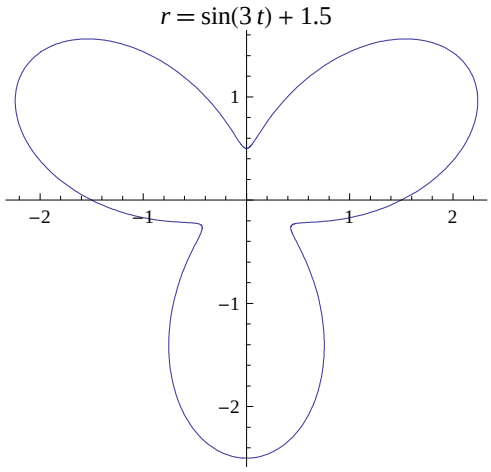
```
? PolarPlot
```

PolarPlot[r, {θ, θ_{min}, θ_{max}}] generates a polar plot of a curve with radius r as a function of angle θ.
PolarPlot[{f_1, f_2, …}, {θ, θ_{min}, θ_{max}}] makes a polar plot of curves with radius functions f_1, f_2, …. ≫

```
Plot[1.5 + Sin[3 * t], {t, 0, 2 π}, AxesOrigin → {0, 0},
 Ticks → {{0, π / 6, π / 2 , 5 π / 6, 7 π / 6, 3 π / 2, 11 π / 6, 2 π} , Automatic},
 Filling → Axis, FillingStyle → {LightBlue, LightPurple}]
```



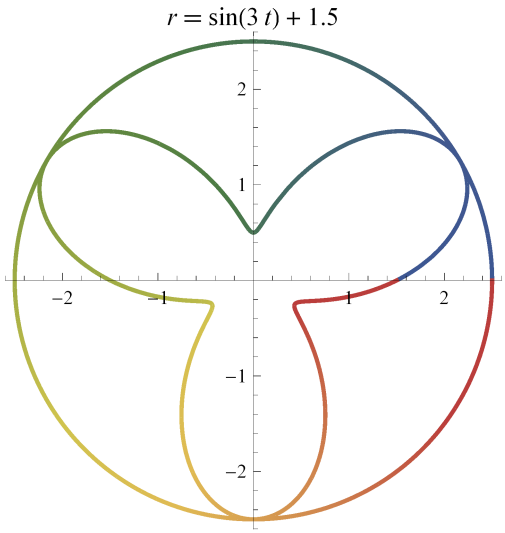

```
PolarPlot[{1.5 + Sin[3 * t]}, {t, 0, 2 π}, PlotLabel → r == 1.5 + Sin[3 * t]]
```



▼ Opciones de PolarPlot[ ]

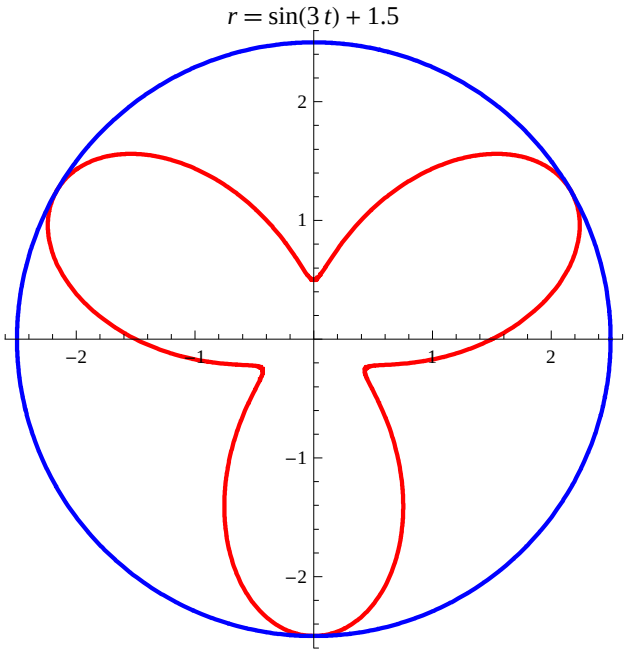
★ ColorFunction

```
PolarPlot[{1.5 + Sin[3 * t], 2.5}, {t, 0, 2 π},
 ColorFunction → "DarkRainbow", PlotLabel → r == 1.5 + Sin[3 * t]]
```



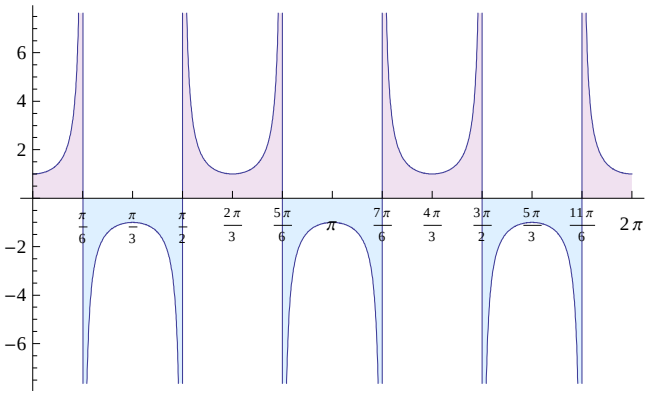
★ PlotStyle

```
PolarPlot[{1.5 + Sin[3 * t], 2.5}, {t, 0, 2 π},
 PlotStyle → {Directive[Red, Thick], Directive[Blue, Thick]},
 PlotLabel → r == 1.5 + Sin[3 * t]]
```

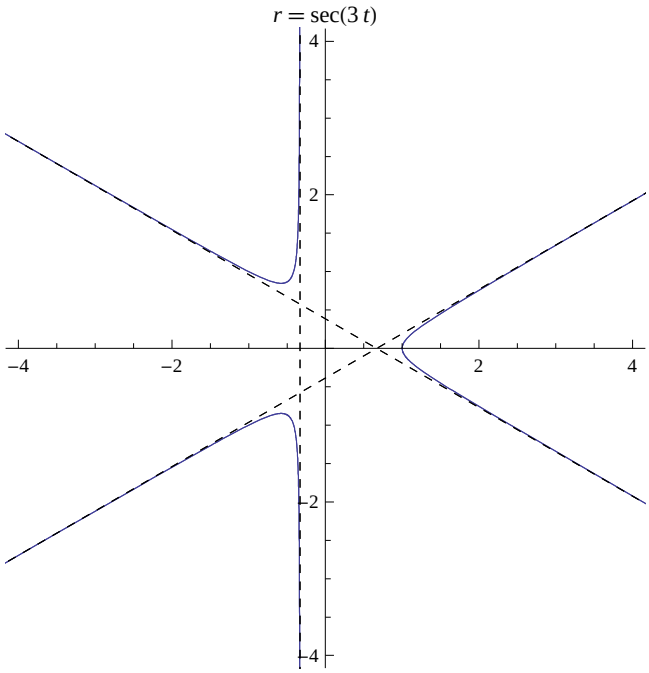


★ Exclusions

```
Plot[1 / Cos[3 t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
 Ticks → {Table[k * π / 6, {k, 0, 12}] , Automatic}, AxesOrigin → {0, 0}]
```

