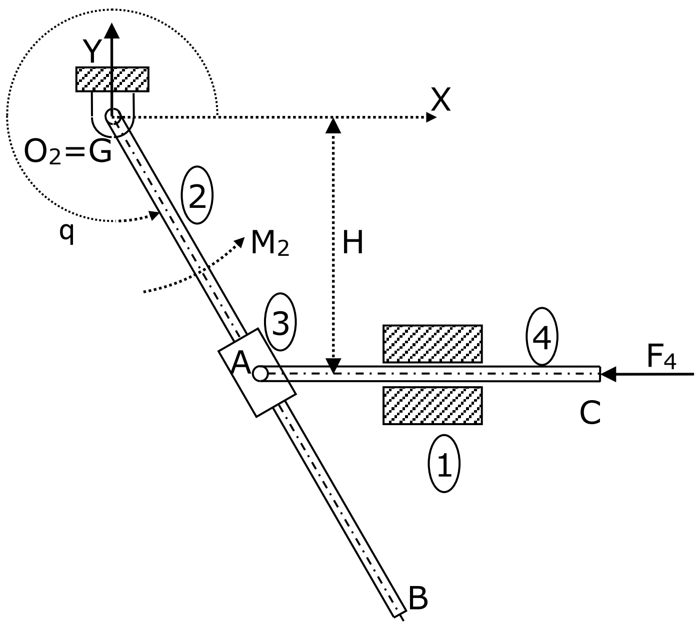
7. GAIA. DINAMIKAKO PROBLEMA ZUZENA

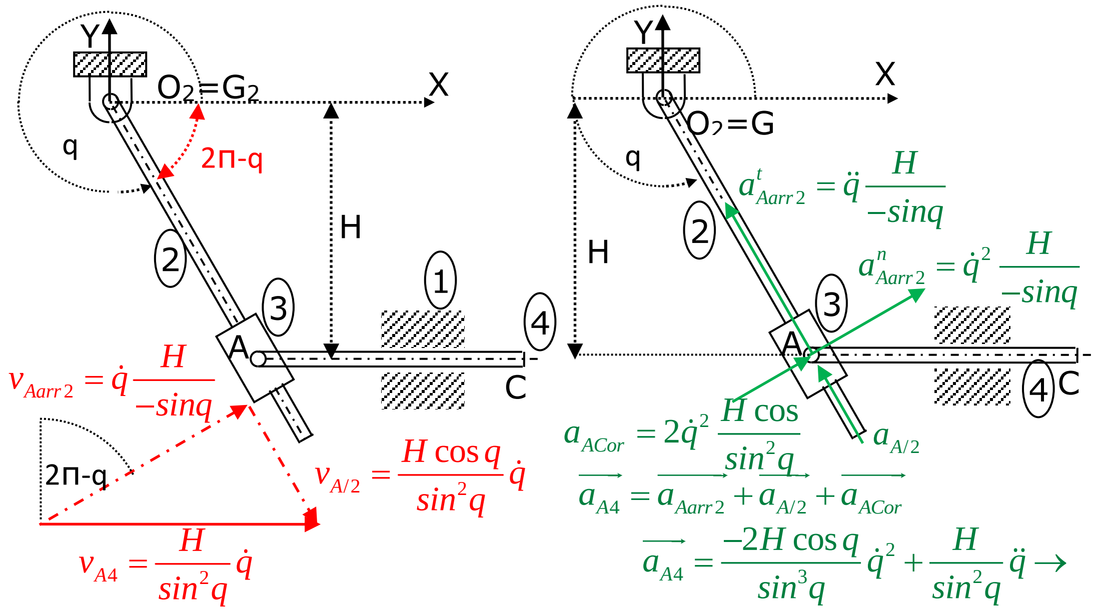
7.1 PROBLEMA 7.1

7.1.1 ENUNTZIATUA

Irudiko mekanismoa plano horizontal baten gainean kokatuta dago, eta, atsedenaldian egonda, martxan jartzen da q = 300º denean. Mekanismoa higitzen da "2" barran aplikaturiko momentu motorra konstanteari esker, $\overrightarrow{M_2} = 80\, N{\cdot}m\, \vec{k}$. "4" barraren gainean $\overrightarrow{F_4} = -100\, N\, \vec{i}$ indarra konstantea gertatzen da. Mekanismoaren ezaugarri geometriko eta inerzialak honakoak dira:
H = 0,45m, I_{G2}=0,005 kg·m^2, m_4 = 50 kg
Honakoa kalkulatzea eskatzen da:
a) Mekanismoaren higidurako ekuazio diferentziala.
b)"2" barraren azelerazioa martxan jartzerakoan.



7.1.2 EBAZPENA 1



1) $T_{Mek}=T_{EB}\rightarrow\begin{cases}T_{Mek}=\frac{1}{2}I_{O2}\dot q^2+\frac{m_4H^2}{\sin^4 q}\\ T_{EB}=\frac{1}{2}I^*{}_{O2}(q)\dot q^2\end{cases}$ -> $I^*{}_{O2}(q)=I_{G2}+\frac{m_4H^2}{\sin^4 q}$

2) $P_{mek}=P_{EB}$ -> $M^*(q)=M_2-(F_4H/\sin^2 q)$

3) $\Delta T_{EB}=W_{EB}\begin{cases}\Delta T_{EB}=\frac{1}{2}I^*{}_{O2}(q)\dot q^2-\frac{1}{2}I^*{}_{O2}(q_0)\dot q_0^2\\ W_{EB}=\int_{q0}^{q}M^*(q)\cdot dq\end{cases}\dot q^2=\frac{I^*_{O2}(q_0)\dot q_0^2+2\int_{q0}^{q}M^*(q)dq}{I^*{}_{O2}(q)}$

4) $\frac{d\Delta T_{EB}}{dt}=\frac{dW_{EB}}{dt}\begin{cases}\frac{d}{dt}(\Delta T_{EB})=\frac{1}{2}\frac{dI^*_{O2}}{dq}\dot q\dot q^2-\frac{1}{2}I^*_{O2}(q)2\dot q\ddot q\\ \frac{dW_{EB}}{dt}=\frac{d}{dq}\left(\int_{q_0}^{q}M^*(q)dq\right)\frac{dq}{dt}=M^*(q)\dot q\end{cases}\ddot q=\frac{M^*(q)-\frac{dI^*_{O2}(q)}{dq}\frac{\dot q^2}{2}}{I^*{}_{O2}(q)}$

$\frac{dI^*_{O2}(q)}{dq}=\frac{d}{dq}\left(I_{G2}+\frac{m_4H^2}{\sin^4 q}\right)=\frac{-4m_4H^2}{\sin^5 q}\cos q$ -> ED: $I^*{}_{G2}(q)\ddot q+\frac{1}{2}\frac{dI^*{}_{O2}(q)}{dq}\dot q^2=M^*(q)$

$$\left(I_{G2}+\frac{m_4H^2}{\sin^4 q}\right)\ddot q+\left(\frac{-2m_4H^2}{\sin^5 q}\cos q\right)\dot q^2=M_2-\frac{F_4H}{\sin^2 q}$$

t=0 -> q= 300º -> $\dot q=0$ -> $\ddot q=$1,11 rads^{-2} -> $\vec\alpha_2=1{,}11s^{-2}(\vec k)$

