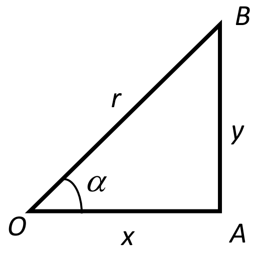
K-III BLOKEA: Graduak, radianak eta oinarrizko funtzio trigonometrikoak

Sinu, kosinu eta tangente arrazoi trigonometrikoak ondoko eran definitzen dira:

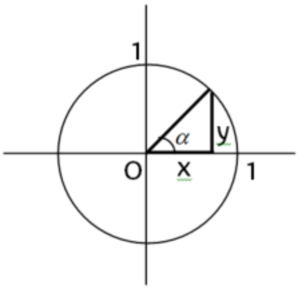
$$\sin(\alpha)=\frac{AB}{OB}=\frac{y}{r}$$
$$\cos(\alpha)=\frac{OA}{OB}=\frac{x}{r}$$
$$\operatorname{tg}(\alpha)=\frac{AB}{OA}=\frac{y}{x}$$



eta alderantzizko arrazoi trigonometrikoak:

$$\operatorname{cosec}(\alpha)=\frac{1}{\sin(\alpha)}=\frac{r}{y}\ ;\ \sec(\alpha)=\frac{1}{\cos(\alpha)}=\frac{r}{x}\ ;\ \operatorname{ctg}(\alpha)=\frac{1}{\operatorname{tg}(\alpha)}=\frac{x}{y}$$

O zentroko eta $r=1$ erradioko zirkunferentzia trigonometrikoan, edozein α angeluri dagokien x eta y zuzenkien balioak $\cos(\alpha)$ eta $\sin(\alpha)$ balioak hurrenez hurren dira.



non koadrante bakoitzean arrazoi trigonometrikoen zeinuak honako hauek diren:

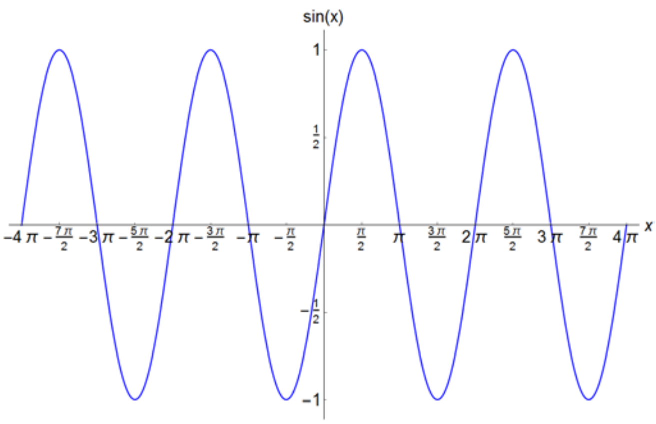
	Lehenengo koadrantea	Bigarren koadrantea	Hirugarren koadrantea	Laugarren koadrantea
$\sin(\alpha)$	+	+	–	–
$\cos(\alpha)$	+	–	–	+
$\operatorname{tg}(\alpha)$	+	–	+	–

Arrazoi Trigonometriko Erabilienak

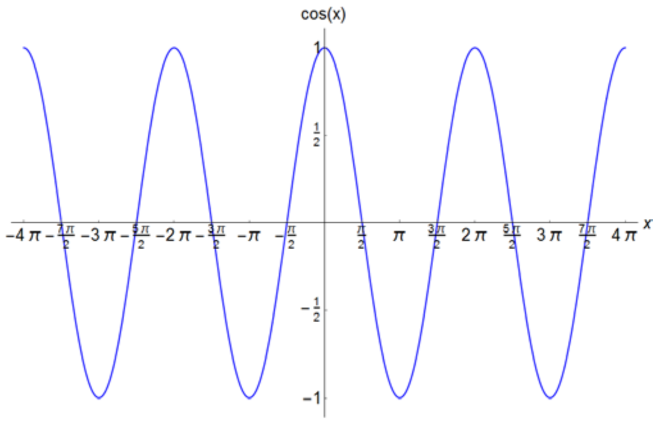
Ondoko taulan arrazoi trigonometriko erabilienak laburbiltzen dira. Ikusten denez, angeluak gradutan eta radianetan adierazten dira, bi unitate horien arteko erlazioa "$360^{\circ} = 2\pi$ rad" izanik.