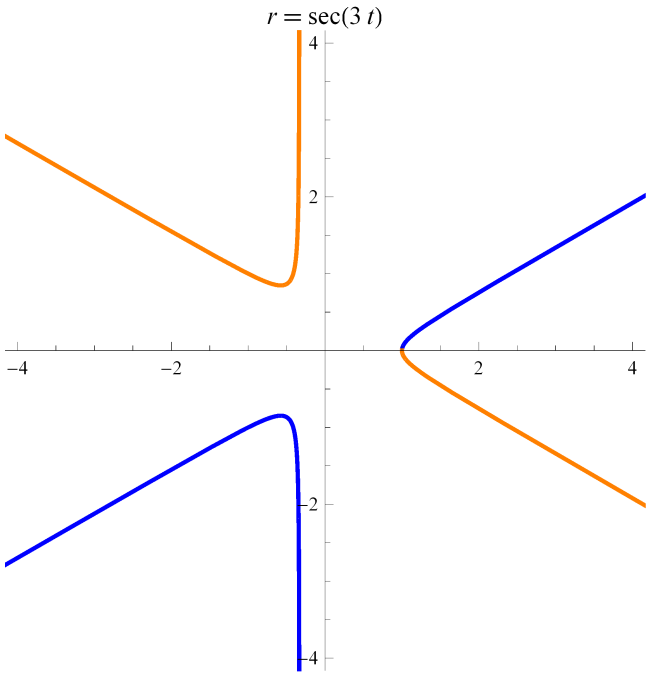


```
PolarPlot[1 / Cos[3 t], {t, 0, 2 π}, PlotLabel → r == Sec[3 t],
 Exclusions → {Cos[3 t] == 0}, ExclusionsStyle → Dashed, PlotRange → 4]
```



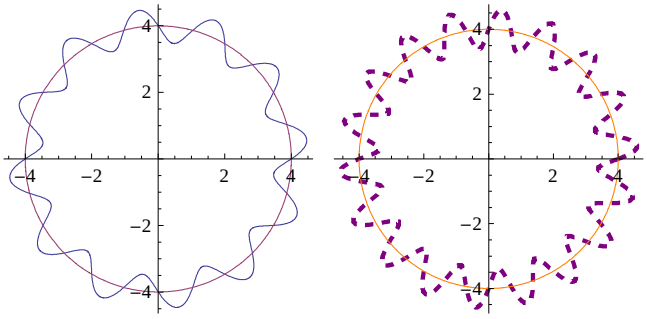
★ Otras Opciones de Estilo

```
PolarPlot[1 / Cos[3 t], {t, 0, Pi}, PlotLabel → r == Sec[3 t],
 ColorFunction → Function[{x, y, t, r}, If[ Cos[3 t] < 0, Orange, Blue]],
 PlotStyle → Thick, Exclusions → {Cos[3 t] == 0}, PlotRange → 4]
```



★ Otras opciones de PolarPlot[ ]

```
g1 = PolarPlot[{4 + 0.5 * Sin[12 * t], 4}, {t, 0, 2 π}];
g2 = PolarPlot[{4 + 0.6 * Sin[18 * t], 4}, {t, 0, 2 π},
  PlotStyle → {Directive[Dashed, Thick, Purple], Orange}]; GraphicsGrid[{{g1, g2}}]
```



5.2. Las funciones más relevantes en forma polar

▼ Círculos

★ Ecuación general de la circunferencia: centro (a,b) y radio c

```
ec = (x - a) ^ 2 + (y - b) ^ 2 == c ^ 2
```

$$(-a+x)^2+(-b+y)^2==c^2$$

★ Círculo1: con centro en el eje OY ((a,b)=(0,b), a=0 y c=b)

```
ec1 = ec /. {a → 0, c → b}
```

$$x^2+(-b+y)^2==b^2$$

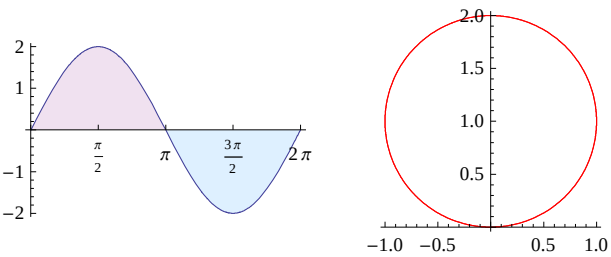
```
polar1 = ec1 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$$r[t]^2==2\,b\,r[t]\,\text{Sin}[t]$$

```
Solve[polar1, r[t]]
```

$$\{\{r[t]\to 0\},\{r[t]\to 2\,b\,\text{Sin}[t]\}\}$$

```
circulo1[t_, b_] = 2 * b Sin[t];
g1 = Plot[circulo1[t, 1], {t, 0, 2 π}, Ticks → {{0, π / 2 , π, 3 π / 2, 2 π} , Automatic},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
c1 = PolarPlot[circulo1[t, 1], {t, 0, 2 π}, PlotStyle → Red]; GraphicsGrid[{{g1, c1}}]
```



★ Círculo2: con centro en el eje OX; tal que (a,b)=(a,0), b=0 y c=a

```
ec2 = ec /. {b → 0, c → a}
```

$$(-a+x)^2+y^2 == a^2$$

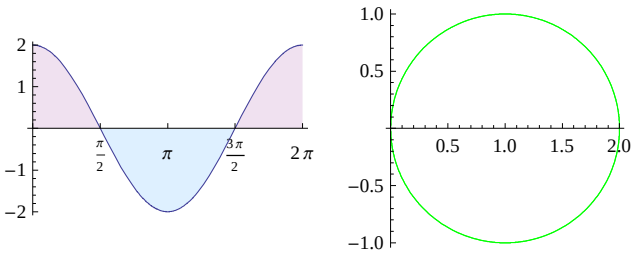
```
polar2 = ec2 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$$2\,a\,\text{Cos}[t]\,r[t] == r[t]^2$$

```
Solve[polar2, r[t]]
```

$$\{\{r[t] \to 0\}, \{r[t] \to 2\,a\,\text{Cos}[t]\}\}$$

```
circulo2[t_, a_] = 2 * a Cos[t];
g2 = Plot[circulo2[t, 1], {t, 0, 2 π}, Ticks → {{0, π / 2 , π, 3 π / 2, 2 π} , Automatic},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
c2 = PolarPlot[circulo2[t, 1], {t, 0, 2 π}, PlotStyle → Green];
GraphicsGrid[{{g2, c2}}]
```



★ Círculo3: con centro en el origen ; tal que (a,b)=(0,0), a=0 y b=0

```
ec3 = ec /. {a → 0, b → 0}
```

$$x^2+y^2 == c^2$$

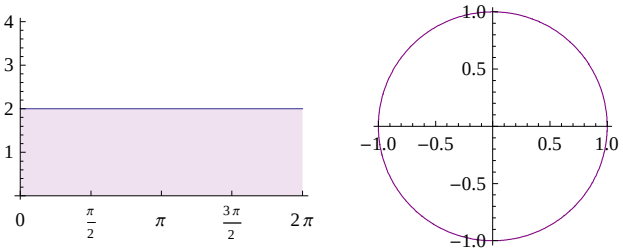
```
polar3 = ec3 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$$c^2 == r[t]^2$$