Ox-N2sin(2?-q)=m2·0=0
Oy-N2cos(2?-q)=m2·0=0
M2+(N2H/sinq)=IG2d2q/dt2

7.1.3 EBAZPENA 2

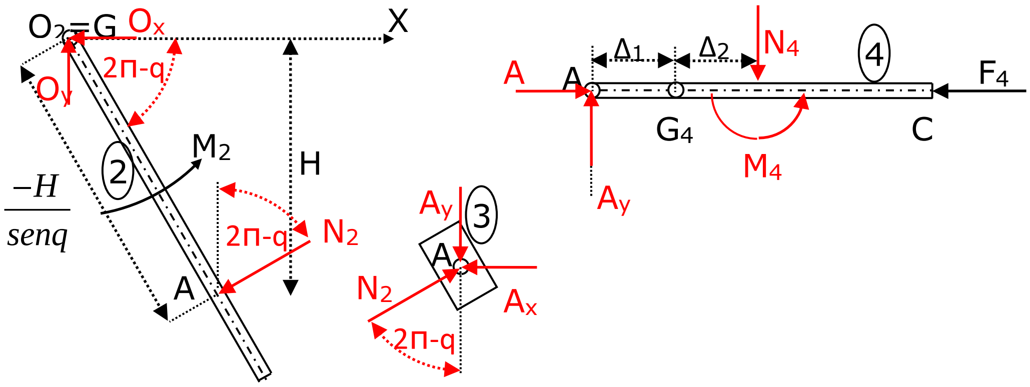
POTENTZI BIRTUALEN METODOA

$$\vec{M}_2\cdot\vec{\omega}_2+\underbrace{(-m_2\vec{a}_{G2})\cdot\vec{v}_{G2}}_{v_{G2}=a_{G2}=0}+(-I_{G2}\vec{\alpha}_2)\cdot\vec{\omega}_2+\vec{F}_4\cdot\vec{v}_C+(-m_4\vec{a}_{G4})\cdot\vec{v}_{G4}+\underbrace{(-I_{G4}\vec{\alpha}_4)\cdot\vec{\omega}_4}_{(\omega_4=\alpha_4=0)}=0$$

$$M_2\cdot\dot{q}-I_{G2}\ddot{q}\dot{q}-F_4\cdot\frac{H}{sin^2 q}\dot{q}-m_4\left(\frac{-2H\cos q}{sin^3 q}\dot{q}^2+\frac{H}{sin^2 q}\ddot{q}\right)\frac{H}{sin^2 q}\dot{q}=0$$

$$\boxed{\left(I_{G2}+\frac{m_4H^2}{sin^4 q}\right)\ddot{q}+\left(\frac{-2m_4H^2}{sin^5 q}\cos q\right)\dot{q}^2=M_2-\frac{F_4H}{sin^2 q}}$$

7.1.4 EBAZPENA 3



$$1)\sum F_x=m_2a_{G2x}\rightarrow ($$

[2] $2)\sum F_y=m_2a_{G2y}\rightarrow$

$$3)\sum M_{O2}=I_{O2}\alpha_2\rightarrow$$

O_x-$N_2sin(2п-q)=m_2\cdot 0=0$
O_y-$N_2cos(2п-q)=m_2\cdot 0=0$
$M_2+(N_2H/sinq)=I_{G2}d^2q/dt^2$

[3] $4)\sum F_x=m_3a_{G3x}\rightarrow -A_x+N_2 sin(2\pi-q)=0a_{G3x}=0$
$5)\sum F_y=m_3a_{G3y}\rightarrow -A_y+N_2 cos(2\pi-q)=0a_{G3y}=0$ $\rightarrow$

$$A_y=N_2\cos q$$

$$A_y=\frac{(I_{G2}\ddot{q}-M_2)\,sinq\cos q}{H}$$

$$6)\sum F_x = m_4 a_{G4x} \rightarrow A_x - F_4 = m_4\left(\frac{-2H\cos q}{sin^3 q}\dot{q}^2 + \frac{H}{sin^2 q}\ddot{q}\right)$$

[4] $7)\sum F_y = m_4 a_{G4y} \rightarrow A_y - N_4 = m_4 0 = 0$

$$8)\sum M_{G4} = I_{G4}\alpha_4 \rightarrow M_4 - A_y\Delta_1 - N_4\Delta_2 = I_{G4}0 = 0$$

$$N_4 = \frac{\left(I_{G2}\ddot{q} - M_2\right) sinq\cos q}{H}$$

$$N_4 = 76{,}97N$$

EKUAZIO DIFERENTZIALA

4) $N_2 = \frac{-A_x}{sinq}$

6) $A_x = F_4 + m_4\left(\frac{-2H\cos q}{sin^3 q}\dot{q}^2 + \frac{H}{sin^2 q}\ddot{q}\right)$

$$N_2 = -\frac{F_4}{sinq} + m_4\left(\frac{+2H\cos q}{sin^4 q}\dot{q}^2 - \frac{H}{sin^3 q}\ddot{q}\right)$$

$$N_2\frac{H}{sinq} = -\frac{F_4 H}{sin^2 q} + m_4\left(\frac{+2H^2\cos q}{sin^5 q}\dot{q}^2 - \frac{H^2}{sin^4 q}\ddot{q}\right)$$

3) Ekuazio diferentziala $\boxed{\left(I_{G2} + \frac{m_4 H^2}{sin^4 q}\right)\ddot{q} + \left(\frac{-2m_4 H^2}{sin^5 q}\cos q\right)\dot{q}^2 = M_2 - \frac{F_4 H}{sin^2 q}}$

Oharra: Honako hiru metodoak erabili dira higidurako ekuazio diferentziala lortzeko:
1. Potentzi birtualen metodoa
2. Zhukowsi metodoa
3. Solido askeen diagramak irudikatzea eta dinamikako ekuazioak aplikatzea

7.2 PROBLEMA 7.2

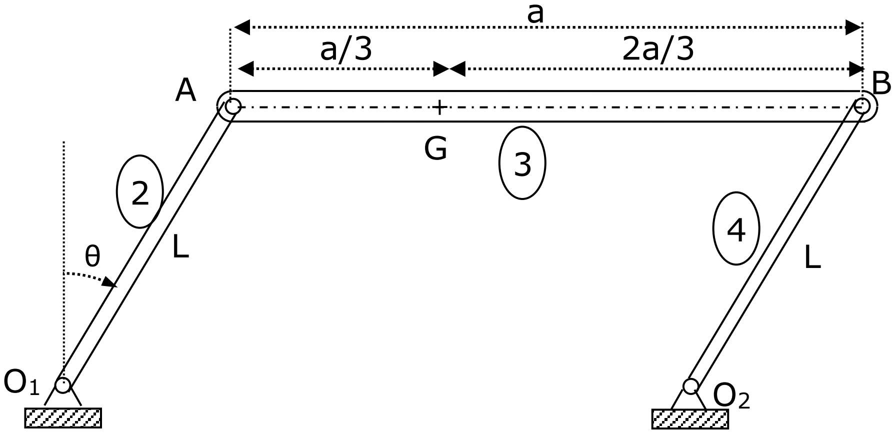
7.2.1 ENUNTZIATUA

Irudiko lauki giltzatua plano bertikalean dago. O_1A (2) eta O_2B (4) barrak L luzerakoak eta masa mespretxagarrikoak dira, eta AB (3) barra m masakoa da eta bere grabitate zentroa G zentratu gabe dago. Hasierako aldiunean (2) eta (4) barrak bertikalak dira eta geldirik dago mekanismoa. Hasierako kokapen ezegonkor honetatik eskubirantz askatzen da eta mekanismoa higitzen hasten da.

