Universidad del País Vasco / Euskal Herriko Unibertsitatea

7. GAIA.

DINAMIKAKO PROBLEMA ZUZENA

Neftalí Carbajal de la Red

AURKIBIDEA

1.- SARRERA

2.- KOORDENATU INDEPENDENTEAK ETA DEPENDENTEAK

3.- POTENTZI BIRTUALEN METODOA ETA D’ALEMBERT METODOA

4.- ZHUKOVSKI METODOA

5.- INERTZIA-BOLANTEAK

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1. SARRERA

Dinamikako problema zuzena

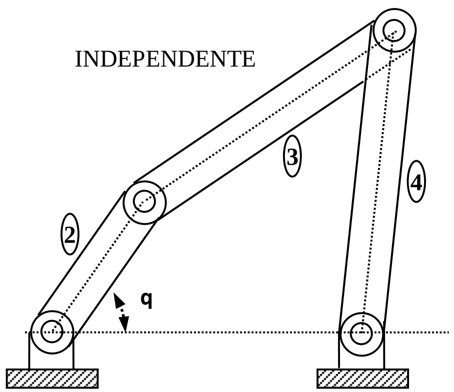
AKZIO ERAGILEAK → MAKINA ← AKZIO ERRESISTENTEAK

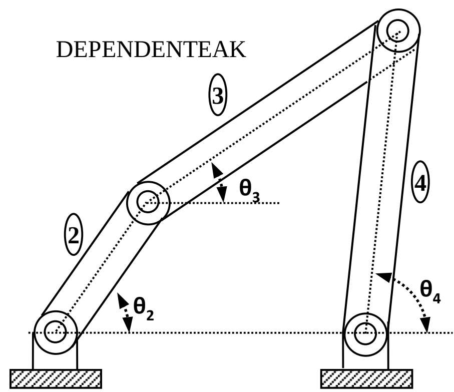
MAKINA → HIGIDURA

MAKINA → ERREAKZIOAK LOTURETAN

Funtzionamendu ziklikoa duten mekanismoak

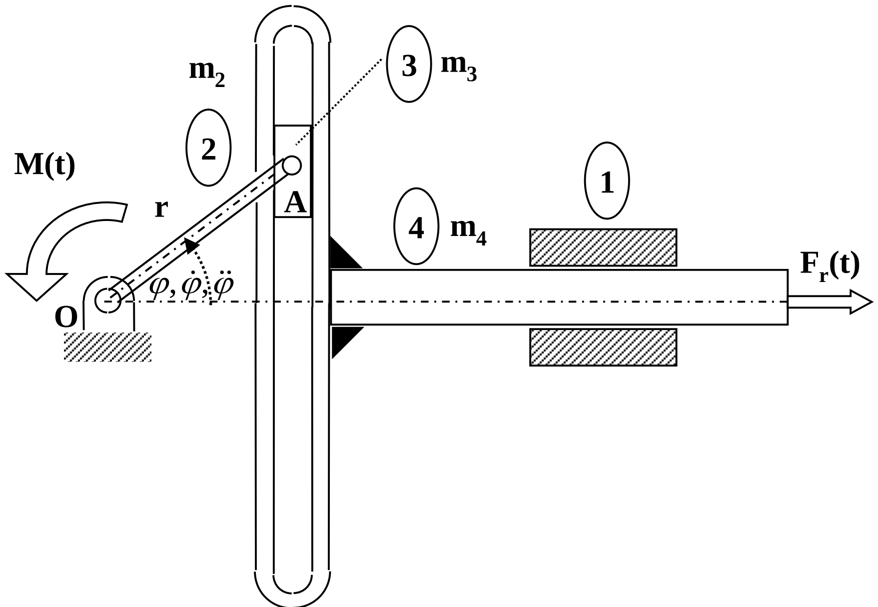
Errobotak eta zinematika paraleloko mekanismoak