```
Solve[polar3, r[t]]
```

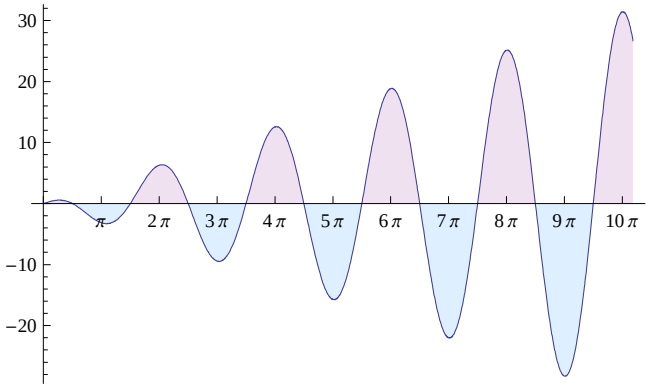
$$\{\{r[t] \to -c\}, \{r[t] \to c\}\}$$

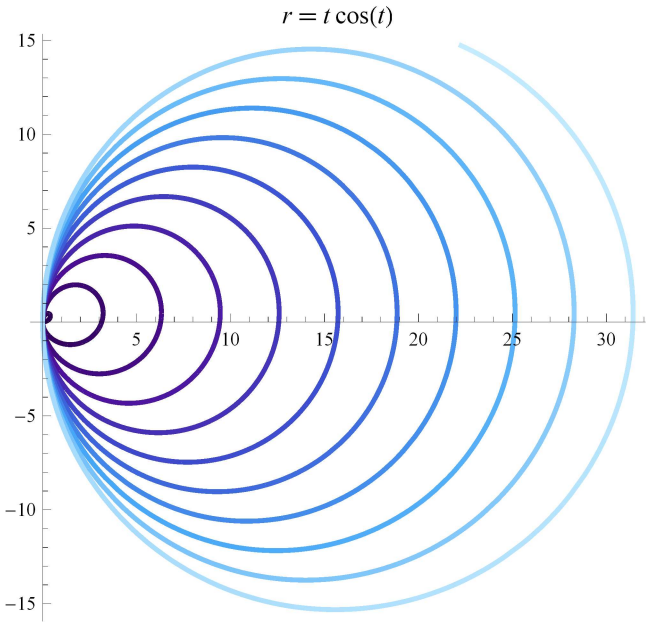
```
circulo3[t_, a_] = a;
g3 = Plot[circulo3[t, 2] , {t, 0, 2 π}, Ticks → {{0, π / 2 , π, 3 π / 2, 2 π} , Automatic},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
c3 = PolarPlot[circulo3[t, 1], {t, 0, 2 π}, PlotStyle → Purple];
GraphicsGrid[{{g3, c3}}]
```



★ ESPIRAL DE CÍRCULOS

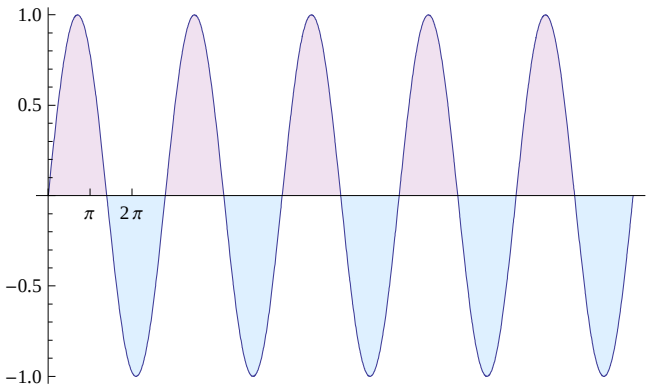
```
g3 = Plot[t Cos[t], {t, 0, 32}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {Table[k * π, {k, 0, 10}] , Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[ t Cos[t], {t, 0, 32}, ColorFunction → "DeepSeaColors",
  PlotStyle → Thickness[0.008], PlotLabel → r == t Cos[t]]
```



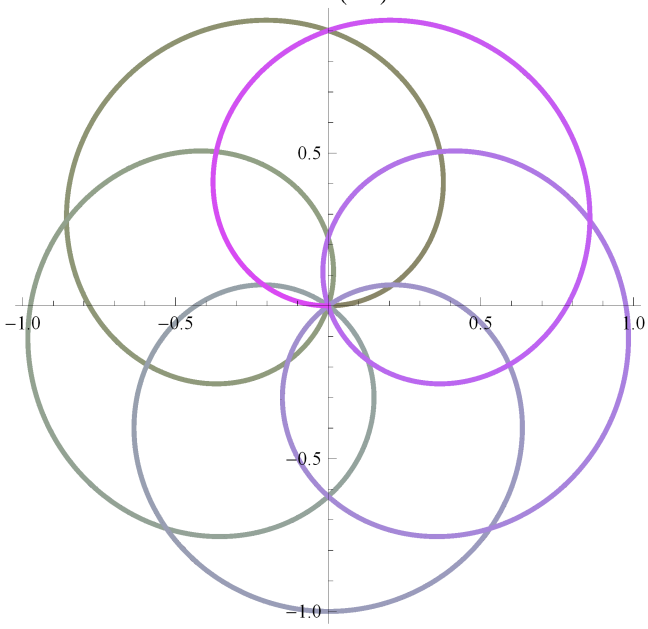


★ PSEUDOCÍRCULOS

