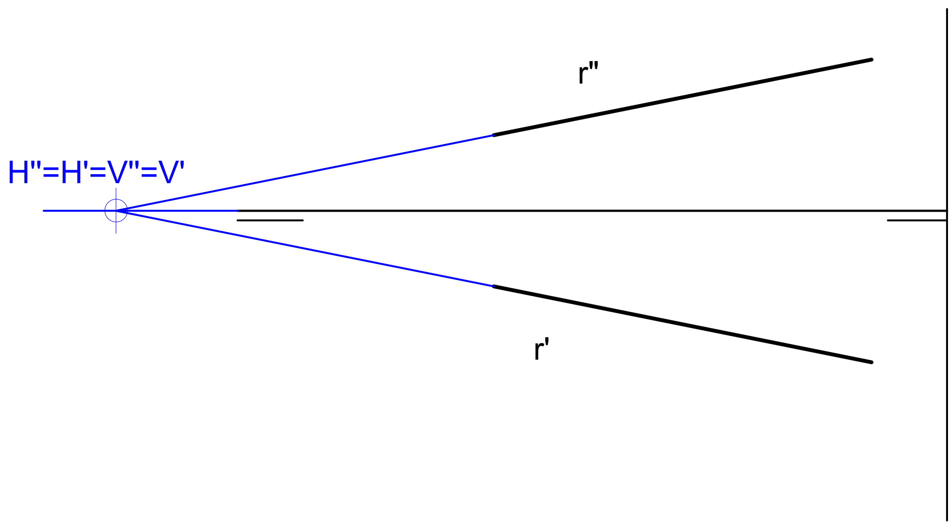
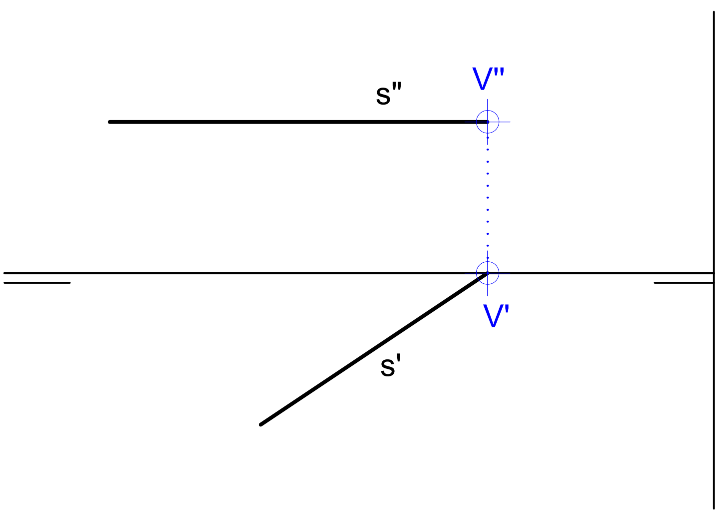
EXERCISE 1

Calculate the intersection between the lines $r: \begin{cases} x + 3y = 13 \\ y = z \end{cases}$ *and* $s: \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ *and the planes XOY and XOZ.*

Find the traces of the lines r and s.





THERE IS NO HORIZONTAL TRACE BECAUSE THE LINE IS PARALLEL TO PH.

Elisabete Alberdi Celaya, Irantzu Álvarez González, Aitziber Unzueta Inchaurbe, Mª Isabel Eguia Ribero and Mª José García López

EXERCISE 1

Calculate the intersection between the lines $r: \begin{cases} x + 3y = 13 \\ y = z \end{cases}$ *and* $s: \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ *and the planes XOY and XOZ.*

<u>*Solution:*</u>

To obtain the intersection between a line and a plane we only have to solve the system formed by their equations. Next, we calculate the requested intersections:

- Intersection between the line r and the plane XOY:

$$\begin{cases} x + 3y = 13 \\ y = z \\ z = 0 \end{cases} \Longrightarrow \begin{cases} x = 13 \\ y = 0 \\ z = 0 \end{cases}$$

The intersection between the line r and the plane XOY is the point $(13{,}0{,}0)$.

- Intersection between the line r and the plane XOZ:

$$\begin{cases} x + 3y = 13 \\ y = z \\ y = 0 \end{cases} \Longrightarrow \begin{cases} x = 13 \\ y = 0 \\ z = 0 \end{cases}$$

The intersection between the line r and the plane XOZ is the point $(13{,}0{,}0)$.

