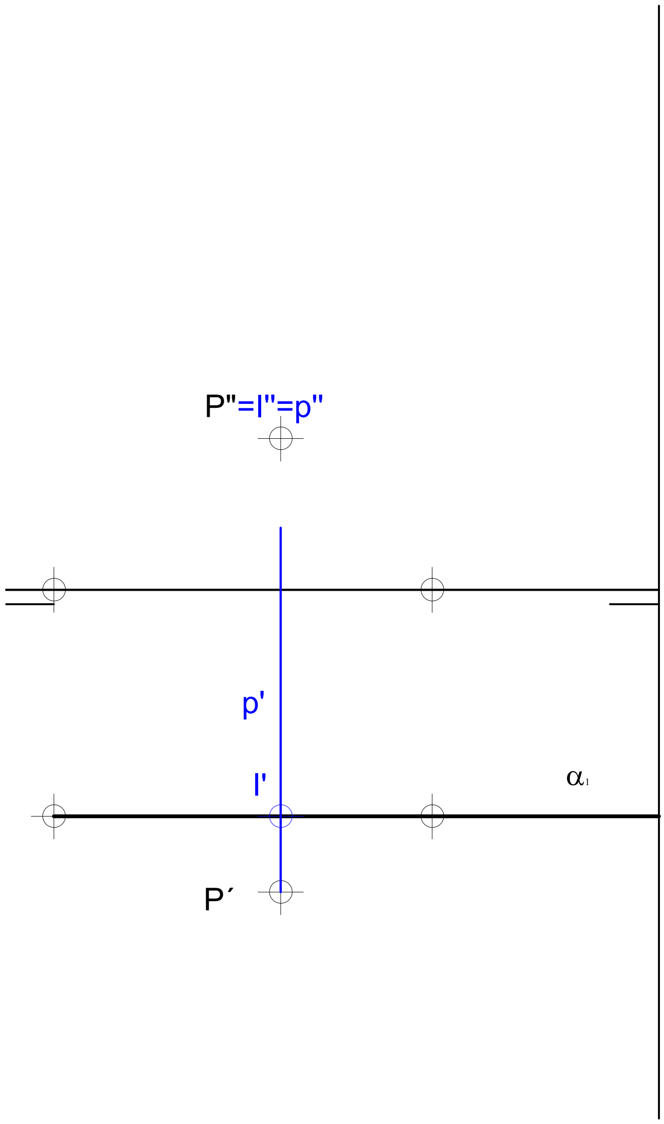
EXERCISE 1

Find the equation of the line that containing the point $P(5,3,2)$ *is perpendicular to the plane that passes through the points* $(7,0,0)$ *and* $(4,3,0)$ *and is perpendicular to the plane* XOY*. Obtain the intersection.*

Draw from the point P a line p that is perpendicular to α. Find also the intersection between the line and the plane (I).



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EXERCISE 1

Find the equation of the line that containing the point $P(5,3,2)$ is perpendicular to the plane that passes through the points $(7,0,0)$ and $(4,3,0)$ and is perpendicular to the plane XOY. Obtain the intersection.

<u>Solution:</u>

First, we obtain the equation of the plane α that passes through the points $(7,0,0)$ and $(4,3,0)$ using a point and two vectors included in it.

For example, the vector $\vec{u} = (7,0,0) - (4,3,0) = (3,-3,0)$ is included in the plane. On the other hand, the plane α is perpendicular to the plane XOY, so the vector $(0,0,1)$ is also included in it. Therefore, the implicit equation of the plane α is:

$$\alpha: \begin{vmatrix} x-7 & 0 & 3 \\ y & 0 & -3 \\ z & 1 & 0 \end{vmatrix} = 0 \Rightarrow \alpha: x + y = 7$$

As we can observe, the normal vector of the plane α is $\vec{n} = (1,1,0)$.

The line we are looking for is perpendicular to the plane calculated above, hence its direction vector is $\vec{n} = (1,1,0)$. Furthermore, as the line passes through the point $P = (5,3,2)$ the parametric equations of the line r are:

$$r: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

And the implicit equations are:

$$\begin{cases} x = 5 + \lambda \\ y = 3 + \lambda \\ z = 2 \end{cases} \Rightarrow \begin{cases} x - 5 = y - 3 \\ z = 2 \end{cases} \Rightarrow r: \begin{cases} x - y = 2 \\ z = 2 \end{cases}$$

Finally, the requested intersection is: $\begin{cases} x - y = 2 \\ z = 2 \\ x + y = 7 \end{cases} \Rightarrow \begin{cases} x = \frac{9}{2} \\ y = \frac{5}{2} \\ z = 2 \end{cases}$