```
g3 = Plot[Sin[5 t / 7], {t, 0, 14 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[Sin[5 t / 7], {t, 0, 14 π}, ColorFunction → "AuroraColors",
  PlotStyle → Thickness[0.008], PlotLabel → r == Sin[5 t / 7]]
```



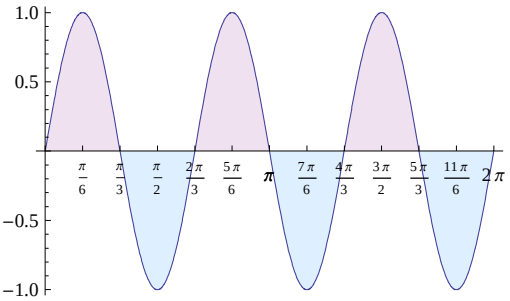
$$r = \sin\left(\frac{5\,t}{7}\right)$$

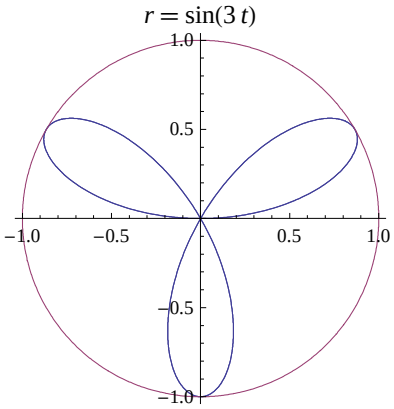


▼ Rosáceas

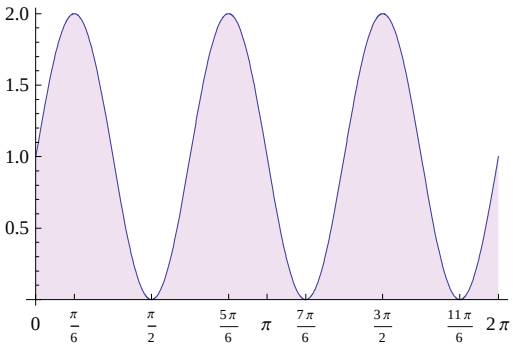
★ Trébol de tres hojas

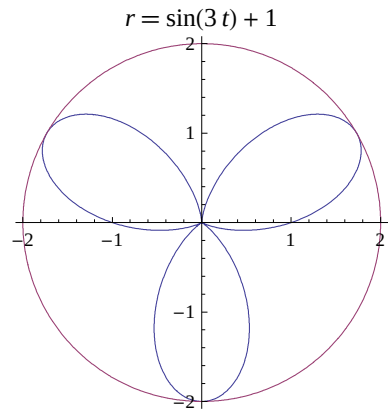
```
g1 = Plot[Sin[3 * t], {t, 0, 2 π}, Ticks →
     {{0, π / 6, π / 3 , 2 π / 3, π / 2 , π, 5 π / 6, π , 7 π / 6, 4 π / 3, 3 π / 2, 5 π / 3, 11 π / 6, 2 π} ,
      Automatic}, Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
g2 = PolarPlot[{Sin[3 * t], 1}, {t, 0, 2 π}, PlotLabel → r == Sin[3 * t]];
GraphicsGrid[{{g1, g2}}]
```



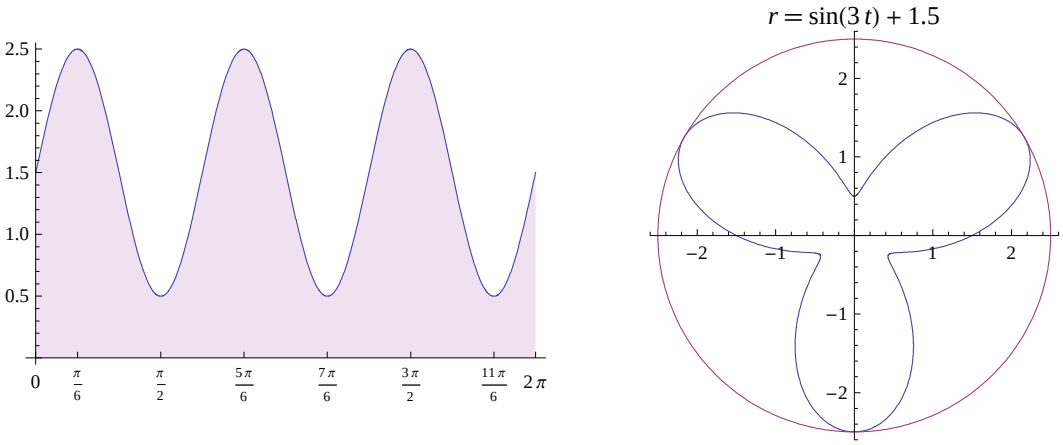



```
g1 = Plot[1 + Sin[3 * t], {t, 0, 2 π},
    Ticks → {{0, π / 6, π / 2 , 5 π / 6, π, 7 π / 6, 3 π / 2, 11 π / 6, 2 π} , Automatic},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
g2 = PolarPlot[{1 + Sin[3 * t], 2}, {t, 0, 2 π}, PlotLabel → r == 1 + Sin[3 * t]];
GraphicsGrid[{{g1, g2}}]
```

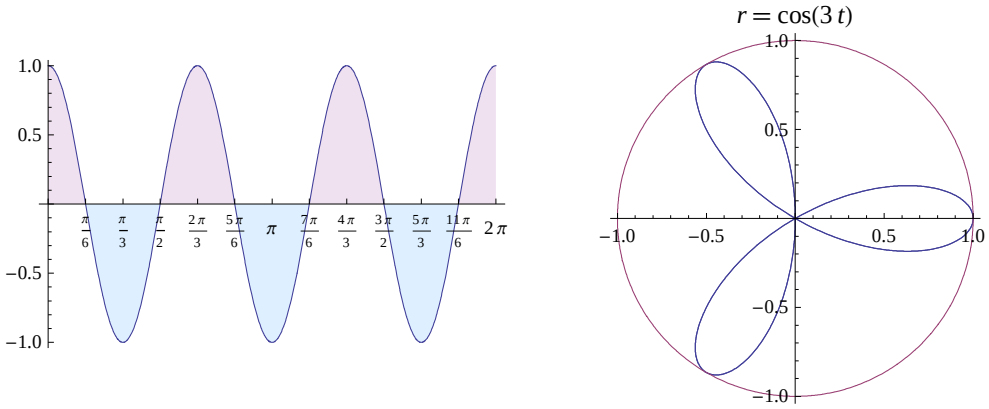




```
g1 = Plot[1.5 + Sin[3 * t], {t, 0, 2 π}, AxesOrigin → {0, 0},
    Ticks → {{0, π / 6, π / 2 , 5 π / 6, 7 π / 6, 3 π / 2, 11 π / 6, 2 π} , Automatic},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
g2 = PolarPlot[{1.5 + Sin[3 * t], 2.5}, {t, 0, 2 π}, PlotLabel → r == 1.5 + Sin[3 * t]];
GraphicsGrid[{{g1, g2}}]
```

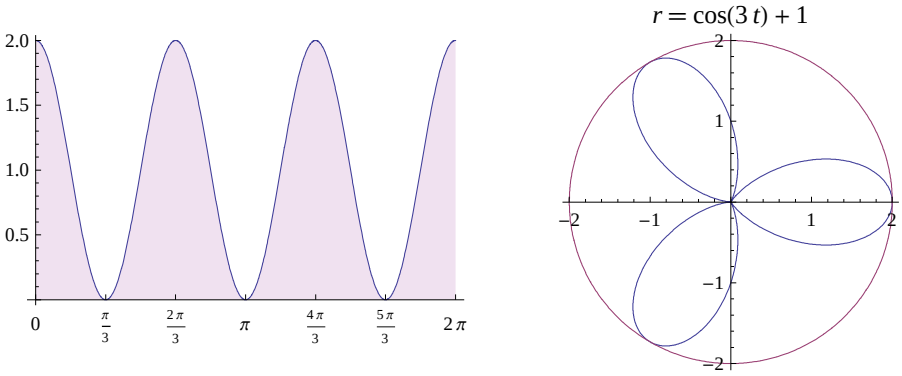



```
g1 = Plot[Cos[3 * t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 6, π / 3 , π / 2 , 2 π / 3, 5 π / 6, π , 7 π / 6, 4 π / 3,
       3 π / 2, 5 π / 3, 11 π / 6, 2 π} , Automatic}, AxesOrigin → {0, 0}];
g2 = PolarPlot[{Cos[3 * t], 1}, {t, 0, 2 π}, PlotLabel → r == Cos[3 * t]];
GraphicsGrid[{{g1, g2}}]
```



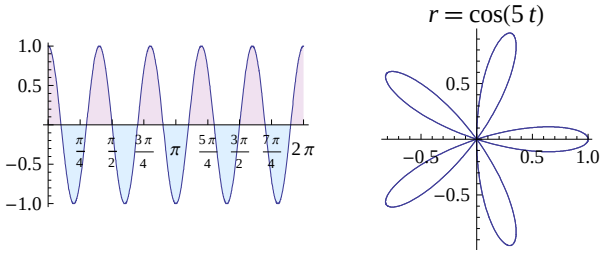

