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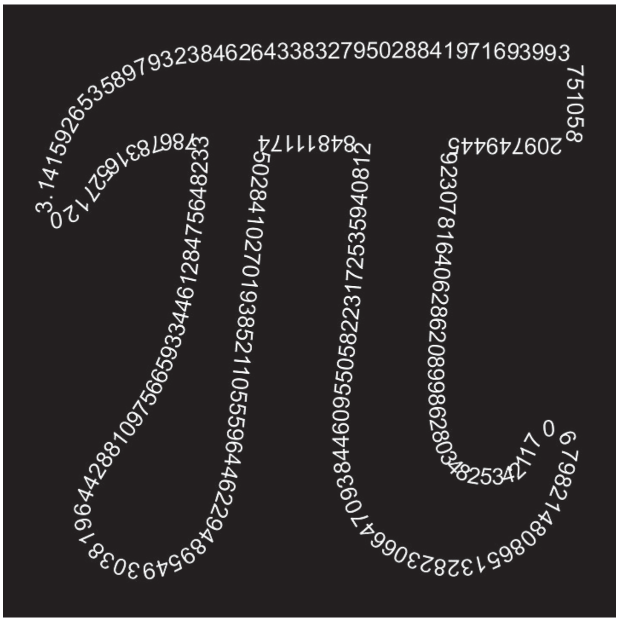
MATHS BASIC COURSE FOR UNDERGRADUATES



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SOLUTIONS: <u>2nd. SUBJECT. COMPLEX NUMBERS</u>

SOLUTION EXERCISE 1: $5i = 5_{\pi/2}; 1+i = (\sqrt{2})_{\pi/4}$ and $-1-i = (\sqrt{2})_{5\pi/4}$.

SOLUTION EXERCISE 2: $(3_{\pi/3})^3 = 27_{\pi}$;
$\frac{2_{\pi/3}}{\sqrt{5}_{\pi/4}} = (\frac{2}{\sqrt{5}})_{\pi/3-\pi/4} = (\frac{2}{\sqrt{5}})_{\pi/12}$.

SOLUTION EXERCISE 3: Observe that $z = -\sqrt{3}+i$ can also be written as $z = 2(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$. Hence,

$$z^7 = 2^7(\cos\frac{35\pi}{6}+i\sin\frac{35\pi}{6}) = 2^7(\cos\frac{-\pi}{6}+i\sin\frac{-\pi}{6}) = 2^7(\frac{\sqrt{3}}{2}+i\frac{-1}{2}) = 2^6(\sqrt{3}-i).$$

SOLUTION EXERCISE 4: First of all, observe that the equation $\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$ is fulfilled. Raising to the third power and matching the real and the imaginary parts of both sides, we conclude that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

SOLUTION EXERCISE 5: The 4-th roots of the unity are $1, e^{i\frac{\pi}{2}}, e^{i\pi}$ and $e^{i\frac{3\pi}{2}}$, which can also be written as $1, i, -1, -i$.
Moreover, the 6-th roots of the unity are $1, e^{i\frac{\pi}{3}}, e^{i\frac{2\pi}{3}}, -1, e^{i\frac{4\pi}{3}}$ eta $e^{i\frac{5\pi}{3}}$, and they are the vertices of a regular hexagon inscribed in a circle.

SOLUTION EXERCISE 6: We have to calculate the fifth roots of the complex number $-\sqrt{3}+i$. To accomplish this, first we have to express $z_0 = -\sqrt{3}+i = 2e^{i\frac{5\pi}{6}}$. Then, a fifth root of z_0 can be $\alpha = 2^{\frac{1}{5}}e^{i\frac{\pi}{6}}$. If w is a fifth root of the unity, then $(\alpha.w)^5 = \alpha^5 w^5 = \alpha^5 = z_0$. Thus, $\alpha.w$ is also a fifth root of z_0. Consequently, the fifth roots of $-\sqrt{3}+i$ are

$$\alpha, \alpha e^{i\frac{2\pi}{5}}, \alpha e^{i\frac{4\pi}{5}}, \alpha e^{i\frac{6\pi}{5}}, \alpha e^{i\frac{8\pi}{5}}.$$

In fact, there are five roots that are the fifth roots of z_0. If β, is another fifth root of z_0, then $\beta^5 = \alpha^5 = z_0$, and from here, $\frac{\beta^5}{\alpha^5} = 1$, which indicates that $\frac{\beta}{\alpha} = w$ is a fifth root of the unity. As a consequence, $\beta = \alpha w$, which is already on the previous list.
Therefore, all the fifth roots of $(-\sqrt{3}+i)$ are

$$2^{\frac{1}{5}}e^{i\frac{\pi}{6}}, 2^{\frac{1}{5}}e^{i\frac{17\pi}{30}}, 2^{\frac{1}{5}}e^{i\frac{29\pi}{30}}, 2^{\frac{1}{5}}e^{i\frac{41\pi}{30}}, 2^{\frac{1}{5}}e^{i\frac{53\pi}{30}}.$$