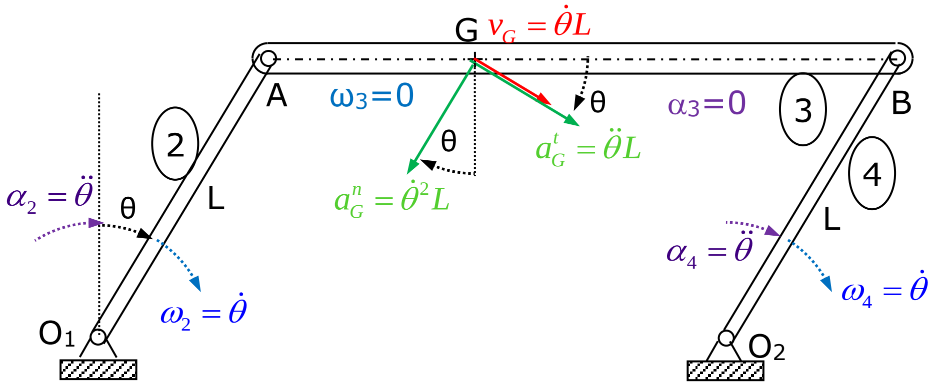
(2) barrak bertikalarekiko θ angelua osatzen duenean honakoak eskatzen da:

a) (2) barraren azelerazio angeluarra

b) O_1A eta O_2B barrek jasaten duten indar axialak



7.2.2 EBAZPENA 1



POTENTZI BIRTUALEN METODOA

$$m\vec{g}\cdot\vec{v}_G - m\vec{a}_G\cdot\vec{v}_G - I_G\vec{\alpha}_3\cdot\vec{\omega}_3 = 0 \rightarrow mg\cdot\dot\theta L\cos(90^o-\theta) - m\ddot\theta L\dot\theta L = 0 \rightarrow \boxed{\ddot\theta = \frac{g}{L}sin\theta}$$

$$\dot\theta ? \rightarrow \frac{d}{dt}\dot\theta = \frac{g}{L}sin\theta \rightarrow \frac{d\dot\theta}{d\theta}\dot\theta = \frac{g}{L}sin\theta \rightarrow \dot\theta d\dot\theta = \frac{g}{L}sin\theta d\theta \rightarrow \int\dot\theta d\dot\theta = \int\frac{g}{L}sin\theta d\theta \rightarrow$$

$$\frac{\dot\theta^2}{2} = -\frac{g}{L}\cos\theta + C \rightarrow \dot\theta(\theta=0^o) = 0 \rightarrow C = \frac{g}{L} \rightarrow \boxed{\dot\theta^2 = \frac{2g}{L}(1-\cos\theta)}$$

7.2.3 EBAZPENA 2

ZHUKOWSKI : Mekanismoa → O_1A barra

1) $T_{Mek} = T_{EB} \rightarrow \left\{ \begin{array}{l} T_{Mek} = \frac{1}{2} m\dot{\theta}^2 L^2 \\ T_{EB} = \frac{1}{2} I^*_{O1}(\theta)\dot{\theta}^2 \end{array} \right\}$ $\boxed{I^*_{O1}(\theta) = mL^2} \rightarrow \frac{dI^*_{O1}}{d\theta} = 0$

2) $P_{Mek} = P_{EB} \rightarrow \left\{ \begin{array}{l} P_{Mek} = m\vec{g} \cdot \vec{v_G} = mg\dot{\theta}L\cos(90^o - \theta) \\ P_{EB} = \overrightarrow{M^*(\theta)} \cdot \overrightarrow{\omega_2} = M^*(\theta) \cdot \dot{\theta} \end{array} \right\}$ -> $\boxed{M^*(\theta) = mgLsin\theta}$

3) $\Delta T_{EB} = W_{EB} \left\{ \begin{array}{l} \Delta T_{EB} = \frac{1}{2} I^*_{O1}(\theta)\dot{\theta}^2 - \frac{1}{2} I^*_{O1}(\theta_0)\dot{\theta}_0^2 \\ W_{EB} = \int_{\theta 0}^{\theta} M^*(\theta) \cdot d\theta \end{array} \right\}$ $\dot{\theta}^2 = \frac{I^*_{O_1}(\theta_0)\dot{\theta}_0^2 + 2\int_{\theta_0}^{\theta} M^*(\theta)d\theta}{I^*_{O_1}(\theta)}$

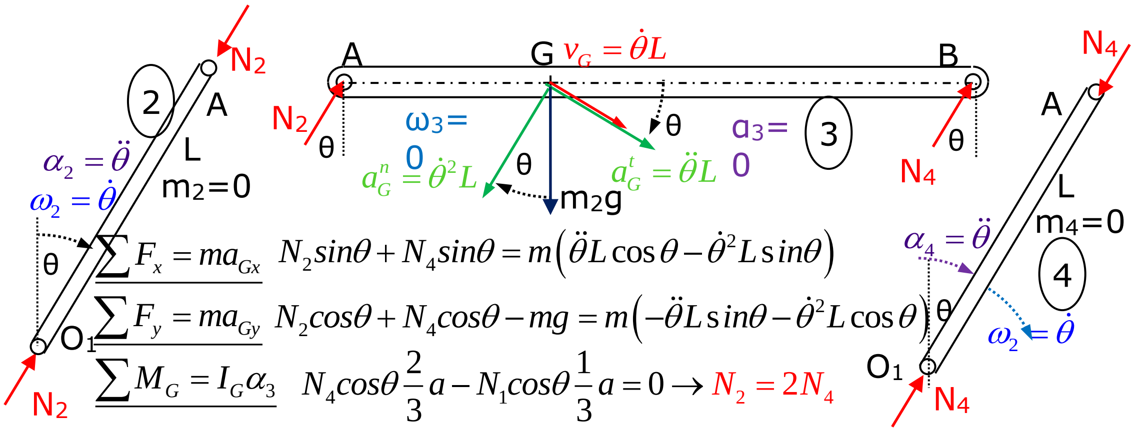
-> $\boxed{\dot{\theta}^2 = \frac{2g}{L}(1 - \cos\theta)}$

4) $\frac{d\Delta T_{EB}}{dt} = \frac{dW_{EB}}{dt} \left\{ \begin{array}{l} \frac{d}{dt}(\Delta T_{EB}) = \frac{1}{2} \frac{dI^*_{O2}}{d\theta} \dot{\theta}\dot{\theta}^2 - \frac{1}{2} I^*_{O1}(\theta) 2\dot{\theta}\ddot{\theta} \\ \frac{dW_{EB}}{dt} = \frac{d}{d\theta}\left(\int_{\theta_0}^{\theta} M^*(\theta) \cdot d\theta \right) \frac{d\theta}{dt} = M^*(\theta)\dot{\theta} \end{array} \right\}$

$$\ddot{\theta} = \frac{M^*(\theta) - \frac{1}{2} \frac{dI^*_{O2}(\theta)}{d\theta} \dot{\theta}^2}{I^*_{O1}(\theta)}$$

$\frac{dI^*_{O1}}{d\theta} = \frac{d}{d\theta}(mL^2) = 0$ -> $\boxed{\ddot{\theta} = \frac{mgLsin\theta^2}{mL^2}}$

7.2.4 EBAZPENA 3



$$\sum F_x = ma_{Gx} \quad N_2 sin\theta + N_4 sin\theta = m\left(\ddot{\theta}L\cos\theta - \dot{\theta}^2 L\sin\theta\right)$$

$$\sum F_y = ma_{Gy} \quad N_2 cos\theta + N_4 cos\theta - mg = m\left(-\ddot{\theta}L\sin\theta - \dot{\theta}^2 L\cos\theta\right)$$

$$\sum M_G = I_G\alpha_3 \quad N_4 cos\theta\frac{2}{3}a - N_1 cos\theta\frac{1}{3}a = 0 \rightarrow N_2 = 2N_4$$

$$N_2 sin\theta(-\cos\theta) + N_4 sin\theta(-\cos\theta) = m\left(-\ddot{\theta}L\cos^2\theta + \dot{\theta}^2 L\sin\theta\cos\theta\right)$$

$$N_2\cos\theta sin\theta + N_4\cos\theta sin\theta - mgsin\theta = m\left(-\ddot{\theta}L\sin^2\theta - \dot{\theta}^2 L\cos\theta sin\theta\right)$$

$$\oplus \rightarrow -mgsin\theta = -m\ddot{\theta}L\left(\cos^2\theta + \sin^2\theta\right) \rightarrow \boxed{\ddot{\theta} = \frac{g}{L}sin\theta}$$

$$N_4 = \frac{m}{3sin\theta}\left(\ddot{\theta}L\cos\theta - \dot{\theta}^2 L\sin\theta\right) \text{->}$$

$$N_4 = \frac{m}{3sin\theta}\left[gsin\theta\cos\theta - 2g\left(1-\cos\theta\right)\sin\theta\right] \rightarrow N_4 = \frac{m}{3}\left[g\cos\theta - 2g\left(1-\cos\theta\right)\right]$$

$$\boxed{N_4 = \frac{mg}{3}\left[3\cos\theta - 2\right]} \quad \boxed{N_2 = \frac{2mg}{3}\left[3\cos\theta - 2\right]}$$

Oharra: Honako hiru metodoak erabili dira (2) barraren azelerazio angeluarra kalkulatzeko:

1. Potentzi birtualen metodoa
2. Zhukowsi metodoa
3. Solido askeen diagramak irudikatzea eta dinamikako ekuazioak aplikatzea. Metodo honez bidez O_1A eta O_2B barrek jasaten duten indar axialak kakulatu dira.