```
g1 = Plot[1 + Cos[3 * t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 3 , 2 π / 3, π , 4 π / 3, 5 π / 3, 2 π} , Automatic}, AxesOrigin → {0, 0}];

g2 = PolarPlot[{1 + Cos[3 * t], 2}, {t, 0, 2 π}, PlotLabel → r == 1 + Cos[3 * t]];
```

```
GraphicsGrid[{{g1, g2}}]
```

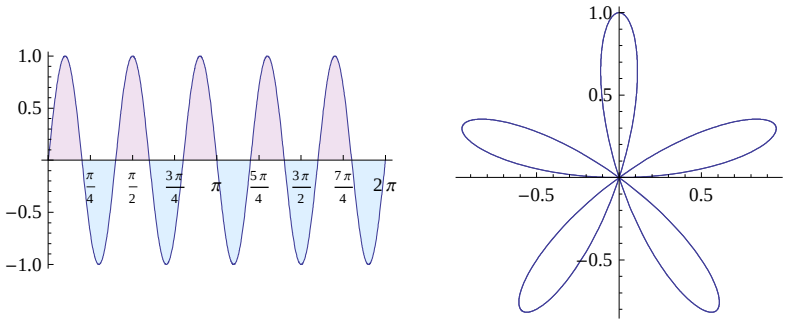


★ Rosas de 5 pétalos

```
g1 = Plot[Cos[5 * t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
   Ticks → {{0, π / 4 , π / 2, 3 π / 4, π , 5 π / 4 , 3 π / 2, 7 π / 4, 2 π} , Automatic},
   AxesOrigin → {0, 0}];

g2 = PolarPlot[Cos[5 * t], {t, 0, 2 π}, PlotLabel → r == Cos[5 * t]];

GraphicsGrid[{{g1, g2}}]
```



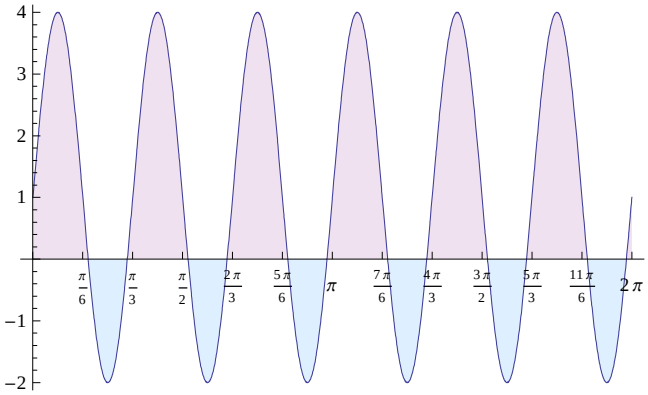

```
g1 = Plot[Sin[5 * t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
   Ticks → {{0, π / 4 , π / 2, 3 π / 4, π , 5 π / 4 , 3 π / 2, 7 π / 4, 2 π} , Automatic},
   AxesOrigin → {0, 0}];

g2 = PolarPlot[Sin[5 * t], {t, 0, 2 Pi}];

GraphicsGrid[{{g1, g2}}]
```



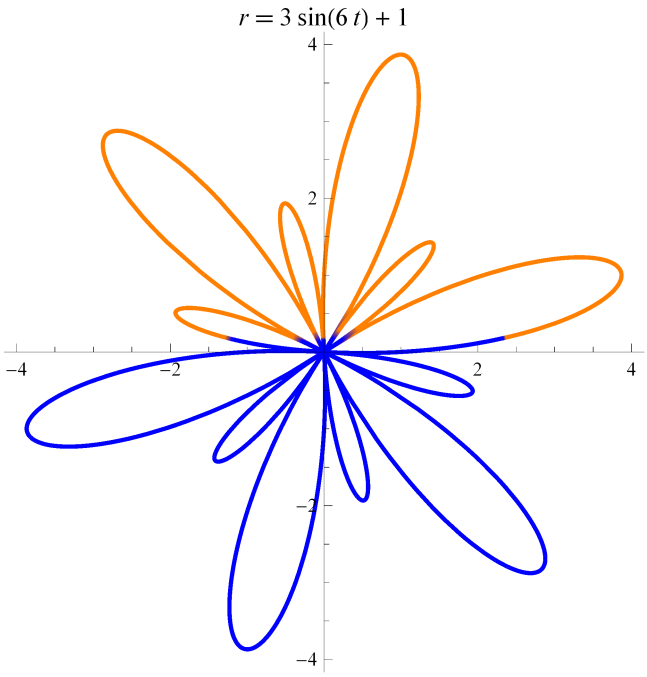
▼ Otras Rosáceas

★ Ejemplo 1

```
g3 = Plot[1 + 3 Sin[6 t], {t, 0, 2 π},
   Filling → Axis, Ticks → {Table[k * π / 6, {k, 0, 12}] , Automatic},
   FillingStyle → {LightBlue, LightPurple},
   Ticks → {{0,  π , 2 π} , Automatic}, AxesOrigin → {0, 0}]
```

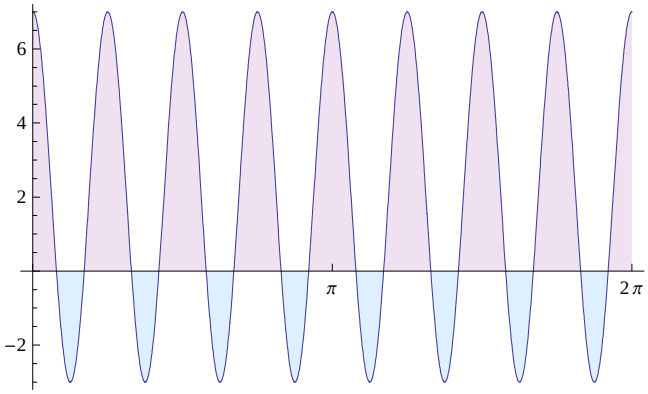



```
PolarPlot[1 + 3 Sin[6 t], {t, 0, 2 Pi},
 ColorFunction → Function[{x, t}, If[Pi / 6 < t < Pi / 3, Orange, Blue]],
 PlotRange → {{-4, 4}, {-4, 4}}, PlotStyle → Thick, PlotLabel → r == 1 + 3 Sin[6 t]]
```

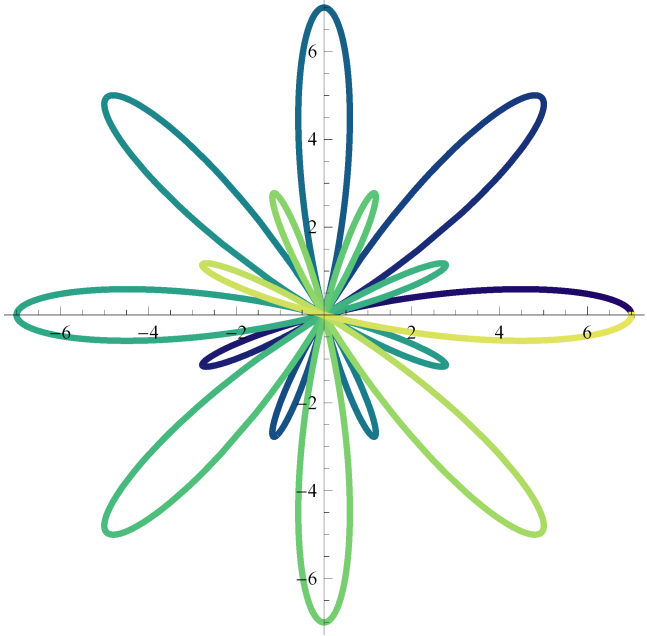


★ Ejemplo 2