- Intersection between the line s and the plane XOY:

First of all we calculate the implicit equation of the line s:

$$s: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} x = 3 + 3\mu \\ y = 2\mu \\ z = 2 \end{cases}$$

Making the expressions of the parameter μ equal:

$$\frac{x-3}{3} = \frac{y}{2} \Rightarrow 2x - 6 = 3y$$

Therefore, the implicit equations of the line s are $\begin{cases} 2x - 3y = 6 \\ z = 2 \end{cases}$

We solve the following system to obtain the intersection between the line s and the plane XOY:

EXERCISE 1

$$\begin{cases} 2x-3y=6 \\ z=2 \\ z=0 \end{cases}$$ ⇒ The system is incompatible, so the plane and the line do not intersect ⇒ the line and the plane are parallel.

- *Intersection between the line* s *and the plane* XOZ*:*

Similarly, we solve the system formed by the implicit equations of the line and the plane:

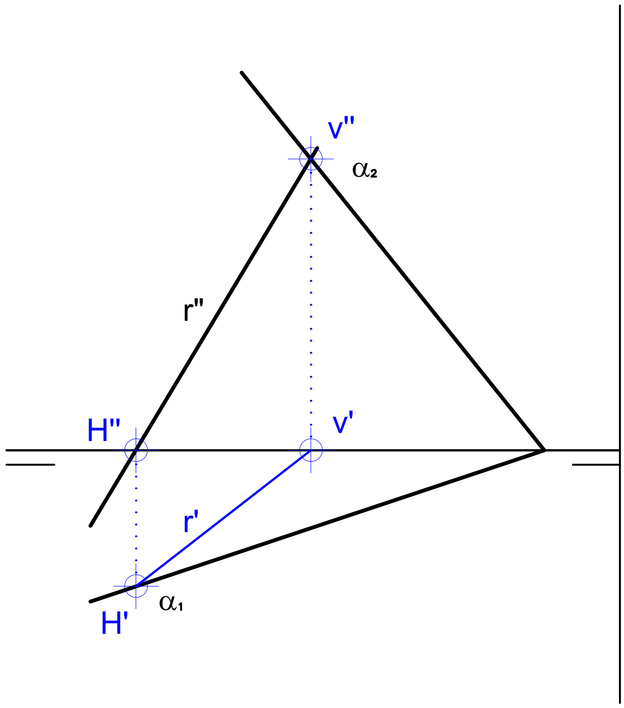
$$\begin{cases} 2x-3y=6 \\ z=2 \\ y=0 \end{cases} \Rightarrow \begin{cases} x=3 \\ y=0 \\ z=2 \end{cases}$$

The intersection between the line s and the plane XOZ is the point $(3,0,2)$.

EXERCISE 2

Calculate the values of the real parameters a and b so that the plane $\alpha: 10x - 30y - 8z = 10$ contains the line that passes through the points $(4, a, 4)$ *and* $(7, b, -1)$.

Find the horizontal projection of the line r so that it is included in the plane α .



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EXERCISE 2

Calculate the values of the real parameters a and b so that the plane $\alpha: 10x - 30y - 8z = 10$ contains the line that passes through the points $(4, a, 4)$ *and* $(7, b, -1)$.

<u>*Solution:*</u>

The line that passes through the points $A(4, a, 4)$ and $B(7, b, -1)$ will lie in the plane α that contains the points A and B.

The plane α contains the point $A(4, a, 4)$ if:

$$10 \cdot 4 - 30a - 8 \cdot 4 = 10 \Rightarrow -30a = 2 \Rightarrow a = -\frac{1}{15}$$

The plane α contains the point $B(7, b, -1)$ if:

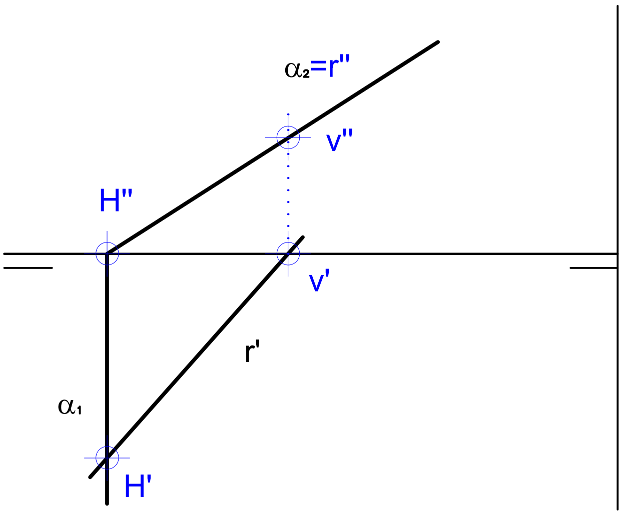
$$10 \cdot 7 - 30b + 8 = 10 \Rightarrow -30b = -68 \Rightarrow b = \frac{34}{15}$$

Therefore, the line that passes through the points $(4, a, 4)$ and $(7, b, -1)$ lies in the plane α if $a = -\frac{1}{15}$ and $b = \frac{34}{15}$.

EXERCISE 3

Calculate the values of the parameters a *and* b *so that the plane that passes through the points* $P=(6,0,0)$, $Q=(2,0,2)$ *and* $R=(6,3,0)$ *contains the line* $r:\begin{cases} 3x-3y-12=0 \\ y(b-a)-3z+3a=0 \end{cases}$.

Find the vertical projection of the line r so that it is included in the plane α .



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EXERCISE 3

Calculate the values of the parameters a *and* b *so that the plane that passes through the points* $P=(6,0,0)$, $Q=(2,0,2)$ *and* $R=(6,3,0)$ *contains the line* $r:\begin{cases} 3x-3y-12=0 \\ y(b-a)-3z+3a=0 \end{cases}$.

<u>Solution:</u>

First, we obtain the plane π that contains the points P, Q and R. To obtain the equation of the plane we use two vectors included in it, for example $\overrightarrow{PQ}=(-4,0,2)$ and $\overrightarrow{PR}=(0,3,0)$.

$$\begin{vmatrix} x-6 & -4 & 0 \\ y & 0 & 3 \\ z & 2 & 0 \end{vmatrix}=0 \Rightarrow (x-6)\begin{vmatrix} 0 & 3 \\ 2 & 0 \end{vmatrix}-y\begin{vmatrix} -4 & 0 \\ 2 & 0 \end{vmatrix}+z\begin{vmatrix} -4 & 0 \\ 0 & 3 \end{vmatrix}=0$$

$$\Rightarrow -6(x-6)-12z=0$$

$$\pi: x+2z-6=0$$

Next, we consider the following linear equations system:

$$r\cap\pi: \begin{cases} 3x-3y-12=0 \\ y(b-a)-3z+3a=0 \\ x+2z-6=0 \end{cases}$$

being M the coefficient matrix and M' the augmented matrix.

$$(M|M')=\left(\begin{array}{ccc|c} 3 & -3 & 0 & -12 \\ 0 & b-a & -3 & 3a \\ 1 & 0 & 2 & -6 \end{array}\right)$$

The line r lies in the plane π if $rank(M)=rank(M')=2<$ number of unknowns$=3$.

The rank of the matrix M is greater or equal to 2, since $\begin{vmatrix} 3 & -3 \\ 1 & 0 \end{vmatrix}=3\neq 0$.

So, the following three minors must be null:

$$\begin{vmatrix} 3 & -3 & 0 \\ 0 & b-a & -3 \\ 1 & 0 & 2 \end{vmatrix}=0 \Rightarrow 2b-2a+3=0 \quad \text{eta} \quad \begin{vmatrix} 3 & -3 & -12 \\ 0 & b-a & 3a \\ 1 & 0 & -6 \end{vmatrix}=0 \Rightarrow$$

$$2b+a=0$$

Solving the system $\begin{cases} 2b-2a+3=0 \\ 2b+a=0 \end{cases}$ formed by the two equations we obtain that values of the parameters are $a=1$ and $b=-\frac{1}{2}$.