Askatasun maila kopurua	**KOORDENATU KOPURUA**	Askatasun maila kopurua baino gehiago
Askatasun maila kopurua	**EKUAZIO KOPURUA**	Askatasun maila kopurua baino gehiago (koordenatu dependenteen arteko erlazioak)
Zaila	**PLANTEAMENDUA**	Erraza
Erraza	**EBAZPENA**	Zaila (Ordenagailuz)
Zinematika sinpleko makina	**APLIKAZIOA**	Zinematika konplexuko makina

ADIBIDEA. UZTARRI ESKOZIARRA.



$$M_m(t) - F_r(t) r sin\varphi = I_O \ddot{\varphi} + m_3 r^2 \ddot{\varphi} + m_4 r^2 sin\varphi\left(\ddot{\varphi} sin\varphi + \dot{\varphi}^2 cos\varphi\right)$$

Higidurako Ekuazio diferentziala

\+

$$\left.\begin{array}{l}\varphi(t=0)=0\\ \dot{\varphi}(t=0)=0\end{array}\right\}$$

Hasierako baldintzak

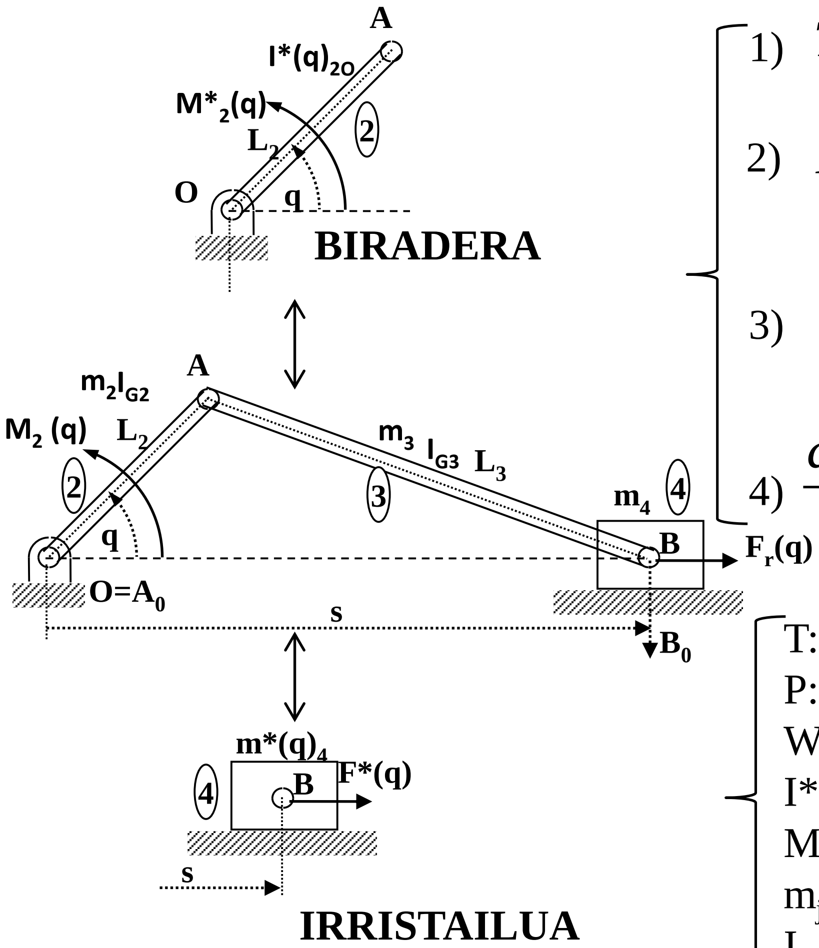
↓

Soluzioa: φ (t)