So, the intersection between the line r and the plane α is:

$$\left(\frac{9}{2}, \frac{5}{2}, 0\right)$$

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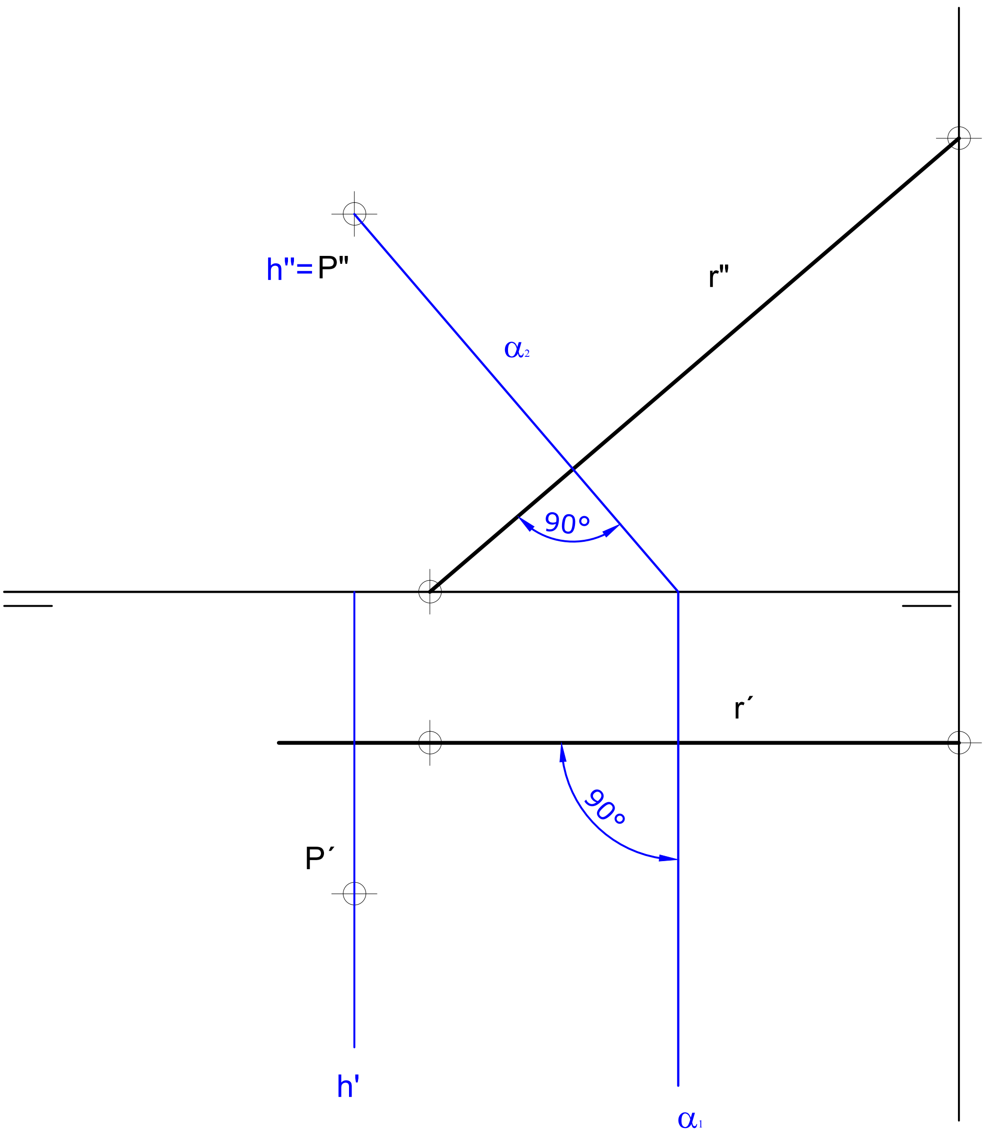
EXERCISE 2

Find the plane that containing the point $P(8,5,4)$ is perpendicular to the line that passes through the points $(7,2,0)$ and $(0,2,6)$. Obtain the intersection.

Draw from the point P a plane α that is perpendicular to r. Find also the intersection point between the line and the plane (I).



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EXERCISE 2

Find the plane that containing the point $P(8,5,4)$ is perpendicular to the line that passes through the points $(7,2,0)$ and $(0,2,6)$. Obtain the intersection.

<u>Solution:</u>

Let r be the line that passes through the points $(7,2,0)$ and $(0,2,6)$. Hence, the direction vector of the line is: $\vec{v}_r = (7,2,0) - (0,2,6) = (7,0,-6)$.

