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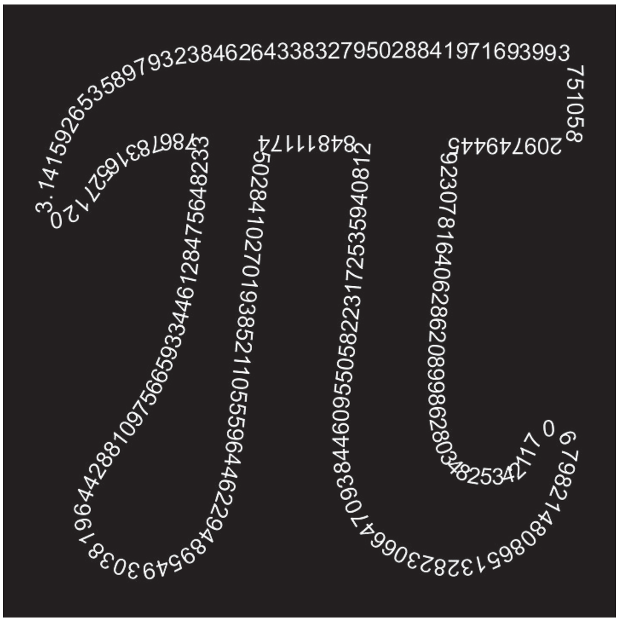
MATHS BASIC COURSE FOR UNDERGRADUATES



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SOLUTIONS: <u>MATHEMATICAL LANGUAGE</u>

SOLUTION EXERCISE 1: Assume by way of contradiction that the result is false, i.e, assume that the set S is finite. $(p \wedge \neg q)$. Consider $S = \{p_1, p_2, \ldots, p_k\}$. Since S is a finite set, we can compute the product of all the elements $p_1, p_2, \ldots, p_k$ of S. Consider now the element $b = (p_1.p_2 \ldots p_k) + 1$. Therefore, there exists a prime number p' which is a divisor of b. (Call this proposition by r.) On the other hand, since p' is a prime number and S is the set formed by all the prime numbers, it follows that p' belongs to the set S. However, neither of the elements of S divides the number b. It means, that p' is not a divisor of b $(\neg r)$.
Thus, we get a contradiction: $(r \wedge \neg r)$, with the hypothesis S is not an infinite set. $(p \wedge \neg q) \implies (r \wedge \neg r)$, which is false. In conclusion, the set of all prime numbers S is an infinite set.

SOLUTION EXERCISE 2: It does not. For instance, the number 12 fulfills at the same time p and $\neg q$, since 12 is divisible by 6 and 4, but 12 is not divisible by 24. Thus, p does not imply q.

SOLUTION EXERCISE 3:

(i) Basic step. First of all, notice that $p(1)$ fulfills: $2^1 \leq 2^{1+1}$, since $2^1 = 2$, $2^{1+1} = 4$, and $2 \leq 4$.

(ii) Step of induction. Prove that for all $k, [p(k) \implies p(k+1)]$. Assume that $p(k)$ fulfills, in other words, assume that $2^k \leq 2^{k+1}$ (hypothesis). Now prove that $p(k+1)$ fulfills, in other words, prove that $2^{k+1} \leq 2^{k+1+1} = 2^{k+2}$. To get that, multiply both sides of the previous inequality by 2 and we have that $2^k.2 \leq 2^{k+1}.2$, which corresponds to $2^{k+1} \leq 2^{k+2}$, as required.

SOLUTION EXERCISE 4: First of all, the statement is proved for the value $n = 1$:

$$1.1! = 1 = (1+1)! - 1 = 2 - 1 = 1.$$

Assume now that the statement fulfills for $k \in \mathbb{N}$, i.e. assume that $1.1! + 2.2! + 3.3! + \ldots + k.k! = (k+1)! - 1$, and let us prove the statement for the value $k+1$:

$$[\mathbf{1.1! + 2.2! + 3.3! + \ldots + k.k!}] + (k+1).(k+1)! = (\mathbf{k+1})! - \mathbf{1} + (k+1).(k+1)! =$$

$$(k+1)![1 + (k+1)] - 1 = (k+1)!(k+2) - 1 = (k+2)! - 1 = ((k+1)+1)! - 1.$$

In this way, the statement is proved for the case $k+1$, and thus the initial statement is proved for any $n \in \mathbb{N}$.