non $I_O = \dfrac{m_2 r^2}{3}$

4.- ZHUKOVSKI METODOA

Elementu baliokidea: BIRADERA



1) $T_{mekanismoa} = T_{elementu\ baliokidea}$ $\longrightarrow$ $I^*(q)$

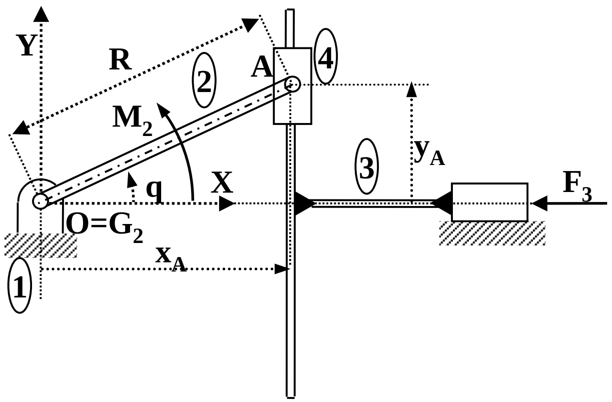
2) $P_{mekanismoa} = P_{elementu\ baliokidea}$ $\longrightarrow$ $M^*(q)$

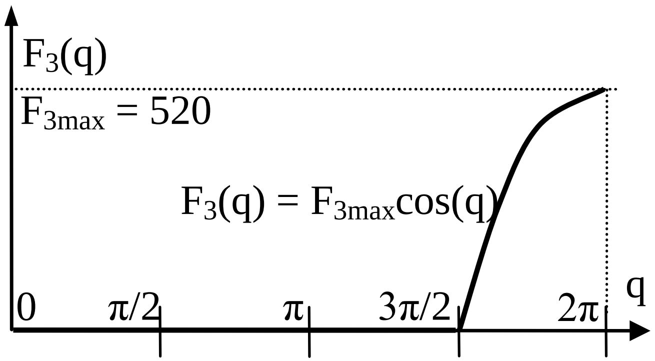
3) $\Delta T = W$ Elementu baliokidean $\longrightarrow$ $\dot{q}^2 = \dfrac{I^*(q_0)\dot{q}_0^2 + 2\int_{q0}^{q} M^*(q)\,dq}{I^*(q)}$

4) $\dfrac{d\Delta T}{dt} = \dfrac{dW}{dt}$ Elementu baliokidean $\longrightarrow$ $\ddot{q} = \dfrac{M^*(q) - \frac{1}{2}\frac{dI^*(q)}{dq}\dot{q}^2}{I^*(q)}$

- T: Energia zinetikoa
- P: Potentzia
- W: Lana
- I*(q): Inertzi murriztua
- M*: Momentu murriztua
- m_j: elementuaren masa
- I_{Gj}: elementuaren inertzia GZrekiko

- q aldagai independentea
- $\dot{q}$ abiadura angeluarra
- $\ddot{q}$ azelerazio angeluarra
- F_j: j elementuan indarra
- M_j: j elementuan momentua





Datuak

$R = 0,3m$

$I_O = I_{G2} = 0,3 kg \cdot m^2$

$m_3 = 5kg$

$m_4 = 0$

$M_2 = 15 N \cdot m = kte$

Emaitzak

$$\omega_2 = \left[\frac{2\int_{q_0}^{q} (M_2 + F_3 R sinq) dq + I^*_{O_A}(q_0)\omega_{20}^2}{I^*_{O_A}(q)} \right]$$

$$\alpha_2 = \frac{(M_2 + F_3 R sinq) - \frac{1}{2}\left[m_3 R^2 sin 2q\right]\omega_2^2}{\left[I_O + m_3 R^2 \sin^2 q\right]}$$

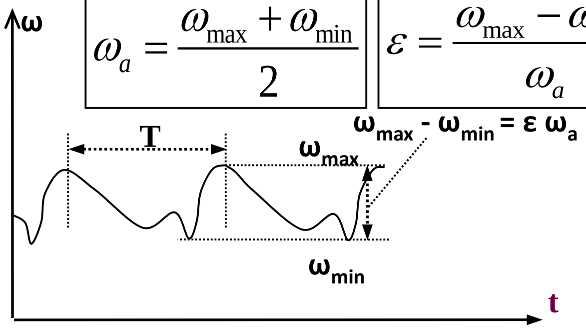
$$I^*_{O_A}(q) = I_O + m_3 R^2 \sin^2 q$$

$$M^*(q) = M_2 + F_3 R \sin q$$

q (rad)	0	π/2	π	3π/2	2π
ω_2 (s^{-1})	0.00	7.93	17.72	13.73	10.41
α_2 (s^{-2})	50.00	20.00	50.00	20.00	50.00