7.3 PROBLEMA 7.3

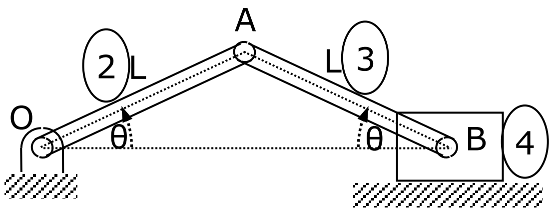
7.3.1 ENUNTZIATUA

Irudiko mekanismoaren OA eta AB barrak homogeneoak eta berdinak dira, **m** masakoak eta **L** luzerakoak eta irristailuaren masa mespretxagarria da. Hasierako aldiunean OA barrak 30º osatzen du horizontalarekiko eta geldirik dago mekanismoa. Orduan, mekanismoa askatzen da.

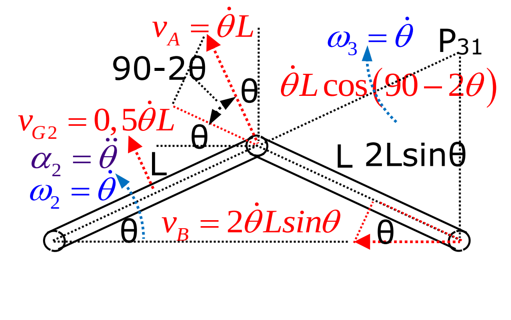
Mekanismoa plano bertikalean dagoela kontuan hartuz, kalkulatu:

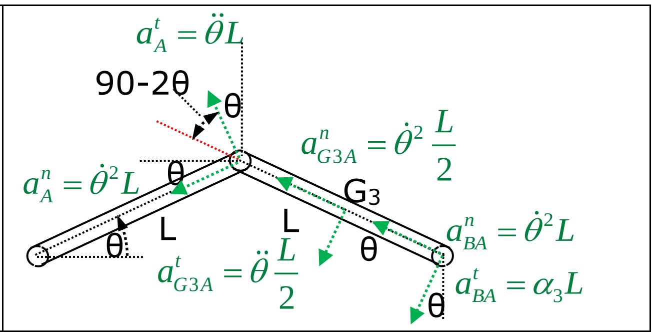
a) Mekanismoaren higidurako ekuazio diferentziala

b) OA barraren azelerazio angeluarra θ = 0 denean.



7.3.2 EBAZPENA 1





$$\overrightarrow{v_{G3}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & -\dot{\theta} \\ -\frac{L}{2}\cos\theta & -\frac{3L}{2}\sin\theta & 0 \end{vmatrix}$$

$$\boxed{\overrightarrow{v_{G3}} = \dot{\theta}\frac{L}{2}\left(-3\sin\theta\vec{i} + \cos\theta\vec{j}\right)}$$

$$a_{By} = 0 \rightarrow$$

$$-\dot{\theta}^2 L\sin\theta + \ddot{\theta}L\cos\theta + \dot{\theta}^2 L\sin\theta - \alpha_3 L\cos\theta = 0$$

$$\alpha_3 = \ddot{\theta} \rightarrow \overrightarrow{\alpha_3} = \ddot{\theta}\vec{k}$$

$$a_{G3x} = -\frac{3}{2}\dot{\theta}^2 L\cos\theta - \frac{3}{2}\ddot{\theta}L\sin\theta$$

$$a_{G3y} = -\frac{1}{2}\dot{\theta}^2 L\sin\theta + \frac{1}{2}\ddot{\theta}L\cos\theta$$

POTENTZI BIRTUALEN METODOA

$$m_2\vec{g}\cdot\overrightarrow{v_{G2}} - m\overrightarrow{a_{G2}}\cdot\overrightarrow{v_{G2}} - I_{G2}\overrightarrow{\alpha_2}\cdot\overrightarrow{\omega_2} + m_3\vec{g}\cdot\overrightarrow{v_{G3}} - m\overrightarrow{a_{G3}}\cdot\overrightarrow{v_{G3}} - I_{G3}\overrightarrow{\alpha_3}\cdot\overrightarrow{\omega_3} = 0$$

$$mg\cdot\dot{\theta}\frac{L}{2}\cos(180-\theta) - m\ddot{\theta}\frac{L}{2}\dot{\theta}\frac{L}{2} - I_{G2}\ddot{\theta}\dot{\theta} + mg\cdot\overrightarrow{v_{G3}} - m\overrightarrow{a_{G3}}\cdot\overrightarrow{v_{G3}} - I_{G3}\overrightarrow{\alpha_3}\cdot\overrightarrow{\omega_3} = 0$$

$$-m\overrightarrow{a_{G3}}\cdot\overrightarrow{v_{G3}} = -m\dot{\theta}\frac{L}{2}\left(\frac{3}{2}\dot{\theta}^2 L\cos\theta 3\sin\theta + \frac{3}{2}\ddot{\theta}L\sin^2\theta - \frac{1}{2}\dot{\theta}^2 L\sin\theta\cos\theta + \frac{1}{2}\ddot{\theta}L\cos^2\theta\right)$$

$$-m\overrightarrow{a_{G3}}\cdot\overrightarrow{v_{G3}} = -m\dot{\theta}\frac{L}{2}\left(4\dot{\theta}^2 L\cos\theta\sin\theta + \frac{1}{2}\ddot{\theta}L + 2\ddot{\theta}L\sin^2\theta\right)$$

$$-mg\cdot\dot{\theta}\frac{L}{2}\cos\theta - m\ddot{\theta}\frac{L}{2}\dot{\theta}\frac{L}{2} - \frac{mL^2}{12}\ddot{\theta}\dot{\theta}$$

$$-mg\dot{\theta}\frac{L}{2}\cos\theta - m\dot{\theta}\frac{L}{2}\left(4\dot{\theta}^2 L\cos\theta\sin\theta + \frac{1}{2}\ddot{\theta}L + 2\ddot{\theta}L\sin^2\theta\right) - \frac{mL^2}{12}\ddot{\theta}\cdot\dot{\theta} = 0$$

$$\boxed{\left(\frac{2L}{3} + L\sin^2\theta\right)\ddot{\theta} + 2L\sin\theta\cos\theta\dot{\theta}^2 + g\cos\theta = 0} \quad \theta = 0^\circ \rightarrow \boxed{\ddot{\theta} = -\frac{3}{2}\frac{g}{L}} \rightarrow \boxed{\overrightarrow{\alpha_2} = -\frac{3}{2}\frac{g}{L}\vec{k}}$$