	0° 0 rad	30° $\pi/6$ rad	45° $\pi/4$ rad	60° $\pi/3$ rad	90° $\pi/2$ rad
$\sin(\alpha)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(\alpha)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\mathrm{tg}(\alpha)$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$\nexists$

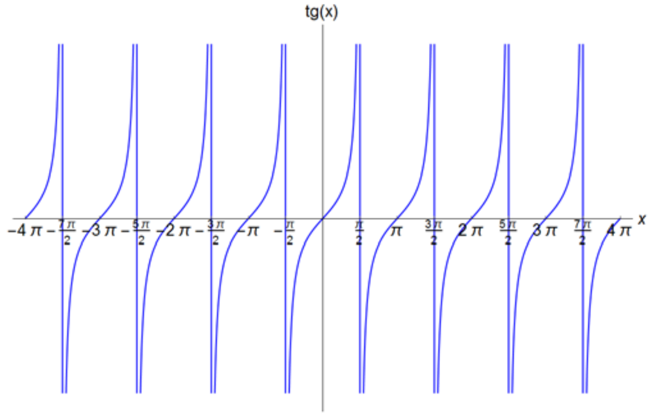
<u>Oinarrizko Funtzio Trigonometrikoen Adierazpen Grafikoak</u>



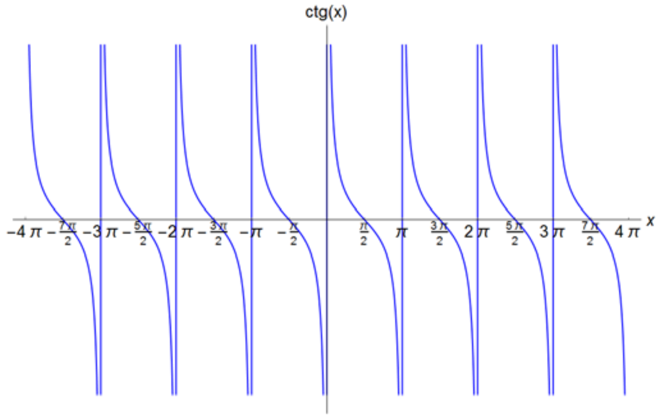
Periodoa: T=2π



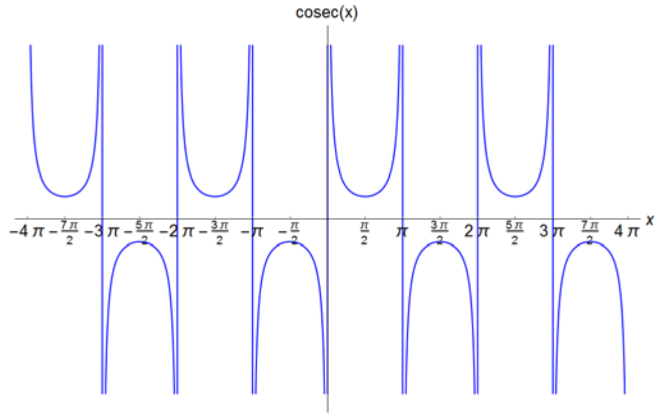
Periodoa: T=2π



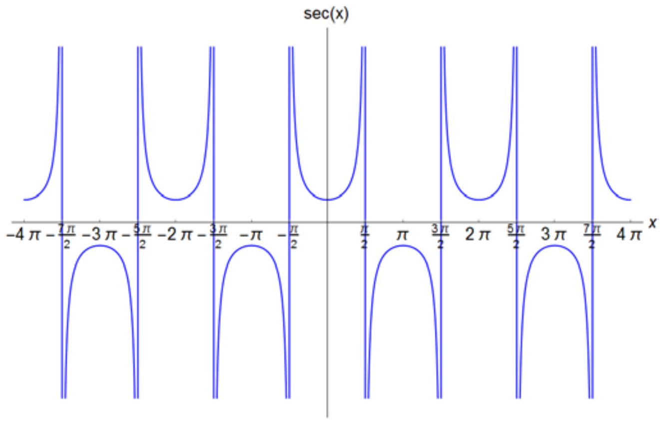
Periodoa: T=π



Periodoa: T=π



Periodoa: T=2π



Periodoa: T=2π

Trigonometriaren Oinarrizko Ekuazioa

Zirkunferentzia trigonometrikotik eta Pitagorasen teorematik abiatuz, honako formula hau defini daiteke:

$$\sin^2(\alpha)+\cos^2(\alpha)=x^2+y^2=1$$

non formula hori ***trigonometriaren oinarrizko ekuazioa*** deritzon.