INERTZIA-BOLANTEAREN KALKULU HURBILA

$$\omega_a = \frac{\omega_{\max} + \omega_{\min}}{2} \qquad \varepsilon = \frac{\omega_{\max} - \omega_{\min}}{\omega_a}$$

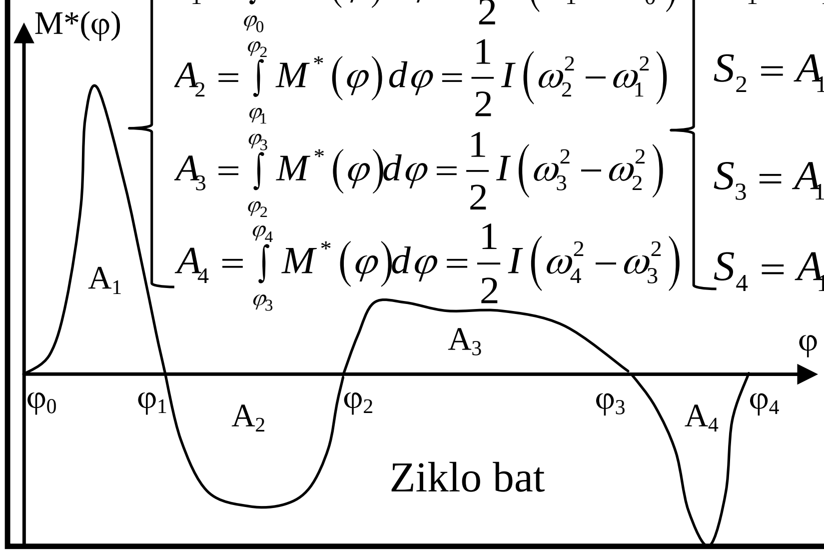


Energia zinetikoaren Teorema bolantearekin $\int_{\varphi_0}^{\varphi} M^*(\varphi)\, d\varphi = \frac{1}{2}\left(I + I^*(\varphi)\right)\omega^2 - \frac{1}{2}\left(I + I^*(\varphi_0)\right)\omega_0^2$

Sinplifikazioa $I >> I^*(\varphi) \rightarrow I + I^*(\varphi) \approx I + I^*(\varphi_0) \approx I$

$$\int_{\varphi_0}^{\varphi} M^*(\varphi)\, d\varphi = \frac{1}{2} I \left(\omega^2 - \omega_0^2\right)$$

$$\left.\begin{array}{l} A_1 = \int_{\varphi_0}^{\varphi_1} M^*(\varphi)\, d\varphi = \frac{1}{2} I \left(\omega_1^2 - \omega_0^2\right) \\ A_2 = \int_{\varphi_1}^{\varphi_2} M^*(\varphi)\, d\varphi = \frac{1}{2} I \left(\omega_2^2 - \omega_1^2\right) \\ A_3 = \int_{\varphi_2}^{\varphi_3} M^*(\varphi)\, d\varphi = \frac{1}{2} I \left(\omega_3^2 - \omega_2^2\right) \\ A_4 = \int_{\varphi_3}^{\varphi_4} M^*(\varphi)\, d\varphi = \frac{1}{2} I \left(\omega_4^2 - \omega_3^2\right) \end{array}\right\} \left\{\begin{array}{l} S_1 = A_1 = \frac{1}{2} I \left(\omega_1^2 - \omega_0^2\right) \\ S_2 = A_1 + A_2 = \frac{1}{2} I \left(\omega_2^2 - \omega_0^2\right) \\ S_3 = A_1 + A_2 + A_3 = \frac{1}{2} I \left(\omega_3^2 - \omega_0^2\right) \\ S_4 = A_1 + A_2 + A_3 + A_4 = \underbrace{\frac{1}{2} I \left(\omega_4^2 - \omega_0^2\right)}_{0} \end{array}\right\} \left.\begin{array}{l} S_{\max} = \frac{1}{2} I \left(\omega_{\max}^2 - \omega_0^2\right) \\ S_{\min} = \frac{1}{2} I \left(\omega_{\min}^2 - \omega_0^2\right) \end{array}\right\} \boxed{I = \frac{S_{\max} - S_{\min}}{\varepsilon \omega_a^2}}$$