7.3.3 EBAZPENA 2

3) ZHUKOWSKI : Mekanismoa → OA barra

$$1)\ T_{Mek}=T_{EB}\left\{\begin{array}{l} T_{Mek}=\frac{1}{2}\left(\frac{mL^2}{3}\right)\dot{\theta}^2+\frac{1}{2}m\dot{\theta}^2\frac{L^2}{4}\left(1+8\sin^2\theta\right)+\frac{1}{2}\left(\frac{mL^2}{12}\right)\dot{\theta}^2 \\ T_{EB}=\frac{1}{2}I^*{}_O(\theta)\dot{\theta}^2 \end{array}\right\}$$

$$\boxed{I^*{}_O(\theta)=mL^2\left(\frac{2}{3}+2\sin^2\theta\right)}$$

2)

$$P_{Mek}=P_{EB}\rightarrow\left\{\begin{array}{l} P_{Mek}=m_2\vec{g}\cdot\overrightarrow{v_{G2}}+m_3\vec{g}\cdot\overrightarrow{v_{G3}}=-mg\dot{\theta}\frac{L}{2}\cos\theta-mg\dot{\theta}\frac{L}{2}\cos\theta=-mg\dot{\theta}L\cos\theta \\ P_{EB}=\overrightarrow{M^*(\theta)}\cdot\overrightarrow{\omega_2}=M^*(\theta)\cdot\dot{\theta} \end{array}\right\}$$

-> $\boxed{M^*(\theta)=-mgL\cos\theta}$

$$3)\ \Delta T_{EB}=W_{EB}\left\{\begin{array}{l} \Delta T_{EB}=\frac{1}{2}I^*{}_O(\theta)\dot{\theta}^2-\frac{1}{2}I^*{}_O(\theta_0)\dot{\theta}_0^2 \\ W_{EB}=\int_{\theta 0}^{\theta}M^*(\theta)\cdot d\theta \end{array}\right\}\dot{\theta}^2=\frac{I_O^*(\theta_0)\dot{\theta}_0^2+2\int_{\theta_0}^{\theta}M^*(\theta)d\theta}{I_O^*(\theta)}$$

$$\int_{\theta_0}^{\theta}M^*(\theta)d\theta=\int_{\pi/6}^{\theta}-mgL\cos\theta d\theta=-mgL\left[\sin\theta\right]_{\pi/6}^{\theta}=mgL\left(\frac{1}{2}-\sin\theta\right)$$

$$\boxed{\dot{\theta}^2=\frac{g}{L}\frac{\left(\frac{1}{2}-\sin\theta\right)}{\left(\frac{1}{3}+\sin^2\theta\right)}}$$

$$4)\ \frac{d\Delta T_{EB}}{dt}=\frac{dW_{EB}}{dt}\left\{\begin{array}{l} \frac{d}{dt}\left(\Delta T_{EB}\right)=\frac{1}{2}\frac{dI^*{}_O}{d\theta}\dot{\theta}\dot{\theta}^2-\frac{1}{2}I^*{}_O(\theta)2\dot{\theta}\ddot{\theta} \\ \frac{dW_{EB}}{dt}=\frac{d}{d\theta}\left(\int_{\theta_0}^{\theta}M^*(\theta)d\theta\right)\frac{d\theta}{dt}=M^*(\theta)\dot{\theta} \end{array}\right\}\ddot{\theta}=\frac{M^*(\theta)-\frac{dI_O^*(\theta)}{d\theta}\frac{\dot{\theta}^2}{2}}{I_O^*(\theta)}$$

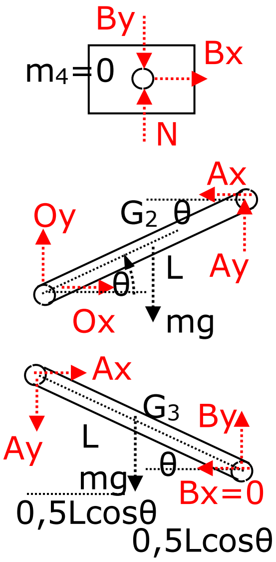
ED $I^*{}_O(\theta)\ddot{\theta}=M^*(\theta)-\frac{1}{2}\frac{dI^*{}_O(\theta)}{d\theta}\dot{\theta}^2$

$$\frac{dI^*{}_O}{d\theta}=\frac{d}{d\theta}\left(mL^2\left(\frac{2}{3}+2\sin^2\theta\right)\right)=mL^2 4\sin\theta\cos\theta$$

$$mL^2\left(\frac{2}{3}+2\sin^2\theta\right)\ddot{\theta}=-mgL\cos\theta-\frac{1}{2}mL^2 4\sin\theta\cos\theta\dot{\theta}^2$$

$$\boxed{\left(\frac{2L}{3}+2L\sin^2\theta\right)\ddot{\theta}+2L\sin\theta\cos\theta\dot{\theta}^2+g\cos\theta=0}$$

7.3.4 EBAZPENA 3



1) $\sum F_x = m_4 a_{B4x} \quad B_x = 0$

2) $\sum F_y = m_4 a_{By} \quad B_y = N$

3) $\sum F_x = m_2 a_{G2x} \quad O_x - A_x = m\left(-\dot{\theta}^2 \frac{L}{2}\cos\theta - \ddot{\theta}\frac{L}{2}\sin\theta\right)$

4) $\sum F_y = m_2 a_{G2y} \quad A_y - O_y - mg = m\left(-\dot{\theta}^2 \frac{L}{2}\sin\theta + \ddot{\theta}\frac{L}{2}\cos\theta\right)$

5) $\sum M_O = I_O \alpha_2 \quad A_x L\sin\theta + A_y L\cos\theta - mg\frac{L}{2}\cos\theta = \frac{mL^2}{3}\ddot{\theta}$

6) $\sum F_x = m a_{G3x} \quad A_x = m\left(-\frac{3}{2}\dot{\theta}^2 L\cos\theta - \frac{3}{2}\ddot{\theta}L\sin\theta\right)$

7) $\sum F_y = m a_{G3y} \quad B_y - A_y - mg = m\left(-\frac{1}{2}\dot{\theta}^2 L\sin\theta + \frac{1}{2}\ddot{\theta}L\cos\theta\right)$

8) $\sum M_{G3} = I_{G3}\alpha_3 \quad A_y\frac{L}{2}\cos\theta + B_y\frac{L}{2}\cos\theta - A_x\frac{L}{2}\sin\theta = -\frac{mL^2}{12}\ddot{\theta}$

6) $\underline{-A_x\sin\theta} = \frac{3}{2}m\dot{\theta}^2 L\cos\theta\sin\theta + \frac{3}{2}m\ddot{\theta}L\sin^2\theta$

5) $\underline{A_y\cos\theta} = -A_x\sin\theta + \frac{mg}{2}\cos\theta + \frac{mL}{3}\ddot{\theta}$

7) $B_y = A_y + mg - \frac{1}{2}m\dot{\theta}^2 L\sin\theta + \frac{1}{2}m\ddot{\theta}L\cos\theta$

$$B_y\cos\theta = A_y\cos\theta + mg\cos\theta - \frac{1}{2}m\dot{\theta}^2 L\sin\theta\cos\theta + \frac{1}{2}m\ddot{\theta}L\cos^2\theta$$

$$\underline{B_y\cos\theta} = -A_x\sin\theta + \frac{3mg}{2}\cos\theta + \frac{mL}{3}\ddot{\theta} - \frac{1}{2}m\dot{\theta}^2 L\sin\theta\cos\theta + \frac{1}{2}m\ddot{\theta}L\cos^2\theta$$

8) $A_y\cos\theta + B_y\cos\theta - A_x\sin\theta = -\frac{mL}{6}\ddot{\theta}$

$$-A_x\sin\theta + \frac{mg}{2}\cos\theta + \frac{mL}{3}\ddot{\theta}$$

$$-A_x\sin\theta + \frac{3mg}{2}\cos\theta + \frac{mL}{3}\ddot{\theta} - \frac{1}{2}m\dot{\theta}^2 L\sin\theta\cos\theta + \frac{1}{2}m\ddot{\theta}L\cos^2\theta$$

$$-A_x\sin\theta = -\frac{mL}{6}\ddot{\theta}$$

$$\frac{9}{2}m\dot{\theta}^2 L\cos\theta\sin\theta + \frac{9}{2}m\ddot{\theta}L\sin^2\theta + 2mg\cos\theta + \frac{2mL}{3}\ddot{\theta} + -\frac{1}{2}m\dot{\theta}^2 L\sin\theta\cos\theta + \frac{1}{2}m\ddot{\theta}L\cos^2\theta = -\frac{mL}{6}\ddot{\theta}$$

$$\boxed{\left(\frac{2L}{3} + 2L\sin^2\theta\right)\ddot{\theta} + 2L\cos\theta\sin\theta\dot{\theta}^2 + g\cos\theta = 0}$$

Oharra: aurreko problemetan erabilitako bideak erabili dira problema honetan higidurako ekuazio diferentziala lortzeko.