Finally, we obtain the equation of the line r using these two values:

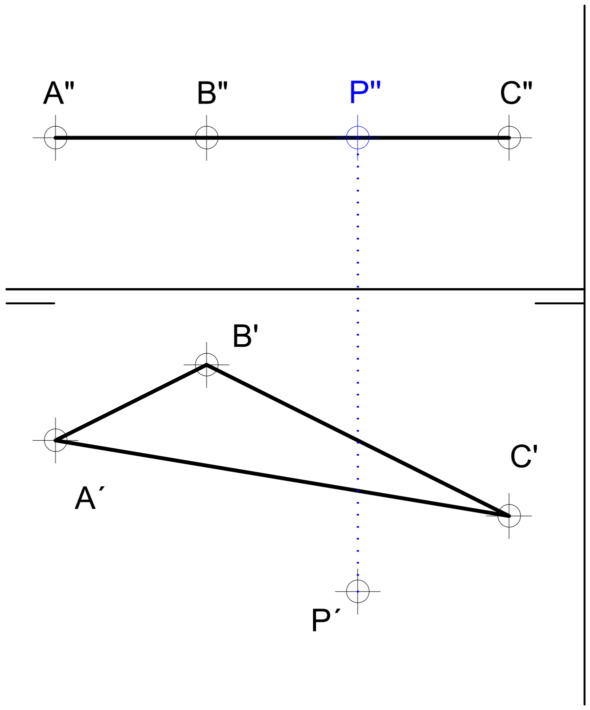
$$r: \begin{cases} 3x-3y-12=0 \\ -\frac{3}{2}y-3z+3=0 \end{cases} \Rightarrow \begin{cases} x-y=4 \\ y+6z=6 \end{cases}$$

EXERCISE 4

Determine the coordinate z *so that the plane* α: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

contains the point $P = (3,4,z)$.

Find the vertical projection of the point P so that it is included in the plane ABC.



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EXERCISE 4

Determine the coordinate z *so that the plane* α: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ *contains the point* $P = (3,4,z)$.

<u>*Solution:*</u>

The plane α contains the point $P = (3,4,z)$, if the point satisfices the equation of the plane. Therefore, the following system must be compatible:

$$\begin{pmatrix} 3 \\ 4 \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 3 = 1 + 6\lambda + 2\mu \\ 4 = 3 - \lambda + \mu \\ \mathrm{z} = 2 \end{cases} \Rightarrow \begin{cases} 1 = 3\lambda + \mu \\ 1 = -\lambda + \mu \\ \mathrm{z} = 2 \end{cases} \Rightarrow \begin{cases} \lambda = 0 \\ \mu = 1 \\ \mathrm{z} = 2 \end{cases}$$

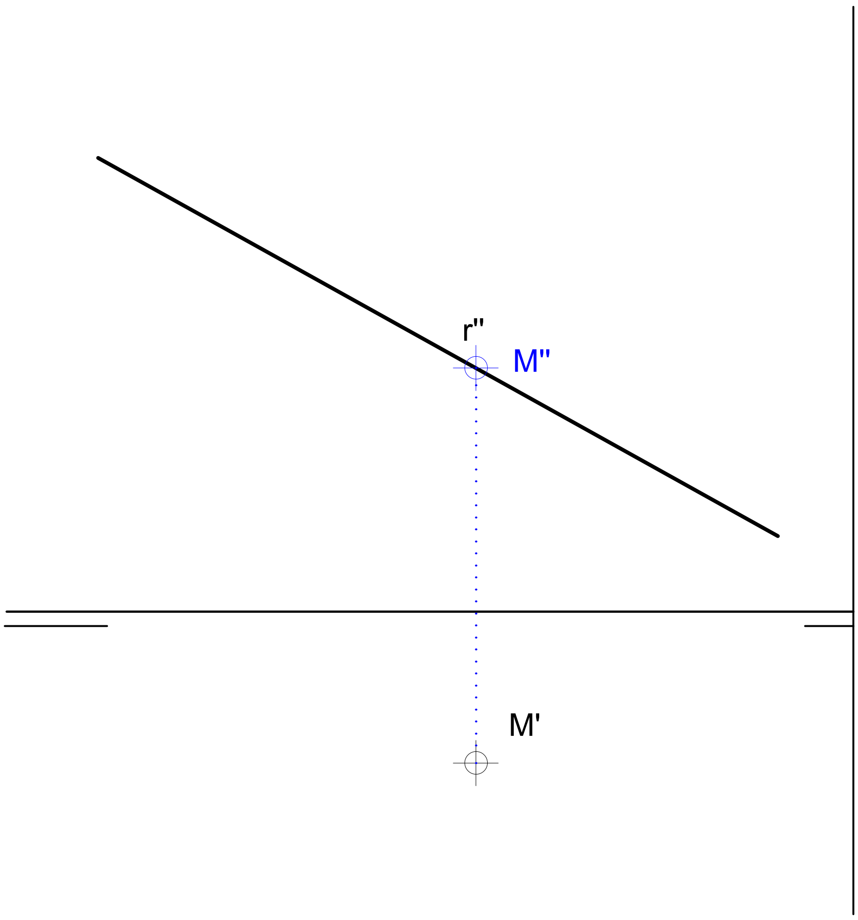
Coordinate $z = 2$, so, the point P that is included in plane α is $(3,4,2)$.

