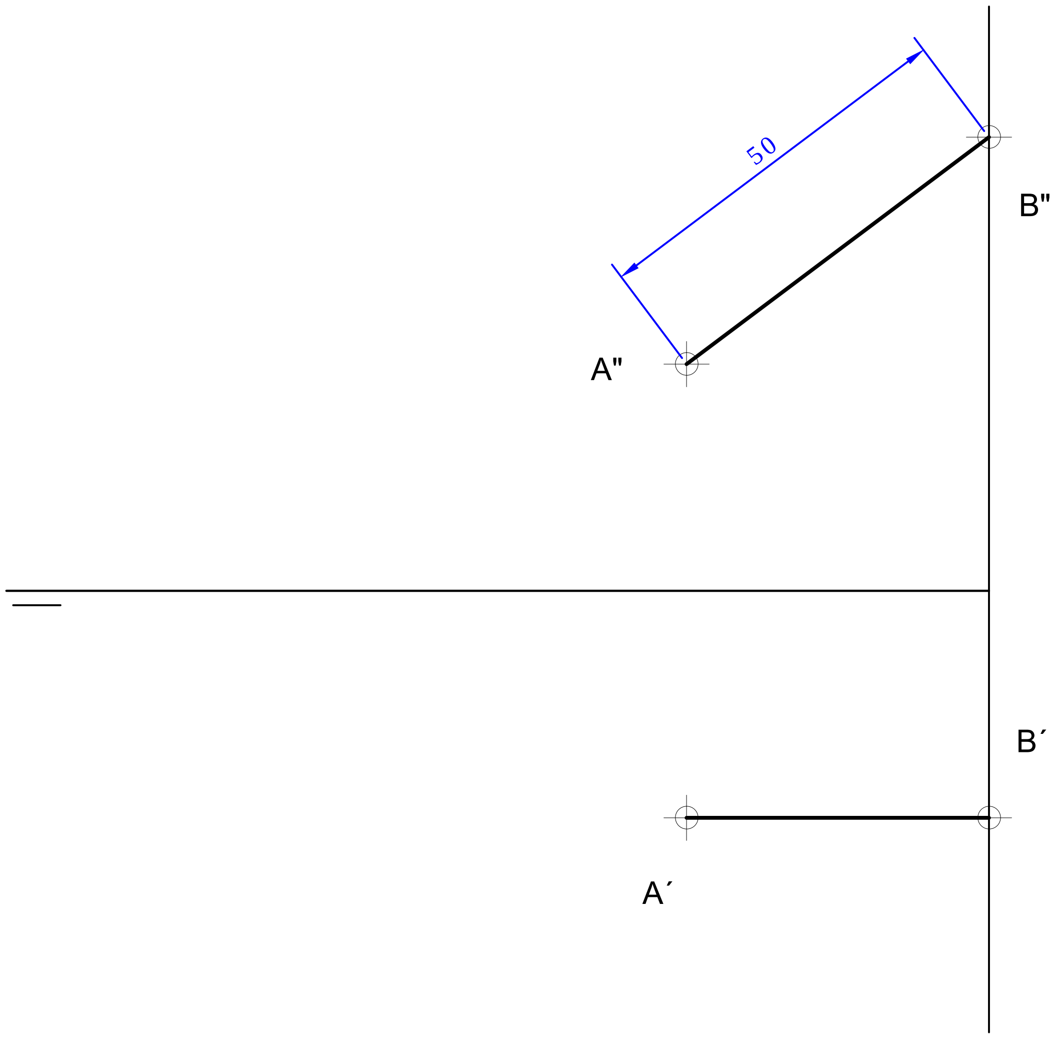
EXERCISE 1

Calculate the distance between the points $A(4,3,3)$ *and* $B(0,3,6)$.

Calculate the distance between the points A and B.

As the line AB is parallel to the PV, we can see the real magnitude of AB in the vertical projection.



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EXERCISE 1

Calculate the distance between the points $A(4,3,3)$ *and* $B(0,3,6)$.