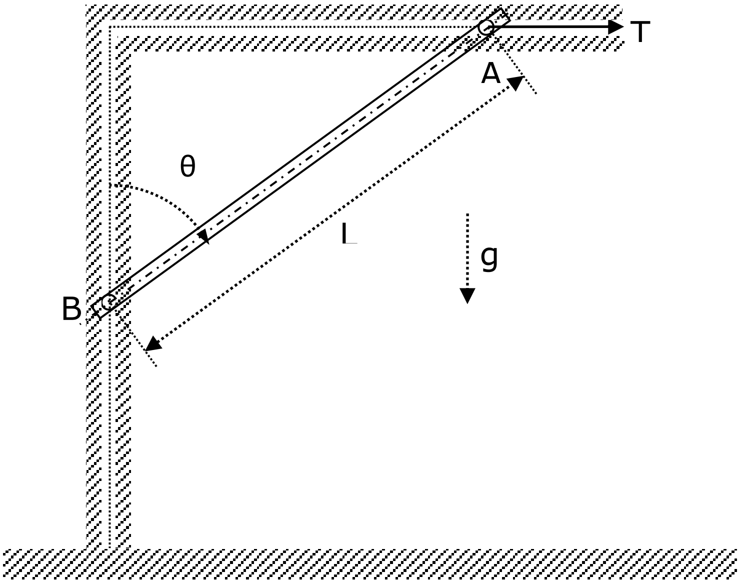
7.4 PROBLEMA 7.4

7.4.1 ENUNTZIATUA

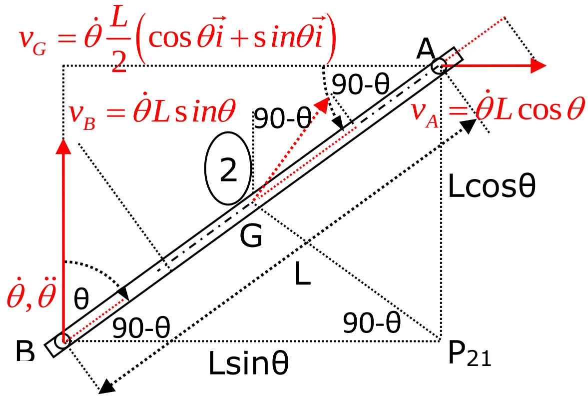
Irudiko mekanismoan AB barra homogeneoa L luzerakoa eta m masakoa da. Barraren A muturrean T indar horizontala aplikatuta dago. A eta B muturrak marruskadurarik gabe irristatzen dira bi arteketan zehar, arteka hauek horizontala eta bertikala izanik hurrenez hurren. Hasierako aldiunean AB barrak bertikalarekiko θ=65º angelua osatzen du eta geldirik dago

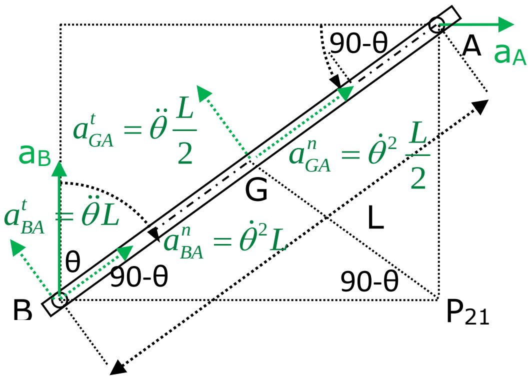
a) Kalkulatu AB barraren higidurako ekuazio diferentziala.

b) Mekanismoa martxan jartzerakoan, AB barraren azelerazio angeluarra kalkulatu m=10 kg, L = 1,2 m eta T = 0 N diren kasuan.



7.4.2 EBAZPENA 1





$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a^n_{BA}} + \overrightarrow{a^t_{BA}}$ ->

$+X \rightarrow)\ 0 = a_A\vec{i} + \dot\theta^2 L sin\theta - \ddot\theta L cos\theta$ -> $\overrightarrow{a_A} = -\dot\theta^2 L sin\theta + \ddot\theta L cos\theta \rightarrow$

$+Y \uparrow)\ a_B = \dot\theta^2 L\cos\theta + \ddot\theta L sin\theta$ -> $\overrightarrow{a_B} = \dot\theta^2 L\cos\theta + \ddot\theta L sin\theta \uparrow$

$\overrightarrow{a_G} = \overrightarrow{a_A} + \overrightarrow{a^n_{GA}} + \overrightarrow{a^t_{GA}}$

$$a_{Gx} = -\dot\theta^2 L sin\theta + \ddot\theta L cos\theta + \dot\theta^2 \frac{L}{2}\sin\theta - \ddot\theta\frac{L}{2}\cos\theta \rightarrow a_{Gx} = -\dot\theta^2\frac{L}{2} sin\theta + \ddot\theta\frac{L}{2} cos\theta$$

$$a_{Gy} = \dot\theta^2\frac{L}{2}\cos\theta + \ddot\theta\frac{L}{2}\sin\theta$$

1) POTENTZI BIRTUALEN METODOA

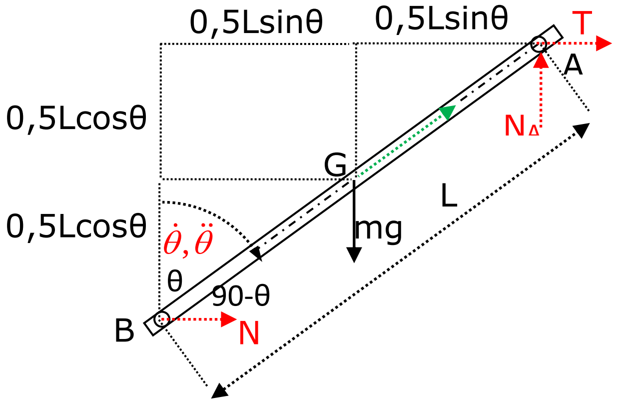
$$\vec{T}\cdot\overrightarrow{v_A} + m\vec{g}\cdot\overrightarrow{v_G} - m\overrightarrow{a_G}\cdot\overrightarrow{v_G} - I_G\vec\alpha\cdot\vec\omega = 0$$

$$-m\overrightarrow{a_G}\cdot\overrightarrow{v_G} = 0 = -m\dot\theta\frac{L}{2}\left(-\dot\theta^2\frac{L}{2} sin\theta cos\theta + \ddot\theta\frac{L}{2} cos^2\theta + \dot\theta^2\frac{L}{2}\cos\theta\sin\theta + \ddot\theta\frac{L}{2}\sin^2\theta\right)$$

$$T\dot\theta L\cos\theta - mg\dot\theta\frac{L}{2} sin\theta - m\dot\theta\frac{L}{2}\left(\ddot\theta\frac{L}{2}\right) - \frac{mL^2}{12}\ddot\theta\dot\theta = 0$$

$$\boxed{\left(\frac{mL}{3}\right)\ddot\theta = T\cos\theta - \frac{mg}{2} sin\theta}$$