EXERCISE 5

Calculate the values of the parameters a and b so that the line that passes through the points $Q=(10,a,6)$ and $R=(1,b,1)$ and the plane XOZ are parallel.

1. *Determine the coordinate z so that the line r contains the point $M=$ υἰςἲΰ 8*
2. *Determine the coordinates x and y so that the line r contains the point P=(x,y,5).*

Find the vertical projection of the point M so that it is included in the line r. Find the horizontal projection of the line r so that it is parallel to the vertical projection plane. Find the projections of a point P with an elevation of 5, so that it is in the line r.



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EXERCISE 5

Calculate the values of the parameters a and b so that the line that passes through the points $Q = (10, a, 6)$ and $R = (1, b, 1)$ and the plane XOZ are parallel.

1. *Determine the coordinate z so that the line r contains the point $M = (5,2,z)$.*
2. *Determine the coordinates x and y so that the line r contains the point $P = (x, y, 5)$.*

<u>Solution:</u>

We calculate the direction vector of the line r: $\vec{v}_r = \overrightarrow{QR} = R - Q = (-9, b-a, -5)$. The line r and the plane OXZ are parallel if the vector $\vec{v}_r$ and the normal vector of the plane OXZ are perpendicular.

Therefore, the scalar product of both vectors must be zero:

$$\vec{v}_r \perp \vec{n}_\pi \Rightarrow \vec{v}_r \cdot \vec{n}_\pi = 0$$

$\vec{n}_\pi = (0,1,0)$ is the normal vector of the plane OXZ, so the scalar product is:

$$\vec{v}_r \cdot \vec{n}_\pi = 0 \Rightarrow = (-9, b-a, -5) \cdot (0,1,0) = b - a = 0 \Rightarrow a = b$$

Then, the vector equation of the line r is:

$$r: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ b \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -9 \\ 0 \\ -5 \end{pmatrix}, \lambda \in \mathbb{R}$$

1.-The line r will contain the point $M = (5,2,z)$ if there exists a value of the parameter $\lambda \in \mathbb{R}$ for which the point satisfies the equation of the line.

So, if $M \in r$ is satisficed, then $\exists \lambda \in \mathbb{R}$: $\begin{pmatrix} 5 \\ 2 \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ b \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -9 \\ 0 \\ -5 \end{pmatrix} \Rightarrow \begin{cases} 5 = 1 - 9\lambda \\ 2 = b \\ z = 1 - 5\lambda \end{cases} \Rightarrow$

$$b = 2, \lambda = -\frac{4}{9}$$

By substituting the obtained values in the point M we obtain that $M = (5,2,\frac{29}{9})$.

2.- Similarly, the line r will contain the point $P = (x, y, 5)$ if there exists a value of the parameter $\lambda \in \mathbb{R}$ for which the point satisfies the parametric equations of the line.

So, if $P \in r$ is satisficed, then $\exists \lambda \in \mathbb{R}$: $\begin{pmatrix} x \\ y \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ b \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -9 \\ 0 \\ -5 \end{pmatrix} \Rightarrow \begin{cases} x = 1 - 9\lambda \\ y = b \\ 5 = 1 - 5\lambda \end{cases} \Rightarrow$

