K-III BLOKEA: Graduak, radianak eta oinarrizko funtzio trigonometrikoak

1. ARIKETA:

Adierazi honako angelu hauek graduetan eta kalkulatu beren sinuaren eta kosinuaren balioak:

a) $\dfrac{5\pi}{12}$ b) $\dfrac{11\pi}{6}$

Ebazpena:

a) $\dfrac{5\pi}{12}\ \text{rad}=\dfrac{5\pi}{12}\ \text{rad}\cdot\dfrac{360^\circ}{2\pi\ \text{rad}}=75^\circ$

$$\sin\left(75^\circ\right)=\sin\left(45^\circ+30^\circ\right)=\sin\left(45^\circ\right)\cos\left(30^\circ\right)+\cos\left(45^\circ\right)\sin\left(30^\circ\right)=\cdots$$

$$\cdots=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)$$

$$\cos\left(75^\circ\right)=\cos\left(45^\circ+30^\circ\right)=\cos\left(45^\circ\right)\cos\left(30^\circ\right)-\sin\left(45^\circ\right)\sin\left(30^\circ\right)=\cdots$$

$$\cdots=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right)$$

b) $\dfrac{11\pi}{6}\ \text{rad}=\dfrac{11\pi}{6}\ \text{rad}\cdot\dfrac{360^\circ}{2\pi\ \text{rad}}=330^\circ$

$$\sin\left(\frac{11\pi}{6}\right)=\sin\left(-\frac{\pi}{6}+\frac{12\pi}{6}\right)=\sin\left(-\frac{\pi}{6}+2\pi\right)=\sin\left(-\frac{\pi}{6}\right)=-\sin\left(\frac{\pi}{6}\right)=-\frac{1}{2}$$

$$\cos\left(\frac{11\pi}{6}\right)=\cos\left(-\frac{\pi}{6}+\frac{12\pi}{6}\right)=\cos\left(-\frac{\pi}{6}+2\pi\right)=\cos\left(-\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$$

2. ARIKETA:

Deduzitu sinuen biderkaduraren adierazpen orokorra, angeluen baturaren eta kenduraren arrazoi trigonometrikoetatik abiatuz.

Ebazpena:

$$\cos(\alpha+\beta)=\cos(\alpha)\cdot\cos(\beta)-\sin(\alpha)\cdot\sin(\beta) \qquad (1)$$

$$\cos(\alpha-\beta)=\cos(\alpha)\cdot\cos(\beta)+\sin(\alpha)\cdot\sin(\beta) \qquad (2)$$

eta (2)-(1) kenduz:

$$\cos(\alpha-\beta)-\cos(\alpha+\beta)=2\sin(\alpha)\cdot\sin(\beta)$$

$$\sin(\alpha)\cdot\sin(\beta)=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$$

3. ARIKETA:

Deduzitu angelu bikoitzaren sinuaren adierazpen orokorra, angeluen baturaren sinuaren adierazpen orokorretik abiatuz.

Ebazpena:

$$\sin(\alpha+\beta)=\sin(\alpha)\cdot\cos(\beta)+\cos(\alpha)\cdot\sin(\beta)$$

non , $\alpha=\beta$ bada, honako hau lortzen den:

$$\sin(\alpha+\alpha)=\sin(\alpha)\cdot\cos(\alpha)+\cos(\alpha)\cdot\sin(\alpha)$$

$$\sin(2\alpha)=2\sin(\alpha)\cdot\cos(\alpha)$$

4. ARIKETA:

Deduzitu angeluen baturaren tangentearen adierazpen orokorra.

Ebazpena:

$$\mathrm{tg}(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin(\alpha)\cdot\cos(\beta)+\cos(\alpha)\cdot\sin(\beta)}{\cos(\alpha)\cdot\cos(\beta)-\sin(\alpha)\cdot\sin(\beta)}=\cdots$$

$$\cdots=\frac{\dfrac{\sin(\alpha)\cdot\cos(\beta)+\cos(\alpha)\cdot\sin(\beta)}{\cos(\alpha)\cdot\cos(\beta)}}{\dfrac{\cos(\alpha)\cdot\cos(\beta)-\sin(\alpha)\cdot\sin(\beta)}{\cos(\alpha)\cdot\cos(\beta)}}=\frac{\dfrac{\sin(\alpha)}{\cos(\alpha)}+\dfrac{\sin(\beta)}{\cos(\beta)}}{1-\dfrac{\sin(\alpha)}{\cos(\alpha)}\cdot\dfrac{\sin(\beta)}{\cos(\beta)}}=\frac{\mathrm{tg}(\alpha)+\mathrm{tg}(\beta)}{1-\mathrm{tg}(\alpha)\cdot\mathrm{tg}(\beta)}$$

Beraz:

$$\mathrm{tg}(\alpha+\beta)=\frac{\mathrm{tg}(\alpha)+\mathrm{tg}(\beta)}{1-\mathrm{tg}(\alpha)\cdot\mathrm{tg}(\beta)}$$

5. ARIKETA:

Kalkulatu $\operatorname{tg}\left(\frac{\pi}{12}\right)$.

Ebazpena:

Dakigunez, $\operatorname{tg}\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}}$

eta $\pi/12$ angelua lehenengo koadrantean dagoenez, angelu horren tangentea positiboa izango da. Beraz:

$$\operatorname{tg}\left(\frac{\pi}{12}\right)=+\sqrt{\frac{1-\cos(\pi/6)}{1+\cos(\pi/6)}}=+\sqrt{\frac{1-\sqrt{3}/2}{1+\sqrt{3}/2}}=+\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}=\cdots$$

$$\cdots=+\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}}=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{\left(2^2-\left(\sqrt{3}\right)^2\right)}}=\left(2-\sqrt{3}\right)$$

6. ARIKETA:

Ebatzi $\sin(2x)=\sin(x)$ ekuazioa.

Ebazpena:

$$\sin(2x)=\sin(x)\rightarrow 2\sin(x)\cdot\cos(x)=\sin(x)\rightarrow 2\sin(x)\cdot\cos(x)-\sin(x)=0$$

$$\sin(x)\cdot\left(2\cos(x)-1\right)=0\rightarrow\begin{cases}\sin(x)=0\rightarrow x=k\pi\\ \cos(x)=\dfrac{1}{2}\rightarrow x=\pm\dfrac{\pi}{3}+2k\pi\end{cases},\quad k\in\mathbb{Z}$$

7. ARIKETA:

Ebatzi $\sqrt{3}\sin(x)+\cos(x)=1$ ekuazioa.

Ebazpena:

$$\sqrt{3}\sin(x)+\cos(x)=1 \rightarrow \frac{\sqrt{3}}{2}\sin(x)+\frac{1}{2}\cos(x)=\frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right)\cdot\sin(x)+\sin\left(\frac{\pi}{6}\right)\cdot\cos(x)=\frac{1}{2} \rightarrow \sin\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$$

Beraz,

$$\begin{cases} x+\frac{\pi}{6}=\frac{\pi}{6}+2k\pi \rightarrow x=2k\pi \\ x+\frac{\pi}{6}=\frac{5\pi}{6}+2k\pi \rightarrow x=\frac{2\pi}{3}+2k\pi \end{cases}, \quad k\in\mathbb{Z}$$

8. ARIKETA:

Zirkunferentzia trigonometrikotik abiatuz, egiaztatu $\sin(\pi-\alpha)=\sin(\alpha)$ betetzen dela.

Ebazpena: