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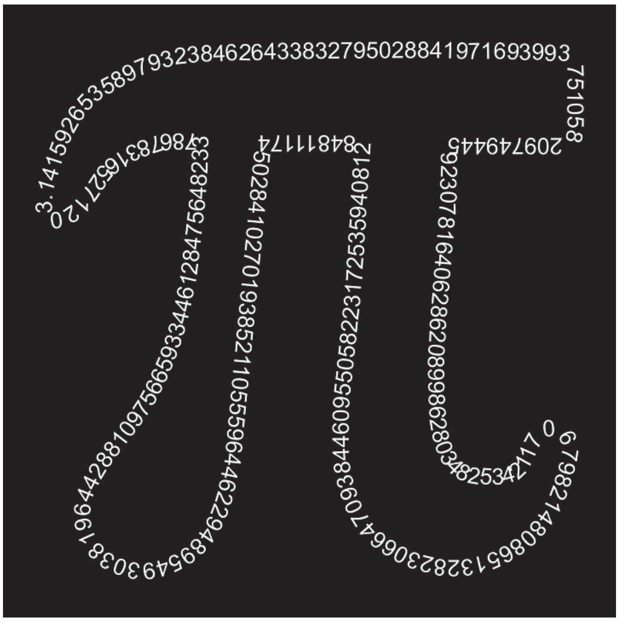
MATHS BASIC COURSE FOR UNDERGRADUATES



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SOLUTIONS: 7th SUBJECT. POLYNOMIAL INEQUATIONS

SOLUTION EXERCISE 1:

(i) $3 - 2x \geq 8 - 7x$

The previous inequality is equivalent to: $-5x+5 \leq 0$, and it is also equivalent to: $-x+1 \leq 0$. Thus, the solution is, $x \geq 1$.

(ii) $\frac{1}{5}(6-2x) > \frac{1}{10}(1-x)$

The previous inequality is equivalent to: $2(6-2x) > (1-x)$, and it is also equivalent to: $12-4x > 1-x$. Therefore, we have $11 > 3x$, and finally we have $x < \frac{11}{3}$.

SOLUTION EXERCISE 2: The previous inequality is equivalent to: $0 \leq 2x^2-3x-5$, and the second degree polynomial can be factorized as $2x^2-3x-5 = (x+1)(2x-5)$. Therefore, the initial inequality is equivalent to: $0 \leq (x+1)(2x-5)$. Now, we analyze the signs by using the following table:

	$(-\infty,-1)$	$(-1,2.5)$	$(2.5,\infty)$
$x+1$	$-$	$+$	$+$
$2x-5$	$-$	$-$	$+$
$(x+1)(2x-5)$	$+$	$-$	$+$

Hence, $x^2+6x-1 \leq 3x^2+3x-6$ if and only if $x \in (-\infty,-1] \cup [2.5,\infty]$.

SOLUTION EXERCISE 3: The previous inequality is equivalent to: $0 < x^4+3x^3-3x^2+3x-4$, and the fourth degree polynomial can be factorized as $x^4+3x^3-3x^2+3x-4 = (x-1)(x+4)(x^2+1)$. Therefore, the initial inequality is equivalent to: $0 < (x-1)(x+4)(x^2+1)$. Now, we analyze the signs by using the following table:

	$(-\infty,-4)$	$(-4,1)$	$(1,\infty)$
$x-1$	$-$	$-$	$+$
$x+4$	$-$	$+$	$+$
x^2+1	$+$	$+$	$+$
$(x-1)(x+4)(x^2+1)$	$+$	$-$	$+$

Hence, $3x^2+4 < x^4+3x^3+3x$ if and only if $x \in (-\infty,-4) \cup (1,-\infty,)$.

SOLUTION EXERCISE 4: The previous inequality is equivalent to: $x^3+x-4x^2+6 \leq 0$, and the third degree polynomial can be factorized as $x^3+x-4x^2+6 = (x+1)(x-2)(x-3)$. Therefore, the initial inequality is equivalent to: $(x+1)(x-2)(x-3) \leq 0$. Now, we analyze the signs by using the following table:

	$(-\infty,-1)$	$(-1,2)$	$(2,3)$	$(3,\infty)$
$x+1$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$(x+1)(x-2)(x-3)$	$-$	$+$	$-$	$+$

Hence, $x^3+x \leq 4x^2-6$ if and only if $x \in (-\infty,-1] \cup [2,3]$.