```
g3 = Plot[2 + 5 Cos[8 t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 + 5 Cos[8 t], {t, 0, 2 π}, PlotLabel → r == 2 + 5 Cos[8 t],
  ColorFunction → "BlueGreenYellow", PlotStyle → Thickness[0.01]]
```

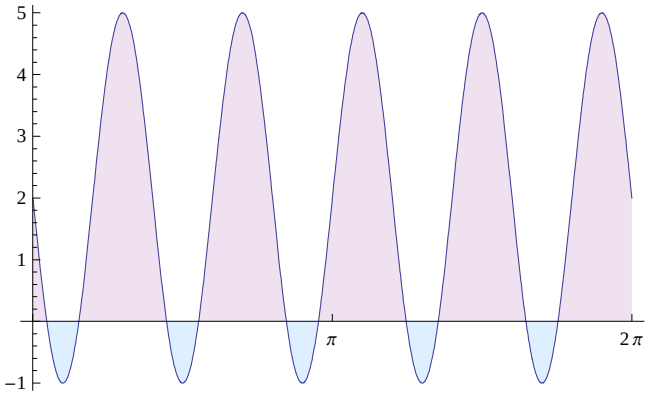


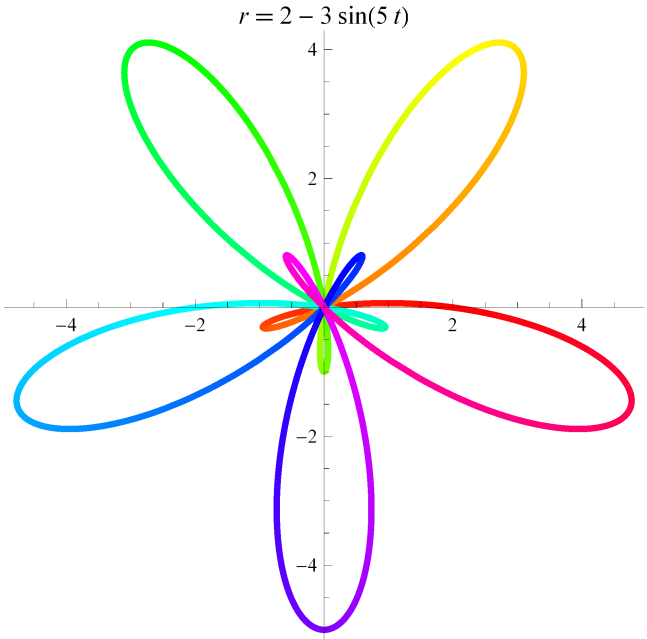
$r = 5\cos(8\,t) + 2$



★ Ejemplo 3

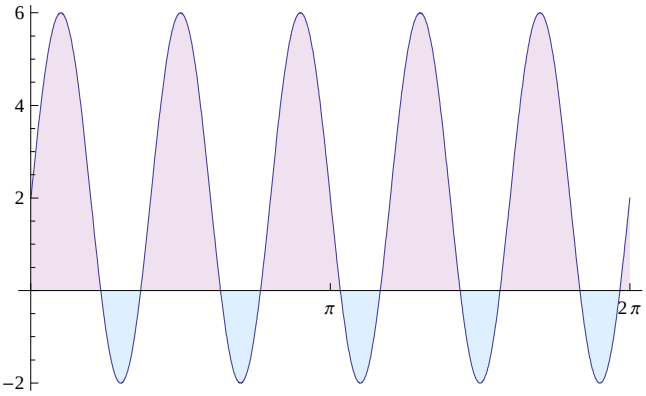
```
g3 = Plot[2 - 3 Sin[5 t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 - 3 Sin[5 t], {t, 0, 2 π}, PlotLabel → r == 2 - 3 Sin[5 t],
  ColorFunction → Function[{x, y, z}, Hue[z]], PlotStyle → Thickness[0.01]]
```

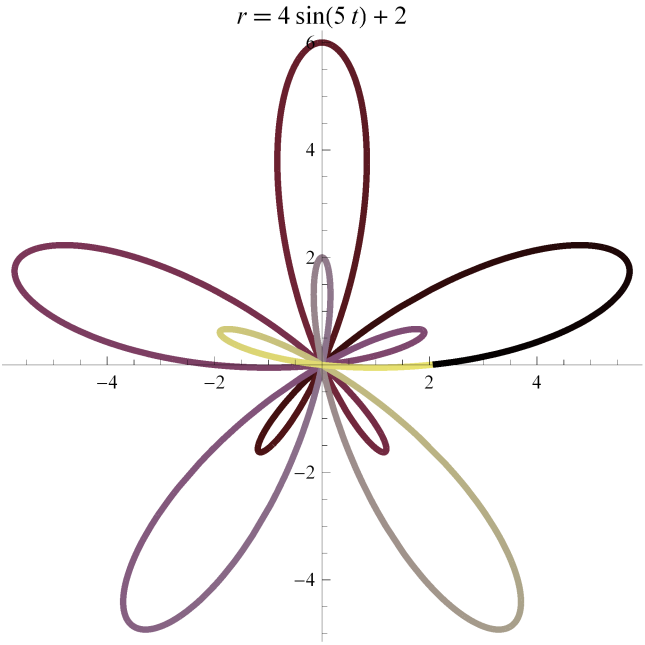




★ Ejemplo 4

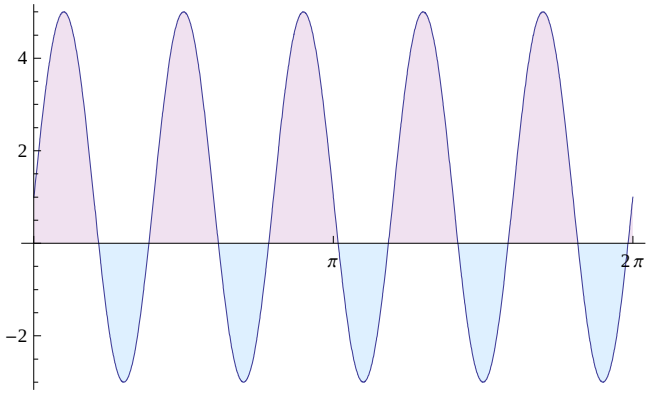
```
g3 = Plot[2 + 4 Sin[5 t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 + 4 Sin[5 t], {t, 0, 2 π}, ColorFunction → "PlumColors",
  PlotStyle → Thickness[0.01], PlotLabel → r == 2 + 4 Sin[5 t]]
```

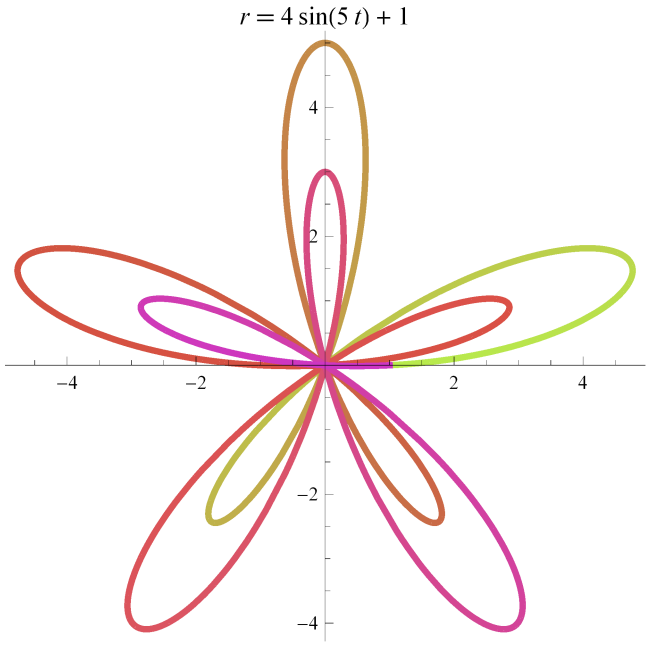




★ Ejemplo 5

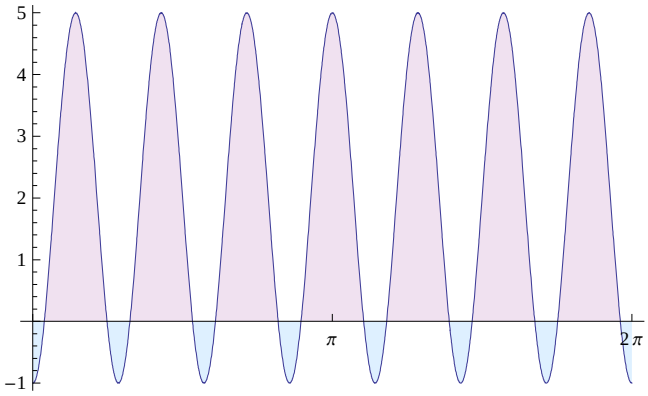
```
g3 = Plot[1 + 4 Sin[5 t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[1 + 4 Sin[5 t], {t, 0, 2 π}, ColorFunction → "NeonColors",
  PlotStyle → Thickness[0.01], PlotLabel → r == 1 + 4 Sin[5 t]]
```

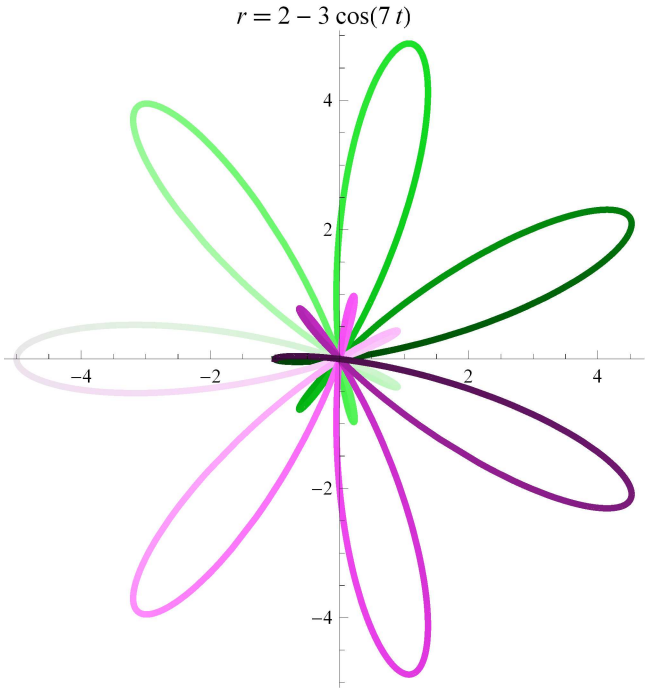




★ Ejemplo 6

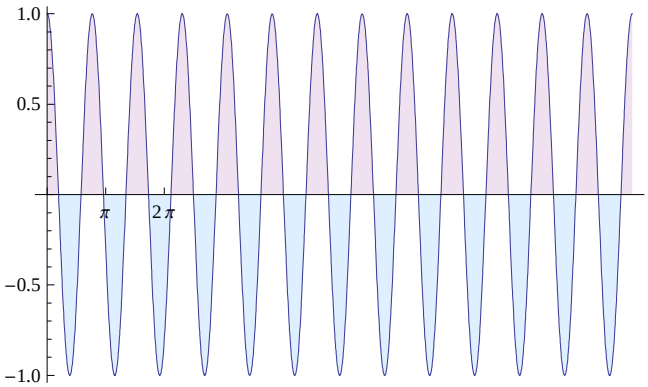
```
g3 = Plot[2 - 3 Cos[7 t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 - 3 Cos[7 t], {t, 0, 2 π}, ColorFunction → "GreenPinkTones",
  PlotStyle → Thickness[0.01], PlotLabel → r == 2 - 3 Cos[7 t]]
```

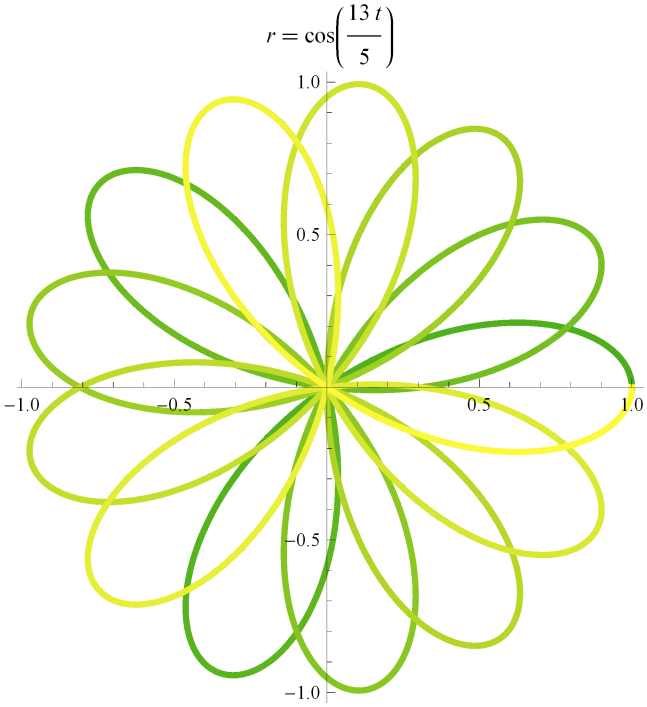




★ Ejemplo 7