Solution:

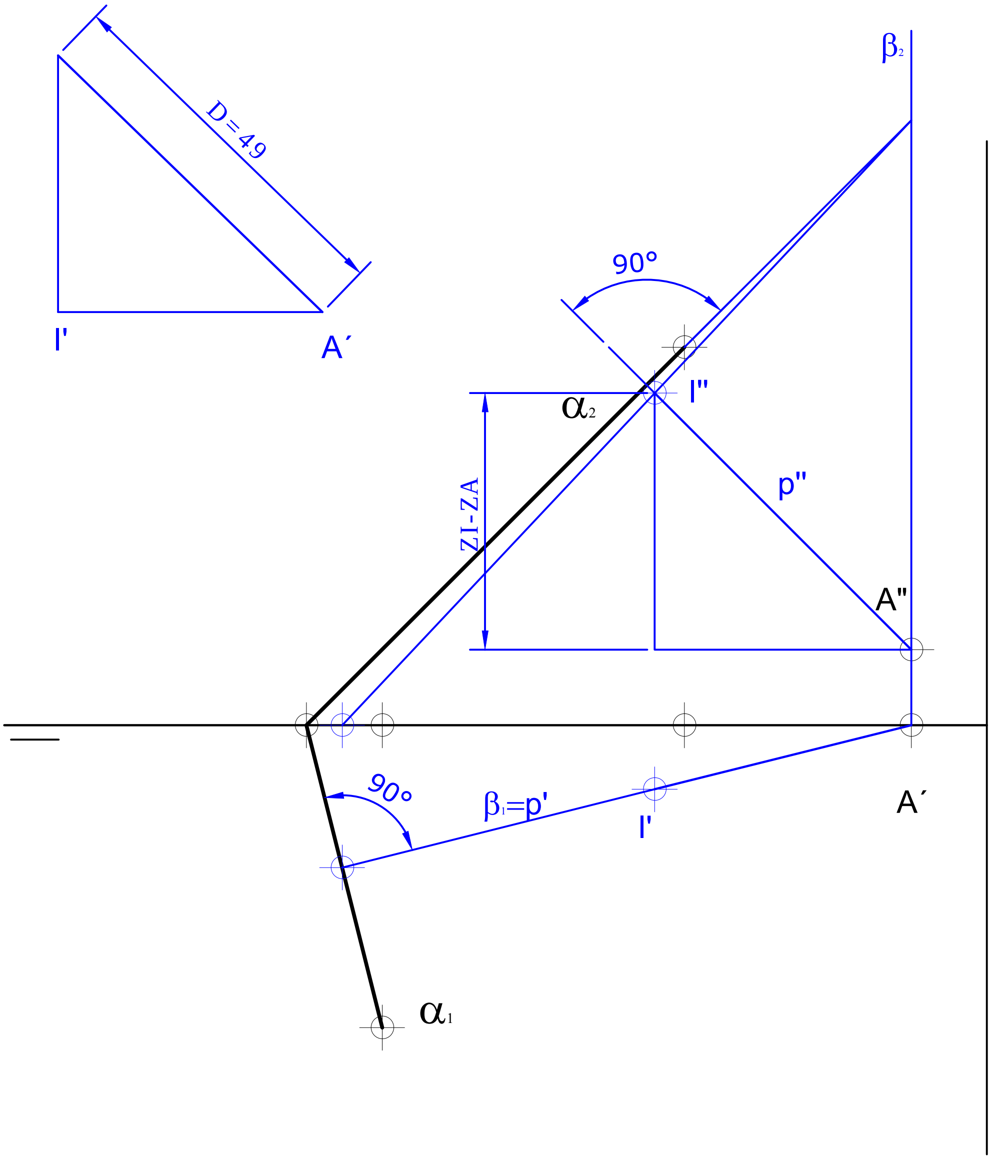
The distance between the points $A(4,3,3)$ *and* $B(0,3,6)$ *can be calculated using the next expression:*

$$d(A,B)=\sqrt{(4-0)^2+(3-3)^2+(3-6)^2}=\sqrt{25}=5$$

EXERCISE 2

Calculate the distance from the point $A(1,0,1)$ *to the plane* $\alpha: 4x + y + 4z = 36$.

Calculate the distance between the point A and the plane α.



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EXERCISE 2

Calculate the distance from the point $A(1,0,1)$ *to the plane* $\alpha: 4x+y+4z=36$.