The normal vector of the plane π and the direction vector of the line r are parallel:

$$\vec{v}_r \parallel \vec{n}_\pi \Rightarrow \vec{n}_\pi = (7,0,-6)$$

So, $\pi: 7x + 0y - 6z + k = 0$.

Furthermore, as the point $P(8,5,4)$ is included in the plane, it has to satisfy the equation of the plane:

$$7 \cdot 8 - 6 \cdot 4 + k = 0 \Rightarrow k = -32$$

In conclusion, the equation of the plane π is:

$$\pi: 7x - 6z - 32 = 0$$

Next, we obtain the parametric equations of the line r to compute the intersection of this line and the plane π.

$$r: \begin{cases} x = 7 + 7\lambda \\ y = 2 \\ z = -6\lambda \end{cases}$$

We consider $r \cap \pi$ by substituting a generic point $(x, y, z) \in r$ in the plane:

$$7(7 + 7\lambda) + 36\lambda - 32 = 0 \Rightarrow 49 + 49\lambda + 36\lambda - 32 = 0 \Rightarrow \lambda = -\frac{1}{5}$$

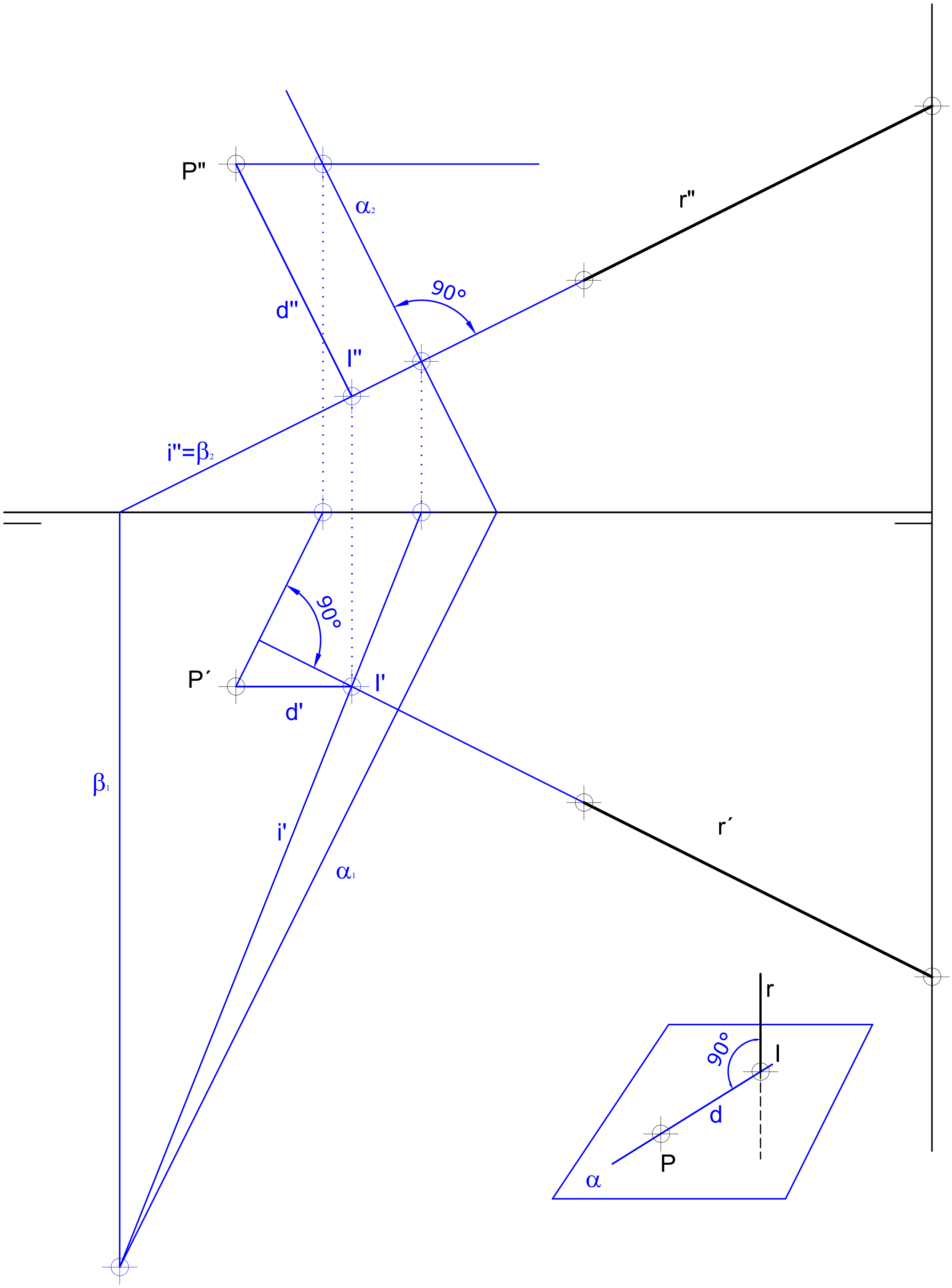
Then the point of intersection is $Q\left(\frac{28}{5}, 2, \frac{42}{5}\right)$.