```
g3 = Plot[Cos[13 t / 5], {t, 0, 10 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[ Cos[13 t / 5], {t, 0, 10 π}, ColorFunction → "AvocadoColors",
  PlotStyle → Thickness[0.01], PlotLabel → r == Cos[13 t / 5]]
```

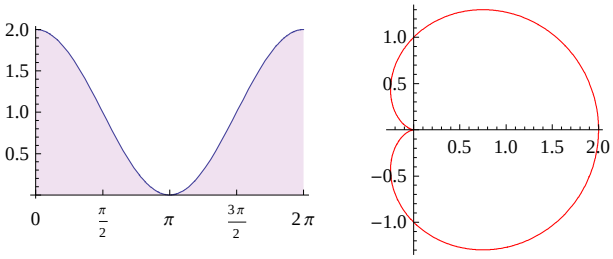




▼ Cardiodes

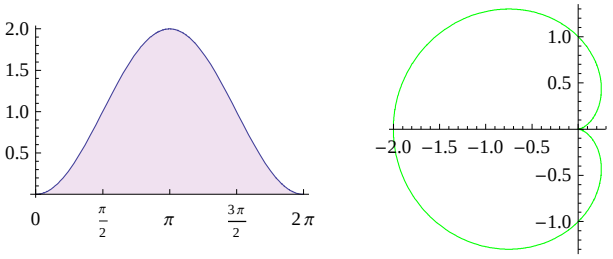
★ Cardioide 1