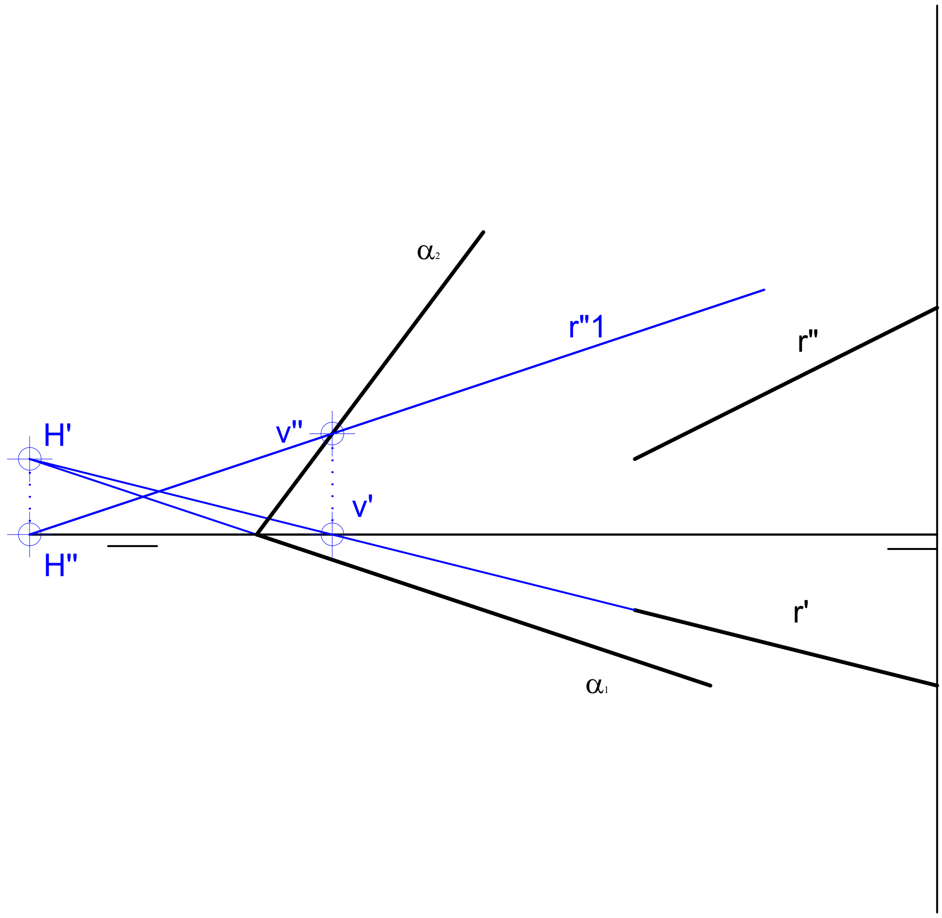
$$\lambda = -\frac{4}{5}$$

By substituting $\lambda = -\frac{4}{5}$ in the point P, we obtain that the line r contains the point $P = (\frac{41}{5}, b, 5)$ $\forall b \in \mathbb{R}$.

EXERCISE 6

Find the relative position of the line r: $\frac{x}{-4} = y - 2 = \frac{z-3}{2}$ and the plane α, being α the plane that passing through the point $P = (3,2,0)$ has as normal vector $\vec{n} = (4,12,3)$.

Define the relative position between the line r and the plane α.



The line r is not in the plane and it is not parallel to the plane, thus, they intersect.

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EXERCISE 6

Find the relative position of the line $r\colon \frac{x}{-4} = y - 2 = \frac{z-3}{2}$ and the plane α, being α the plane that passing through the point $P = (3,2,0)$ has as normal vector $\vec{n} = (4,12,3)$.

Solution:

We obtain the implicit equations of the line r using its parametric equations:

$$r\colon \begin{cases} \dfrac{x}{-4} = y - 2 \\ \dfrac{x}{-4} = \dfrac{z-3}{2} \end{cases} \Rightarrow \begin{cases} x + 4y - 8 = 0 \\ x + 2z - 6 = 0 \end{cases}$$

On the other hand, the implicit equation of a plane with associated vector $\vec{n}$ that passes through the point P is:

$$\alpha\colon 4(x-3) + 12(y-2) + 3(z-0) = 0 \Rightarrow 4x + 12y + 3z - 36 = 0$$

We solve the system $r \cap \alpha$ to determine the relative position of the line r and the plane α:

$$\begin{cases} x + 4y - 8 = 0 \\ x + 2z - 6 = 0 \\ 4x + 12y + 3z - 36 = 0 \end{cases}$$

being M the coefficient matrix and M' the augmented matrix.