Solution:

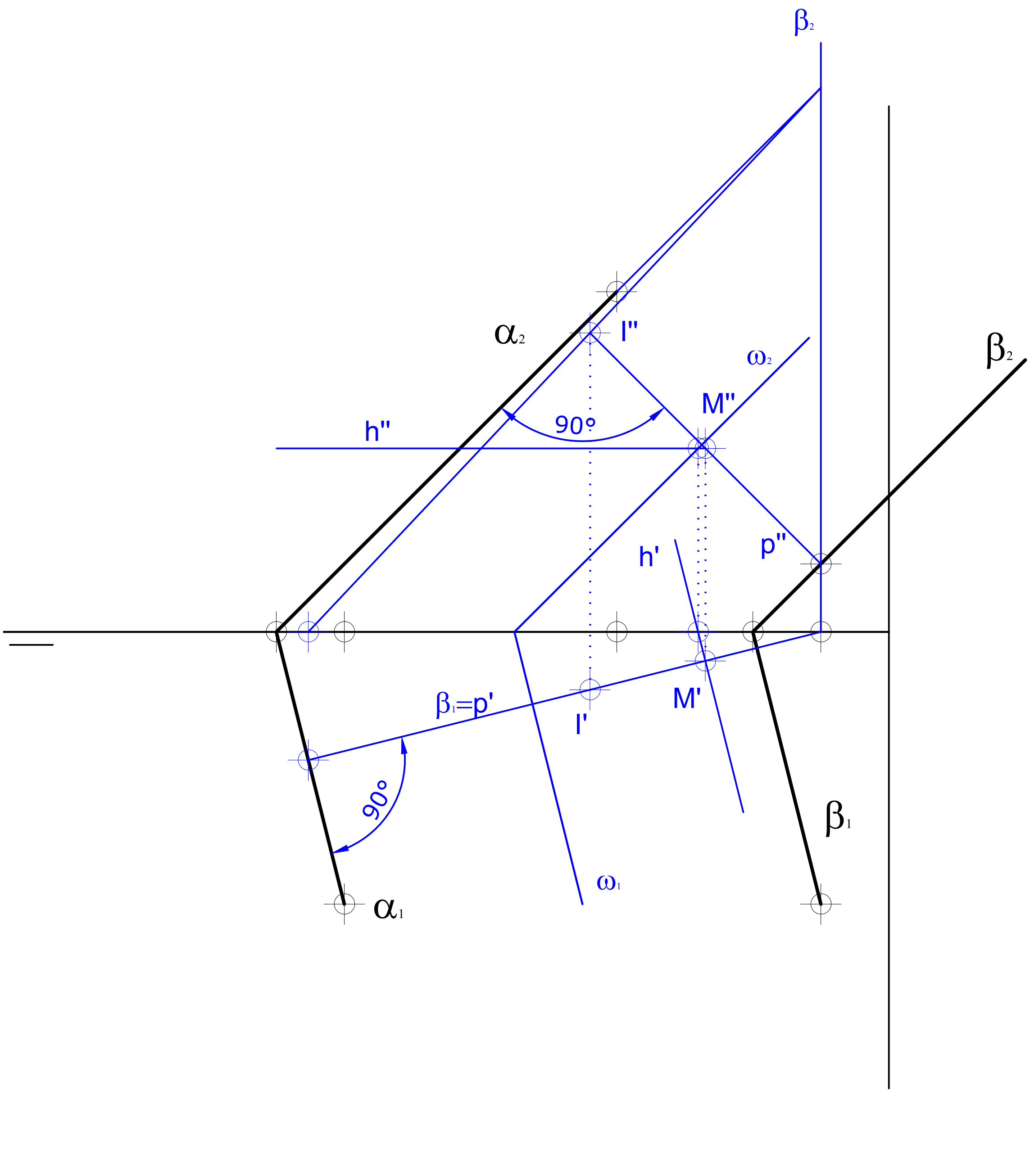
$\vec{n}_\alpha = (4,1,4)$ is the normal vector of the plane α. We obtain the distance between a point and a plane applying the formula:

$$d(A,\alpha) = \frac{|4\cdot 1 + 1\cdot 0 + 4\cdot 1 - 36|}{\sqrt{4^2+1^2+4^2}} = \frac{28}{\sqrt{33}}$$

EXERCISE 3

Calculate the bisector plane of the planes $\alpha: 4x+y+4z=36$ *and* $\beta: 4x+y+4z=8$.

