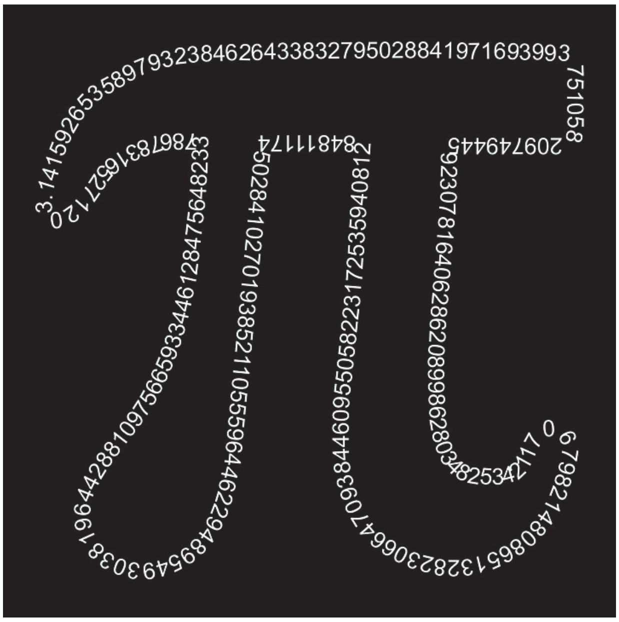
MATHS BASIC COURSE FOR UNDERGRADUATES



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SOLUTIONS: <u>4th SUBJECT. DIVISIBILITY</u>

SOLUTION EXERCISE 1: The only possible remainders of the division of a by 3 are $0, 1$ or 2. Thus, there are three possibilities:

(i) if $a = 3q$, then $a^2 = 9q^2 = 3(3q^2) = 3k$;

(ii) if $a = 3q + 1$, then $a^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3k + 1$;

(iii) if $a = 3q + 2$, then $a^2 = 9q^2 + 12q + 4 = 3(3q^2 + 4q + 1) + 1 = 3k + 1$.

SOLUTION EXERCISE 2: $(3043)_5 = 3 \cdot 5^3 + 0 \cdot 5^2 + 4 \cdot 5 + 3 = 398$.

SOLUTION EXERCISE 3: We divide the number 1025 by 7. Then, $1025 = 146 \cdot 7 + 3$. Latter, we divide the number 146 by 7: $146 = 20 \cdot 7 + 6$, and finally 20 by 7: $20 = 2 \cdot 7 + 6$. Thus,

$$1025 = 146 \cdot 7 + 3 = (20 \cdot 7 + 6) \cdot 7 + 3 = ((2 \cdot 7 + 6) \cdot 7 + 6) \cdot 7 + 3,$$

and taking away common factors, we get

$$1025 = 2 \cdot 7^3 + 6 \cdot 7^2 + 6 \cdot 7 + 3 = (2663)_7.$$

SOLUTION EXERCISE 4: $3027 = 189 \cdot 16 + 3$ and $189 = 11 \cdot 16 + 13$, and consequently $3027 = (BD3)_{16}$.

SOLUTION EXERCISE 5: Observe that the divisors of 12 are: ± 1, ± 2, ± 3, ± 4, ± 6 and ± 12, and that the divisors of 18 are: ± 1, ± 2, ± 3, ± 6 and ± 18. Thus, $\gcd(-12, 18) = 6$.

SOLUTION EXERCISE 6: First, we divide the number 1479 by 272 and we get the expression $1479 = 5 \cdot 272 + 119$. Thus,

$$\gcd(1479, 272) = \gcd(272, 119).$$

Repeating the same argument, we get that $272 = 2 \cdot 119 + 34$. Thus,

$$\gcd(1479, 272) = \gcd(272, 119) = \gcd(119, 34).$$

Again, $119 = 3 \cdot 34 + 17$, and consequently

$$\gcd(1479, 272) = \gcd(272, 119) = \gcd(119, 34) = \gcd(34, 17).$$

Finally, $34 = 2 \cdot 17 + 0$. Thus,

$$\gcd(1479, 272) = \gcd(272, 119) = \cdots = \gcd(17, 0) = 17.$$

Let us move now to express the $\gcd(1479, 272)$ as a combination of the numbers 1479 and 272.

$$1479 = 5 \cdot 272 + 119,$$

$$272 = 2 \cdot 119 + 34,$$

$$119 = 3 \cdot 34 + 17,$$

$$34 = 2 \cdot 17 + 0.$$

We know that $\gcd(1479, 272) = 17$. Therefore,

$$17 = 119 - 3 \cdot 34 = 119 - 3 \cdot (272 - 2 \cdot 119) =$$

$$7 \cdot 119 - 3 \cdot 272 = 7 \cdot (1479 - 5 \cdot 272) - 3 \cdot 272 = 7 \cdot 1479 - 38 \cdot 272.$$