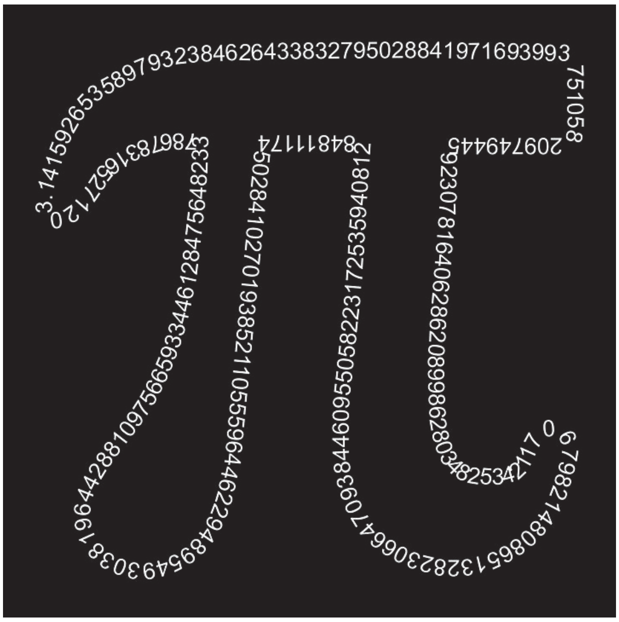
MATHS BASIC COURSE FOR UNDERGRADUATES



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SOLUTIONS: 1st SUBJECT. SET THEORY

SOLUTION EXERCISE 1: It is clear that the relations $\{1,2\} \subseteq A$ and $\{1,4\} \nsubseteq A$ fulfill. These are the subsets of A:

$$\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \text{ and } \{1,2,3\}.$$

Thus, the set A has 8 subsets.

SOLUTION EXERCISE 2:

$$A \times B = \{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y)\}.$$

SOLUTION EXERCISE 3:

$$(A \cup B) \cup (A \cap (C \cup B)) = A \cup B$$

$$(A \cap B) \cup (C \cap A) \cup (A^c \cap B^c)^c = A \cup B$$

SOLUTION EXERCISE 4: To prove that $A \nsubseteq B$, it is enough to find a counterexample; for instance, $16 \in A$ but $16 \notin B$. On the other hand, in an analogous way, since $14 \in B$ and $14 \notin A$, it follows that $B \nsubseteq A$.

SOLUTION EXERCISE 5: First of all, let us observe that $\Re$ satisfies the following three properties: reflexive, symmetric and transitive.

- Reflexive: for any $n \in \mathbb{Z}$, $n \Re n$, since $n - n = 0$ and the number 0 can be considered an even number.
- Symmetric: for any $m, n \in \mathbb{Z}$, if $m \Re n$ then $m - n$ is even, and also $n - m = -(m-n)$ is even. Thus $n \Re m$.
- Transitive: for any $m, n, t \in \mathbb{Z}$ such that $m \Re n$ and $n \Re t$ we have that $m - n = 2t_1$ and $n - t = 2t_2$, for some $t_1, t_2 \in \mathbb{Z}$. Thus, $(m-n) + (n-t) = m - t = 2t_1 + 2t_2 = 2(t_1 + t_2)$, and in particular, it is an even number, i.e $m \Re t$.

On the other hand, the integer numbers that are related through $\Re$ to 2 are $\overline{2} = \{x \in \mathbb{Z} : x \Re 2\} = \{x \in \mathbb{Z} : x - 2 = 2t, t \in \mathbb{Z}\} = \{x \in \mathbb{Z} : x = 2 + 2t, t \in \mathbb{Z}\}$, which coincides with the set formed by all the multiples of 2. The equivalence class of 2008 corresponds to $\overline{2008} = \{x \in \mathbb{Z} : x \Re 2008\} = \{x \in \mathbb{Z} : x - 2008 = 2t, t \in \mathbb{Z}\} = \{x \in \mathbb{Z} : x = 2008 + 2t, t \in \mathbb{Z}\}$, which corresponds to the set formed by all the multiples of 2. Finally, the equivalence class of -11 corresponds to the set formed for all the odd integer numbers.

SOLUTION EXERCISE 6: The proof follows in an analogous way as in Exercise 5.

SOLUTION EXERCISE 7: $\overline{(a,b)} = \{(c,d) \in \mathbb{Z} \times \mathbb{Z}^* \mid \frac{a}{b} = \frac{c}{d}\}$, and the quotient set $(\mathbb{Z} \times \mathbb{Z}^*)/\Re = \mathbb{Q}$.