Draw the bisector plane of the planes α and β .



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EXERCISE 3

Calculate the bisector plane of the planes $\alpha : 4x + y + 4z = 36$ *and* $\beta : 4x + y + 4z = 8$.

<u>*Solution:*</u>

The planes α and β are parallel, being their normal vector $(4,1,4)$. So it is possible to calculate their bisector plane. These are the steps that have to be followed:

- Consider any point of one of the planes. For example, we will choose the point $P_\beta = (0,8,0)$ in the plane β and we will calculate the line r that passing though P_β is perpendicular to both planes. As r is perpendicular to both planes, its direction vector is the normal vector of the planes. The parametric equations of r are the following: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 4 \end{pmatrix} \Rightarrow r : \begin{cases} x = 4\lambda \\ y = 8 + \lambda \\ z = 4\lambda \end{cases}$

- We calculate the point of intersection between r and the plane α:

$$4(4\lambda) + (8 + \lambda) + 4(4\lambda) = 36 \Rightarrow \lambda = \frac{28}{33}$$

The point of intersection is $P_\alpha = \left(\frac{112}{33}, \frac{292}{33}, \frac{112}{33} \right)$.

- We calculate the midpoint of the segment with end points P_β and P_α:

$$P_\gamma = \frac{P_\beta + P_\alpha}{2} = \left(\frac{56}{33}, \frac{273}{33}, \frac{56}{33} \right)$$

- We require P_γ to be included in the bisector plane:

$$4\left(x - \frac{56}{33} \right) + \left(y - \frac{273}{33} \right) + 4\left(z - \frac{56}{33} \right) = 0$$

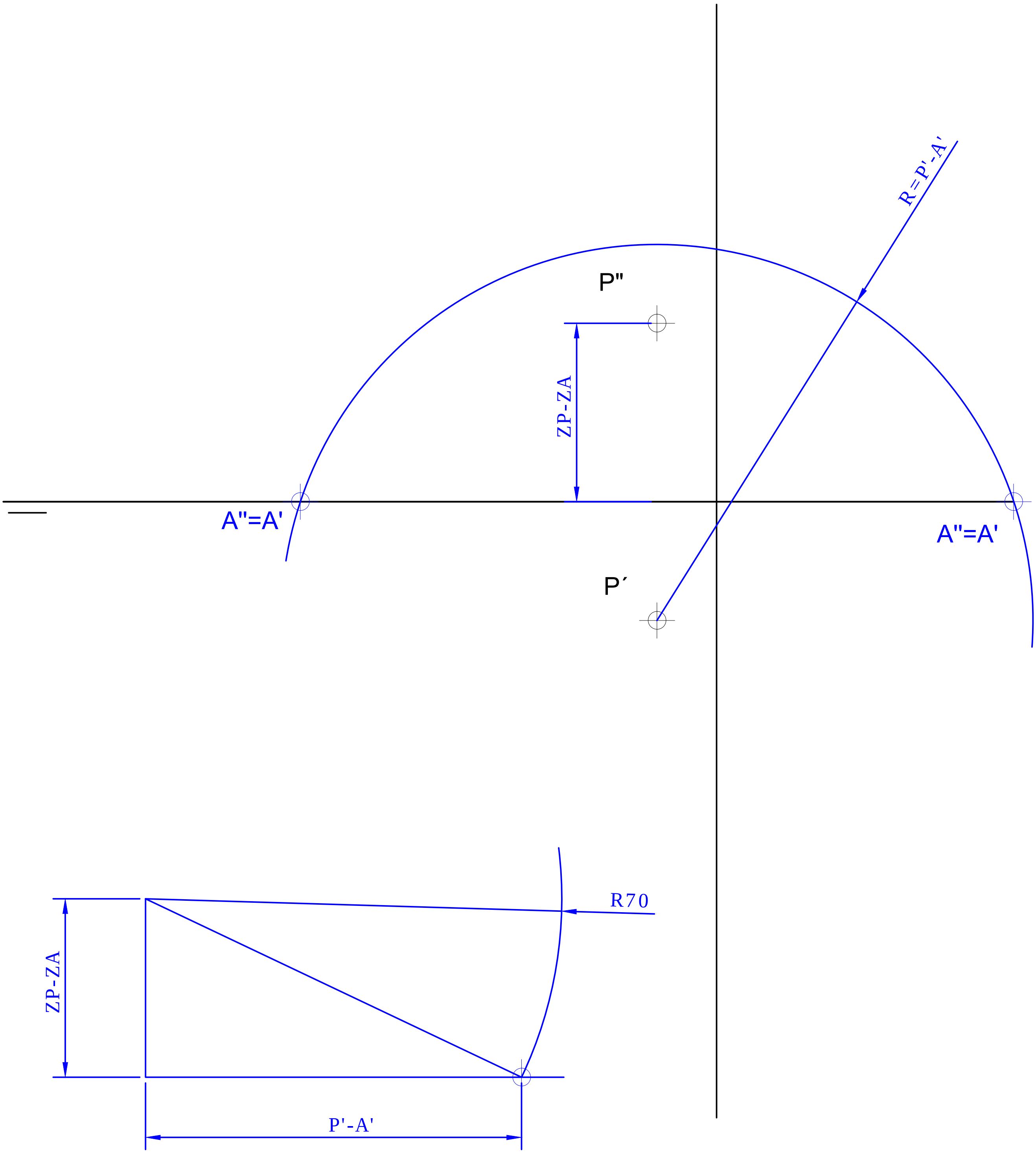
And the bisector plane of α and β is given by the expression:

$$4x + y + 4z - 22 = 0$$

EXERCISE 4

The distance from the point $P(1,2,3)$ *to the point* A *located in the axis of abscissas is 7. Calculate the coordinates of the point* A.

Find the projections of the point A if we know that this is located in the floor-line, being the distance to the point P 70 mm.



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EXERCISE 4

The distance from the point $P(1,2,3)$ *to the point* A *located in the axis of abscissas is 7. Calculate the coordinates of the point* A.

<u>Solution:</u>

As A is located in the axis of abscissas, its y and z coordinates are 0: $A=(x,0,0)$.
It is known that, $d(P,A)=|\overrightarrow{PA}|$ = 7. As a consequence:

$$\sqrt{(x-1)^2+(0-2)^2+(0-3)^2}=7\Rightarrow(x-1)^2+4+9=49\Rightarrow$$

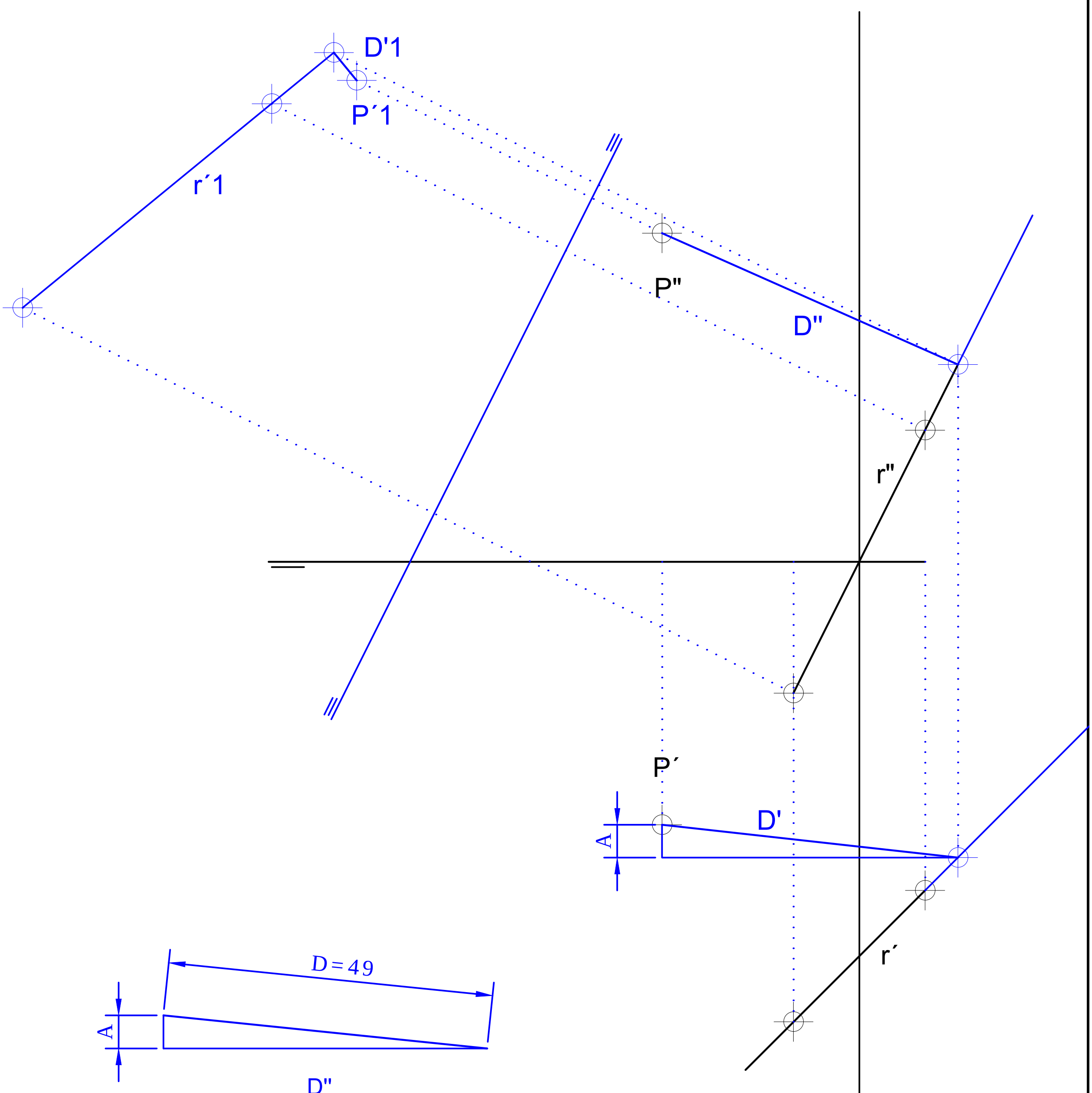
$$(x-1)^2=36\Rightarrow x-1=\pm6\Rightarrow\begin{cases}x-1=6\Rightarrow x=7\\x-1=-6\Rightarrow x=-5\end{cases}$$

Two points that satisfy the requirements are obtained: $A_1=(7,0,0)$ and $A_2=(-5,0,0)$.

EXERCISE 5

Calculate the distance from the point $P(3,4,5)$ *to the line* $r: \frac{x+1}{1} = \frac{y+2}{2} = \frac{z+5}{-1}$.