7.4.3 EBAZPENA 2



1) $\sum F_x = ma_{Gx} \rightarrow N_B + T = m\left(-\dot{\theta}^2 \frac{L}{2} sin\theta + \ddot{\theta}\frac{L}{2} cos\theta\right)$

2) $\sum F_y = ma_{Gy} \rightarrow N_A - mg = m\left(\dot{\theta}^2 \frac{L}{2}\cos\theta + \ddot{\theta}\frac{L}{2} sin\theta\right) \rightarrow$

3) $\sum M_G = I_G\alpha \rightarrow N_B \frac{L}{2}\cos\theta + N_A \frac{L}{2} sin\theta - T\frac{L}{2}\cos\theta = -\frac{mL^2}{12}\ddot{\theta}$

3) $N_B \cos\theta + N_A sin\theta - T\cos\theta = -\frac{mL}{6}\ddot{\theta}$

1) $N_B \cos\theta = -T\cos\theta - m\dot{\theta}^2 \frac{L}{2} sin\theta\cos\theta + m\ddot{\theta}\frac{L}{2}cos^2\theta$

2) $N_A sin\theta = mgsin\theta + m\dot{\theta}^2\frac{L}{2}\cos\theta sin\theta + m\ddot{\theta}\frac{L}{2}sin^2\theta$

3) $\boxed{\frac{mL}{3}\ddot{\theta} = T\cos\theta - \frac{mg}{2}sin\theta}$

T=0, θ=65º L =1,2m, $\dot{\theta} = 0$ ->

$\ddot{\theta} = -\frac{3g}{2L}sin65^o$ $\ddot{\theta} =$ -11,11 rads^{-2}

$N_B = m\ddot{\theta}\frac{L}{2}cos65^o$ -> N_B = 28,2 N

$N_A = mg + m\ddot{\theta}\frac{L}{2}sin65^o$ -> N_A = 37,7 N

Oharra: Potentzi birtualen metodoa eta solido askearen diagramak irudikatuz dinamikako ekuazioak aplikatzea erabili dira higidurako ekuazio diferentziala lortzeko.

7.5 PROBLEMA 7.5

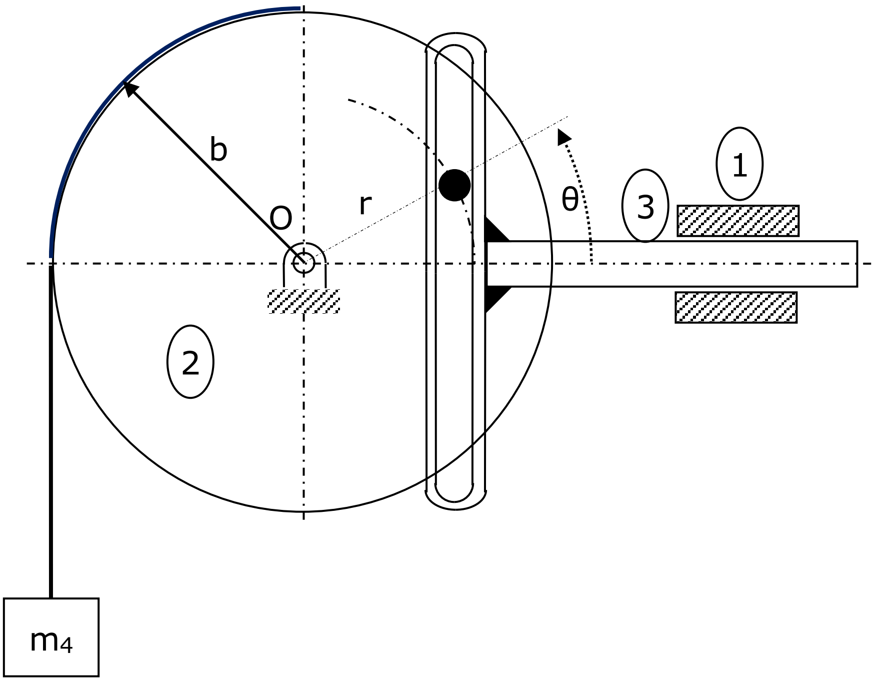
7.5.1 ENUNTZIATUA

Irudiko mekanismoan, O puntuan giltzatua dagoen **b** erradioko txirrikak O-rekiko inertzia-momentua $\mathbf{I_O}$ dauka. P ziria txirrikari zurrunki lotuta eta O-tik **r** distantziara dago. P ziria (3) elementuaren arteka bertikalean zehar irristatzen da marruskadurik gabe eta $\mathbf{m_3}$ masako (3) elementua horizontalki irristazen da (1) elementu finkoan zehar. Masa puntual bat $\mathbf{m_4}$ korda baten muturretik esekitzen da. Txirrikaren gainean biribilduta dagoen korda honen beste muturra txirrikari lotuta dago. **θ** aldagai orokorra erabiliz:

a) Kalkulatu (2) txirrikari dagokion inertzi murriztua.

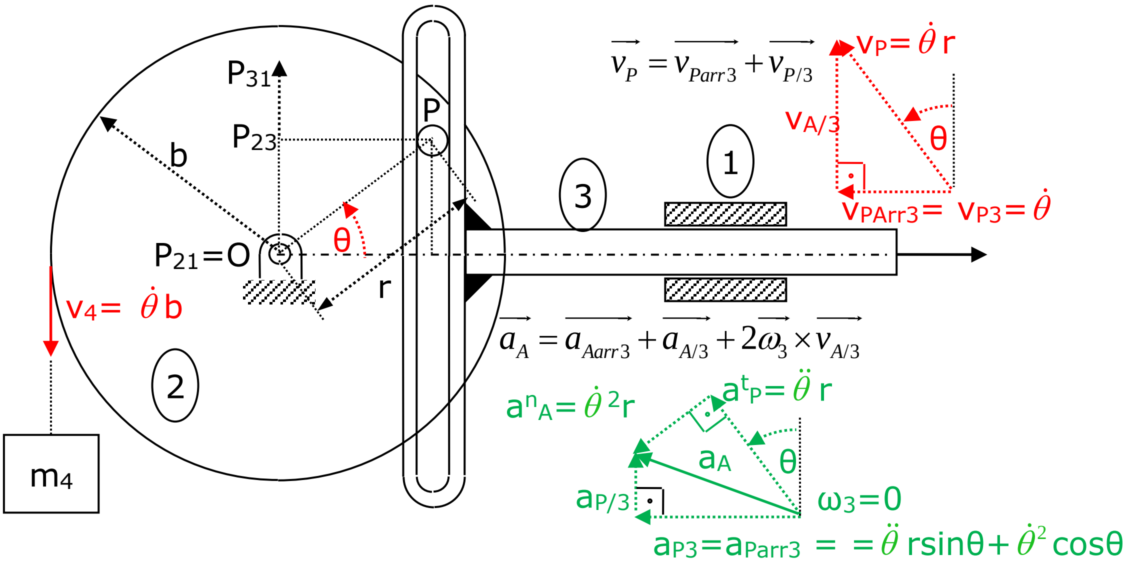
b) Kalkulatu (2) txirrikari dagokion momentu murriztua.

c) Kalkulatu Mekanismoaren higiduraren ekuazio diferentziala θ angeluaren menpe.