Angeluen Baturaren eta Kenduraren Arrazoi Trigonometrikoak

$$\sin(\alpha+\beta)=\sin(\alpha)\cdot\cos(\beta)+\cos(\alpha)\cdot\sin(\beta)$$

$$\sin(\alpha-\beta)=\sin(\alpha)\cdot\cos(\beta)-\cos(\alpha)\cdot\sin(\beta)$$

$$\cos(\alpha+\beta)=\cos(\alpha)\cdot\cos(\beta)-\sin(\alpha)\cdot\sin(\beta)$$

$$\cos(\alpha-\beta)=\cos(\alpha)\cdot\cos(\beta)+\sin(\alpha)\cdot\sin(\beta)$$

$$\mathrm{tg}(\alpha+\beta)=\frac{\mathrm{tg}(\alpha)+\mathrm{tg}(\beta)}{1-\mathrm{tg}(\alpha)\cdot\mathrm{tg}(\beta)}$$

$$\mathrm{tg}(\alpha-\beta)=\frac{\mathrm{tg}(\alpha)-\mathrm{tg}(\beta)}{1+\mathrm{tg}(\alpha)\cdot\mathrm{tg}(\beta)}$$

Sinu eta Kosinuen Batura eta Kendura

$$\sin(\alpha)+\sin(\beta)=2\cdot\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha)-\sin(\beta)=2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha)+\cos(\beta)=2\cdot\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha)-\cos(\beta)=-2\cdot\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)$$

Sinu eta Kosinuen Biderkadura

$$\sin(\alpha)\cdot\sin(\beta)=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}$$

$$\sin(\alpha)\cdot\cos(\beta)=\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$$

$$\cos(\alpha)\cdot\cos(\beta)=\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$$

Beste Arrazoi Trigonometriko Batzuk

$$\sin(2\alpha)=2\cdot\sin(\alpha)\cdot\cos(\alpha) \qquad \cos(2\alpha)=\cos^2(\alpha)-\sin^2(\alpha) \qquad \operatorname{tg}(2\alpha)=\frac{2\operatorname{tg}(\alpha)}{1-\operatorname{tg}^2(\alpha)}$$

$$\sin\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos(\alpha)}{2}} \qquad \cos\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1+\cos(\alpha)}{2}} \qquad \operatorname{tg}\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}}$$

$$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos(\alpha) \qquad \cos\left(\frac{\pi}{2}+\alpha\right)=-\sin(\alpha) \qquad \operatorname{tg}\left(\frac{\pi}{2}+\alpha\right)=-\operatorname{ctg}(\alpha)$$

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos(\alpha) \qquad \cos\left(\frac{\pi}{2}-\alpha\right)=\sin(\alpha) \qquad \operatorname{tg}\left(\frac{\pi}{2}-\alpha\right)=\operatorname{tg}(\alpha)$$

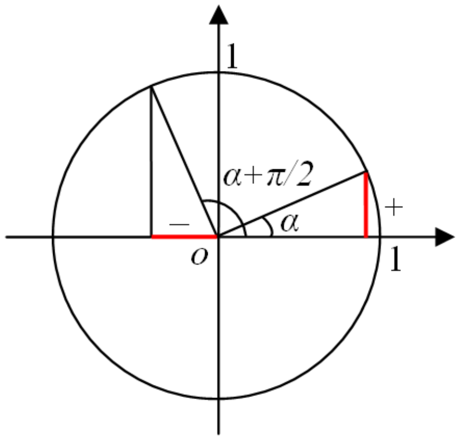
$$\sin(\pi+\alpha)=-\sin(\alpha) \qquad \cos(\pi+\alpha)=-\cos(\alpha) \qquad \operatorname{tg}(\pi+\alpha)=\operatorname{tg}(\alpha)$$

$$\sin(\pi-\alpha)=\sin(\alpha) \qquad \cos(\pi-\alpha)=-\cos(\alpha) \qquad \operatorname{tg}(\pi-\alpha)=-\operatorname{tg}(\alpha)$$

$$\sin(-\alpha)=-\sin(\alpha) \qquad \cos(-\alpha)=\cos(\alpha) \qquad \operatorname{tg}(-\alpha)=-\operatorname{tg}(\alpha)$$

Amaitzeko, aurreko arrazoietako batzuk zirkunferentzia trigonometrikoan argi eta garbi ikusten direla azpimarratu behar da. Adibidez:

$$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin(\alpha)\rightarrow\sin(\alpha)=-\cos\left(\frac{\pi}{2}+\alpha\right)$$



$$\sin(-\alpha)=-\sin(\alpha)\rightarrow\sin(\alpha)=-\sin(-\alpha)$$