INERTZIA-BOLANTEAREN DIMENTSIONAKETA

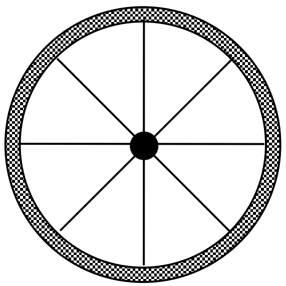
$$\boxed{P \cdot D^2 = 4gI}$$

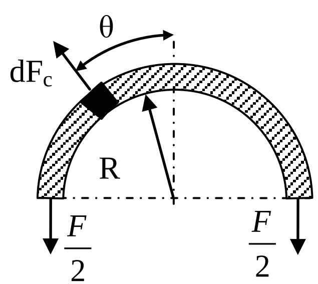
$$F_c = m\omega^2 R \qquad dF_c = dm \cdot \omega^2 R \qquad dF_c \cos\theta = dm \cdot \omega^2 R \cdot \cos\theta$$

$$dm = \rho dV$$

$$dV = (R \cdot d\theta)(R_{ext} - R_{int})s$$

$$dF_c \cos\theta = \rho \cdot (R \cdot d\theta)(R_{ext} - R_{int})s \cdot \omega^2 R \cdot \cos\theta$$



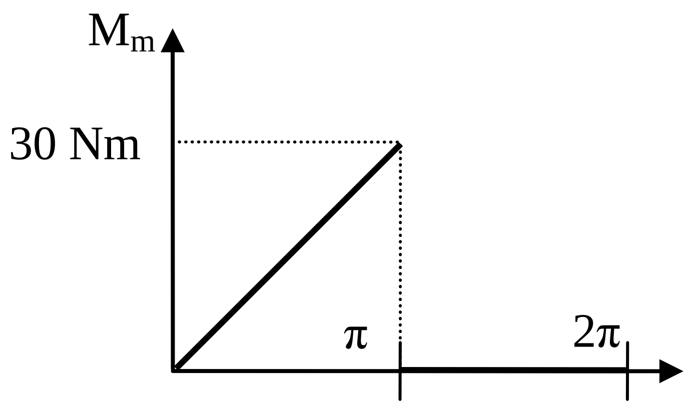


$$\frac{F}{2} + \frac{F}{2} = \int_{-\pi/2}^{\pi/2} dF_c \cos\theta \qquad F = 2\rho \cdot (R_{ext} - R_{int})s \cdot \omega^2 R^2$$

$$\sigma = \frac{\frac{F}{2}}{A} \qquad \left\{ \begin{aligned} \frac{F}{2} &= \rho(R_{ext} - R_{int})s\omega^2 R^2 \\ A &= (R_{ext} - R_{\mathrm{int}})s \end{aligned} \right\} \quad \sigma = \rho\omega^2 R^2 \qquad \boxed{\sigma = \rho v^2}$$

INERTZIA-BOLANTEAREN GAINEKO PROBLEMA

Makina zikliko baten ardatz nagusiak buelta osoa egiten du bere lan-zikloan zehar eta bere momentu eragilearen kurba irudian erakusten da.



Ardatz honen momentu erresistente murriztua konstantea dela jakinik, ardatz nagusian montatu behar den inertzia-bolantearen inertzia-momentua kalkulatu, ardatz nagusiaren abiadura 210±10 bira minutuko izan dadin. Inertzia-bolantea A-42 altzairukoa (σ_y = 42kg/mm^2) bada, zenbatekoa izango da bere bataz besteko erradio maximoa?

Emaitzak: I = 0,57 kg·m^2 R_{max} = 10,5 m

7. BIBLIOGRAFIA

- Hernández, A. Dinámica de maquinaria. Escuela Técnica Superior de Ingeniería. Bilbao, 2007