EXERCISE 3

Calculate the line that passing through the point $P(12,3,6)$ *is perpendicular and intersects the line* r: $\frac{x-6}{-6}=\frac{y-5}{3}=\frac{z-4}{3}$. *Obtain the intersection.*

Draw from the point P a line that intersects the line r and is perpendicular to it.

Find also the intersection between the two lines.



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EXERCISE 3

Calculate the line that passing through the point $P(12,3,6)$ is perpendicular and intersects the line r: $\frac{x-6}{-6}=\frac{y-5}{3}=\frac{z-4}{3}$. Obtain the intersection.

Solution:

First, we define the plane π that passing through the point P has as normal vector the direction vector of the line r:

$$\pi:\ -6(x-12)+3(y-3)+3(z-6)=0\ \Rightarrow\ \pi:\ -2x+y+z+15=0$$

On the other hand, we obtain the parametric equations of the line, using its continuous equation:

$$\frac{x-6}{-6}=\frac{y-5}{3}=\frac{z-4}{3}=\lambda\ \Rightarrow \begin{cases} x=6-6\lambda \\ y=5+3\lambda \\ z=4+3\lambda \end{cases}$$

We obtain the point Q, the intersection between the plane π and the line r by substituting these values in the equation of the plane:

$$-2(6-6\,\lambda)+1(5+3\lambda)+1(4+3\lambda)+15=0\ \Rightarrow \lambda=-2/3$$

$$\begin{cases} x=6-6\,(-2/3) \\ y=5+3\,(-2/3) \\ z=4+3\,(-2/3) \end{cases} \Rightarrow \begin{cases} x=10 \\ y=3 \\ z=2 \end{cases} \Rightarrow Q=(10,3,2)$$

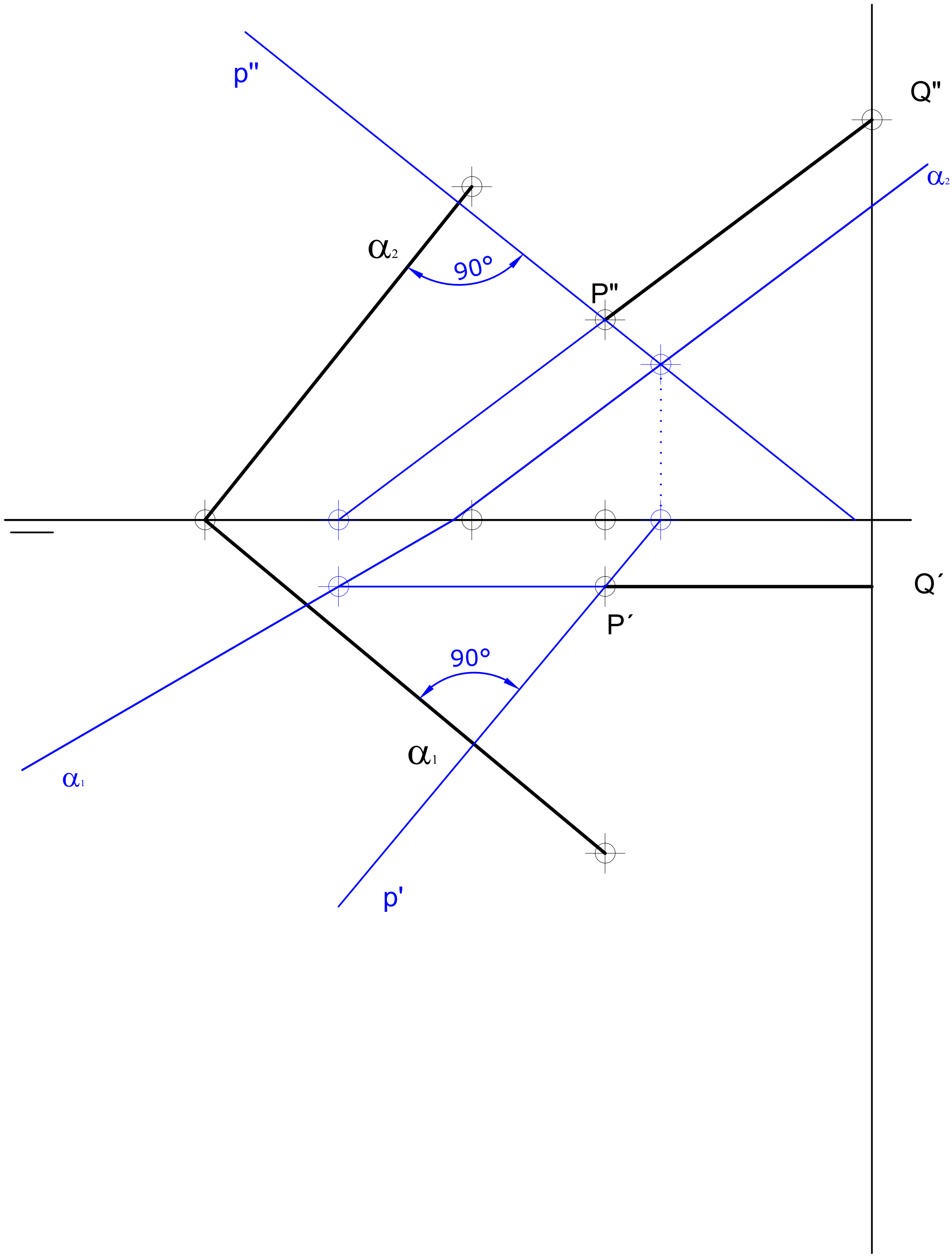
Then, the line we are looking for is the line that passes through the points P and Q:

$$p:\begin{cases} \mathrm{y}=3 \\ 2\mathrm{x}-\mathrm{z}=18 \end{cases}$$

EXERCISE 4

Let r be the line determined by the points $(4,1,3)$ *and* $Q(0,1,6)$, *and* α *the plane determined by the points* $(10,0,0)$, $(6,0,5)$ *and* $(4,5,0)$. *Find the planes that containing the line r are perpendicular to the plane* α.

Draw from the points P and Q perpendicular planes to the plane α.



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EXERCISE 4

Let r be the line determined by the points $(4,1,3)$ *and* $Q(0,1,6)$, *and* α *the plane determined by the points* $(10,0,0)$, $(6,0,5)$ *and* $(4,5,0)$. *Find the planes that containing the line r are perpendicular to the plane* α.

<u>*Solution:*</u>

The procedure to obtain the planes that containing the line r are perpendicular to the plane α is the following:

- Calculate the equation of the line r: We obtain the parametric equations of the line using the point Q and the direction vector $\vec{u} = Q - P = (-4,0,3)$:

$$r: \begin{cases} x = 0 - 4\lambda \\ y = 1 \\ z = 6 + 3\lambda \end{cases}$$

Its implicit equations are: $\begin{cases} \frac{x}{4} = \frac{z-6}{3} \\ y = 1 \end{cases} \Rightarrow r: \begin{cases} 3x - 4z = 24 \\ y = 1 \end{cases}$

- Obtain the sheaf of intersecting planes that contain the line r:

The expression of the planes that contain the line r is:

$$(y - 1) + \lambda(3x + 4z - 24) = 0 \Rightarrow 3\lambda x + y + 4\lambda z - 24\lambda - 1 = 0$$

- Compute the normal vector of the plane α: We consider two direction vectors of the plane, for example $(6,0,5) - (10,0,0) = (-4,0,5)$ and $(4,5,0) - (10,0,0) = (-6,5,0)$, and we compute the vector product of them:

$$\vec{n}_\alpha = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 0 & 5 \\ -6 & 5 & 0 \end{vmatrix} = -25\vec{i} - 30\vec{j} - 20\vec{k}$$

Therefore, the normal vector of the plane α is $\vec{n} = (5,6,4)$.

- Obtain the planes that containing the line r are perpendicular to the plane α:

Since the planes we are looking for are perpendicular to the plane α, their associated vector is perpendicular to the vector $\vec{n}_\alpha$:

$$(3\lambda, 1, 4\lambda) \cdot (5,6,4) = 0 \Rightarrow \lambda = \frac{-6}{31}$$

By substituting this value in the expression obtained in the second point of this exercise, we obtain the plane that containing the line r is perpendicular to the plane α:

$$3\left(\frac{-6}{31}\right)x + y + 4\left(\frac{-6}{31}\right)z - 24\left(\frac{-6}{31}\right) - 1 = 0 \Rightarrow$$

$$18x - 31y + 24z - 113 = 0$$