Find the distance between the point P and the line r.



This exercise has been solved by changing the projection planes.

The distance has been calculated using a triangle of distances.

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EXERCISE 5

Calculate the distance from the point $P(3,4,5)$ *to the line* $r:\frac{x+1}{1}=\frac{y+2}{2}=\frac{z+5}{-1}$.

<u>*Solution:*</u>

The distance from a point to a line can be calculated using the formula:

$$d(P,r)=\frac{\left|\overrightarrow{AP}\wedge\vec{v}_r\right|}{\left|\vec{v}_r\right|}$$

$\vec{v}_r=(1,2,-1)$ is the direction vector of the line r, and $A=(-1,-2,-5)$ is a point included in the line:

$$\overrightarrow{AP}=P-A=(4,6,10)\Rightarrow\overrightarrow{AP}\wedge\vec{v}_r=\begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 4 & 6 & 10\\ 1 & 2 & -1\end{vmatrix}=-26\vec{\imath}+14\vec{\jmath}+2\vec{k}$$

Therefore, the distance between the point and the line is:

$$d(P,r)=\frac{\left|\overrightarrow{AP}\wedge\vec{v}_r\right|}{\left|\vec{v}_r\right|}=\frac{\sqrt{26^2+14^2+2^2}}{\sqrt{1^2+2^2+(-1)^2}}=\sqrt{146}\,.$$