```
cardioide1[t_, a_] = a (1 + Cos[t]);
g1 = Plot[cardioide1[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π , 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
car1 = PolarPlot[cardioide1[t, 1], {t, 0, 2 π}, PlotStyle → Red]; GraphicsGrid[{{g1, car1}}]
```



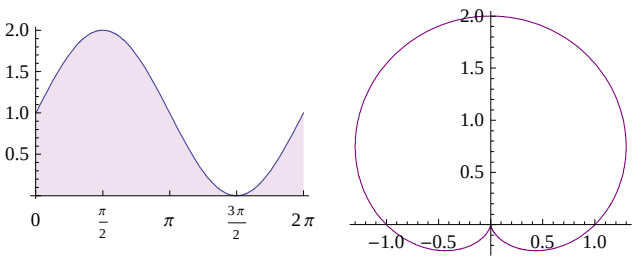
★ Cardioide 2

```
cardioide2[t_, a_] = a (1 - Cos[t]);
g2 = Plot[cardioide2[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π , 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
car2 = PolarPlot[cardioide2[t, 1], {t, 0, 2 π}, PlotStyle → Green];
GraphicsGrid[{{g2, car2}}]
```



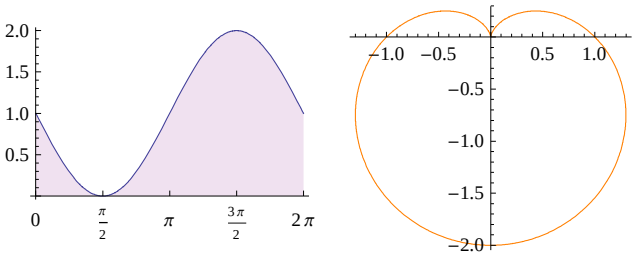
★ Cardioide 3

```
cardioide3[t_, a_] = a (1 + Sin[t]);
g3 = Plot[cardioide3[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π , 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
car3 = PolarPlot[cardioide3[t, 1], {t, 0, 2 π}, PlotStyle → Purple];
GraphicsGrid[{{g3, car3}}]
```



★ Cardioide 4

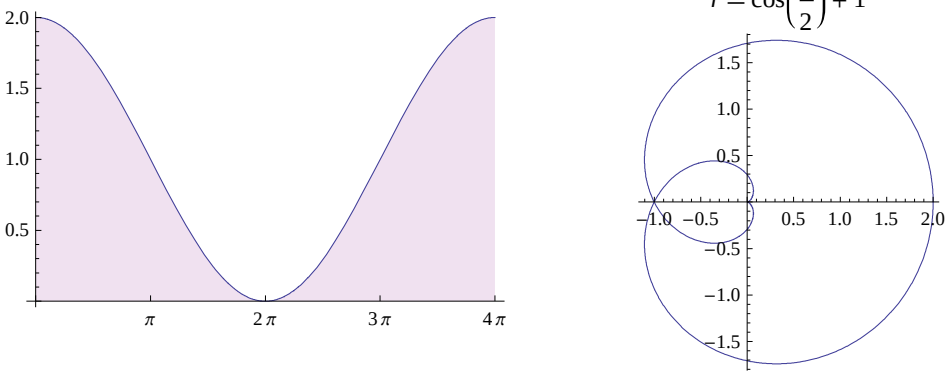
```
cardioide4[t_, a_] = a (1 - Sin[t]);
g4 = Plot[cardioide4[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π, 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
car4 = PolarPlot[cardioide4[t, 1], {t, 0, 2 π}, PlotStyle → Orange];
GraphicsGrid[{{g4, car4}}]
```



▼ Pseudocardiodes

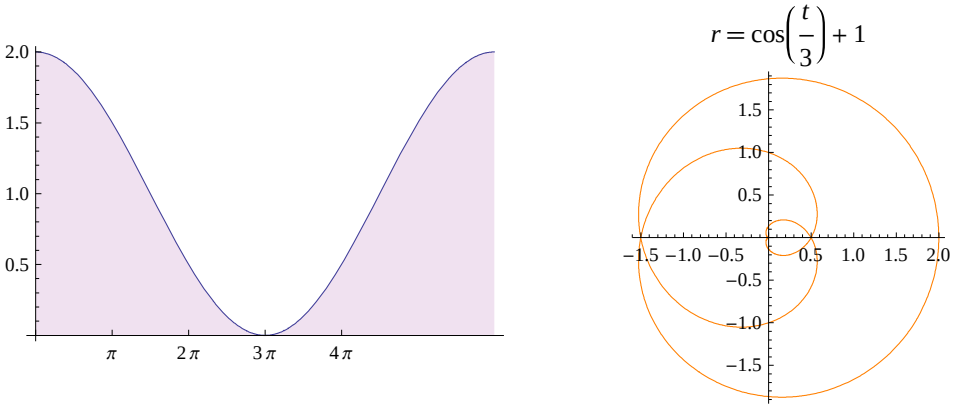
★ Ejemplo 1

```
g1 = Plot[1 + Cos[t / 2], {t, 0, 4 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π , 2 π, 3 π, 4 π} , Automatic}, AxesOrigin → {0, 0}];
c1 = PolarPlot[1 + Cos[t / 2], {t, 0, 4 π}, PlotLabel → r == 1 + Cos[t / 2]];
GraphicsGrid[{{g1, c1}}]
```



★ Ejemplo 2

```
g2 = Plot[1 + Cos[t / 3], {t, 0, 6 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π, 2 π, 3 π, 4 π}, Automatic}, AxesOrigin → {0, 0}];
c2 = PolarPlot[1 + Cos[t / 3], {t, 0, 6 π}, PlotLabel → r == 1 + Cos[t / 3], PlotStyle → Orange];
GraphicsGrid[{{g2, c2}}]
```



★ Ejemplo 3

```
g3 = Plot[ Cos[t / 2], {t, 0, 4 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π, 2 π, 3 π, 4 π}, Automatic}, AxesOrigin → {0, 0}];
c3 = PolarPlot[ Cos[t / 2], {t, 0, 6 π}, PlotLabel → r == Cos[t / 2], PlotStyle → Purple];
GraphicsGrid[{{g3, c3}}]
```