7.5.2 EBAZPENA

AZTERKETA ZINEMATIKOA



ZHUKOWSKI : Mekanismoa → (2) Txirrika

$$1)\ T_{Mek}=T_{EB}\left\{\begin{array}{l} T_{Mek}=\frac{1}{2}I_O\dot{\theta}^2+\frac{1}{2}m\dot{\theta}^2r^2\sin^2\theta+m_4\dot{\theta}^2b^2 \\ T_{EB}=\frac{1}{2}I_O^*(\theta)\dot{\theta}^2 \end{array}\right\}\ \boxed{I_O^*(\theta)=I_O+mr^2\sin^2\theta+m_4b^2}$$

$$\frac{dI^*_O}{d\theta}=\frac{d}{d\theta}\left(I_O+mr^2\sin^2\theta+m_4b^2\right)=mr^2 2\sin\theta\cos\theta$$

$$2)\ P_{Mek}=P_{EB}\rightarrow\left\{\begin{array}{l} P_{Mek}=m_2\vec{g}\cdot\vec{v}_{G2}+m_3\vec{g}\cdot\vec{v}_{G3}+m_4\vec{g}\cdot\vec{v}_4=m_4g\cdot\dot{\theta}^2b^2 \\ P_{EB}=\overrightarrow{M^*(\theta)}\cdot\vec{\omega}_2=M^*(\theta)\dot{\theta} \end{array}\right\}\ \boxed{M^*(\theta)=m_4gb}$$

$$3)\ \Delta T_{EB}=W_{EB}\left\{\begin{array}{l} \Delta T_{EB}=\frac{1}{2}I^*_O(\theta)\dot{\theta}^2-\frac{1}{2}I^*_O(\theta_0)\dot{\theta}_0^2 \\ W_{EB}=\int_{\theta 0}^{\theta}M^*(\theta)\cdot d\theta \end{array}\right\}\ \dot{\theta}^2=\frac{I^*_O(\theta_0)\dot{\theta}_0^2+2\int_{\theta_0}^{\theta}M^*(\theta)\cdot d\theta}{I^*_O(\theta)}$$

$$4)\ \frac{d\Delta T_{EB}}{dt}=\frac{dW_{EB}}{dt}\left\{\begin{array}{l} \frac{d}{dt}(\Delta T_{EB})=\frac{1}{2}\frac{dI_O^*}{d\theta}\dot{\theta}\dot{\theta}^2-\frac{1}{2}I_O^*(\theta)2\dot{\theta}\ddot{\theta} \\ \frac{dW_{EB}}{dt}=\frac{d}{d\theta}\left(\int_{\theta_0}^{\theta}M^*(\theta)d\theta\right)\dot{\theta}=M^*(\theta)\dot{\theta} \end{array}\right\}\ \ddot{\theta}=\frac{M^*(\theta)-\frac{dI_O^*(\theta)}{d\theta}\frac{\dot{\theta}^2}{2}}{I_O^*(\theta)}$$

$$\text{ED}\ \ I^*_O(\theta)\ddot{\theta}+\frac{1}{2}\frac{dI^*_O(\theta)}{d\theta}\dot{\theta}^2=M^*(\theta)$$

$$\boxed{\left(I_O+mr^2\sin^2\theta+m_4b^2\right)\ddot{\theta}+\left(mr^2\sin\theta\cos\theta\right)\dot{\theta}^2=m_4gb\cos\theta}$$

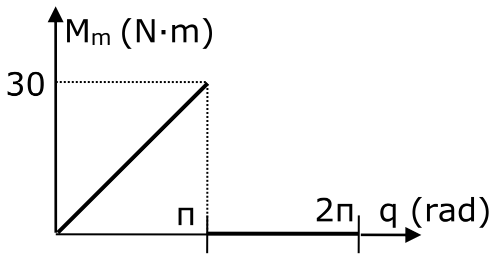
Oharra: Zhukowksi metodoa erabiliz ebatzi da problema dinamikoa.

7.6 PROBLEMA 7.6

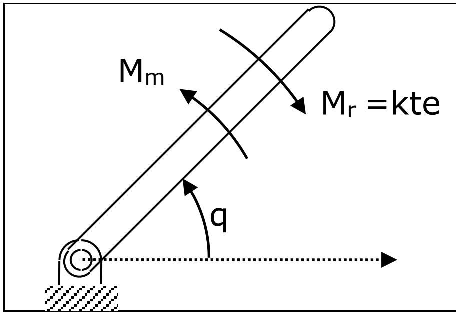
7.6.1 ENUNTZIATUA

Makina zikliko baten ardatz nagusiak buelta osoa egiten du bere lan-zikloan zehar eta bere momentu eragilearen kurba irudian erakusten da.

Ardatz honen momentu erresistente murriztua konstante dela jakinik, ardatz nagusian montatu behar den inertzia-bolantearen inertzia-momentua kalkulatu, ardatz nagusiaren abiadura 210±10 bira minutuko izan dadin.



7.6.2 EBAZPENA

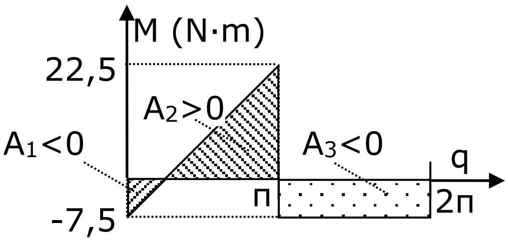


$$P_{Mek} = P_{EB} \rightarrow \begin{cases} P_{Mek} = \overrightarrow{M_m} \cdot \vec{\dot{q}} - \overrightarrow{M_r} \cdot \vec{\dot{q}} \\ P_{Mek} = \left(M_m - M_r\right)\dot{q} \\ P_{EB} = \overrightarrow{M^*(q)} \cdot \vec{\dot{q}} = M^*(\theta) \cdot \dot{q} \end{cases}$$

$$M^*(q) = \left(M_m - M_r\right)$$

$$\int_0^{2\pi} M^*(q)dq = 0 \rightarrow \int_0^{2\pi} \left(M_m - M_r\right)dq = 0 \rightarrow \int_0^{\pi} \left(\frac{30}{\pi}q - M_r\right)dq + \int_{\pi}^{2\pi} \left(0 - M_r\right)dq = 0 \text{->}$$

$$\frac{1}{2}\pi 30 - \pi M_r - \pi M_r = 0 \text{->} \ M_r = \frac{15}{2} \text{Nm}$$



$$A_1 = \int_0^{\pi/4} M^*(q)dq = -\frac{1}{2}\frac{\pi}{4}\frac{15}{2} = -\frac{15}{16}\pi \text{->} S_1 = A_1 = -\frac{15}{16}\pi = S_{min}$$

$$A_2 = \int_{\pi/4}^{\pi} M^*(q)dq = -\frac{1}{2}\frac{3\pi}{4}\frac{45}{2} = \frac{135}{16}\pi \text{->} S_1 = A_1 + A_2 = \frac{120}{16}\pi = S_{max}$$

$$A_3 = \int_{\pi}^{2\pi} M^*(q)dq = -\pi\frac{15}{2} = -\frac{120}{16}\pi \text{->} \ S_3 = A_1 + A_2 + A_3 = 0$$

$$I_v = \frac{S_{max} - S_{min}}{\varepsilon \omega_a^2} \begin{cases} \omega_a = \dfrac{\omega_{max} + \omega_{min}}{2} = \dfrac{210+10+210-10}{2} = 210\dfrac{bira}{\min} = 210\dfrac{2\pi}{60}s^{-1} \\ \varepsilon = \dfrac{\omega_{max} - \omega_{min}}{\omega_a} = \dfrac{210+10-210+10}{210} = \dfrac{2}{21} \end{cases}$$

$$I_v = \frac{\frac{120}{16}\pi - \left(-\frac{15}{16}\pi\right)}{\frac{2}{21}\left(210\frac{2\pi}{60}\right)^2} \text{->} I_v = \frac{\frac{120}{16}\pi - \left(-\frac{15}{16}\pi\right)}{\frac{2}{21}\left(210\frac{2\pi}{60}\right)^2} \text{->} I_v = \frac{405}{224\pi}\frac{Nm}{s^{-2}} \text{->}$$

$$\boxed{I_v = \frac{405}{224\pi} \text{kg·m}^2}$$

Oharra: Inertzia-bolantearen kalkulu hurbila erabili da Bolantearen inertzia-momentua kalkulatzeko.