$$(M|M') = \left(\begin{array}{ccc|c} 1 & 4 & 0 & 8 \\ 1 & 0 & 2 & 6 \\ 4 & 12 & 3 & 36 \end{array}\right)$$

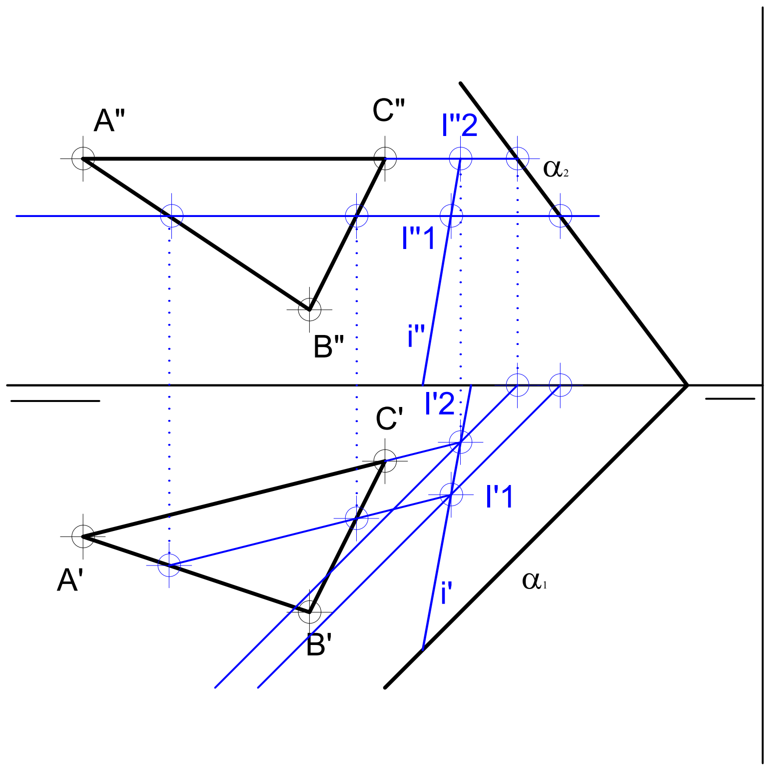
$\begin{vmatrix} 1 & 4 & 0 \\ 1 & 0 & 2 \\ 4 & 12 & 3 \end{vmatrix} = -4 \neq 0,$ therefore $rank(M) = 3 = rank(M') =$ number of unknowns $\Rightarrow$ the system $r \cap \alpha$ is compatible determinate, so the line and the plane intersect in a point.

Solving the system $r \cap \alpha$ we obtain that the intersection point is $I = (-6, 7/2, 6)$.

EXERCISE 7

Find the intersection between β, the plane that contains the points $A=(9,2,3)$, $B=(6,3,1)$ and $C=(5,1,3)$, and the plane $\alpha: 4x-4y-3z=4$.

Find the intersection between the planes ABC and α .



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EXERCISE 7

Find the intersection between β, the plane that contains the points $A = (9,2,3)$, $B = (6,3,1)$ and $C = (5,1,3)$, and the plane $\alpha: 4x - 4y - 3z = 4$.

<u>*Solution:*</u>

We will obtain the equation of the plane with normal vector $\vec{n} = (n_1, n_2, n_3)$ that passes through the point $\mathrm{A} = (9,2,3)$.

First, we determine the direction vectors $\vec{u} = B - A = (-3,1,-2)$ and

$\vec{v} = C - B = (-1,-2,2)$ of the plane β, and we obtain its normal vector as the vector product of them:

$$\vec{n} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -3 & 1 & -2 \\ -1 & -2 & 2 \end{vmatrix} = -2\vec{\imath} + 8\vec{\jmath} + 7\vec{k}$$

So, the equation of β is:

$$-2(x-9) + 8(y-2) + 7(z-3) = 0 \Rightarrow$$

$$\beta:\; -2x + 8y + 7z - 19 = 0$$

To obtain the intersection between the plane α and the plane β, we solve the system formed by their equations:

$\begin{cases} -2x + 8y + 7z = 19 \\ 4x - 4y - 3z = 4 \end{cases}$, being M the coefficient matrix and M' the augmented matrix the followings:

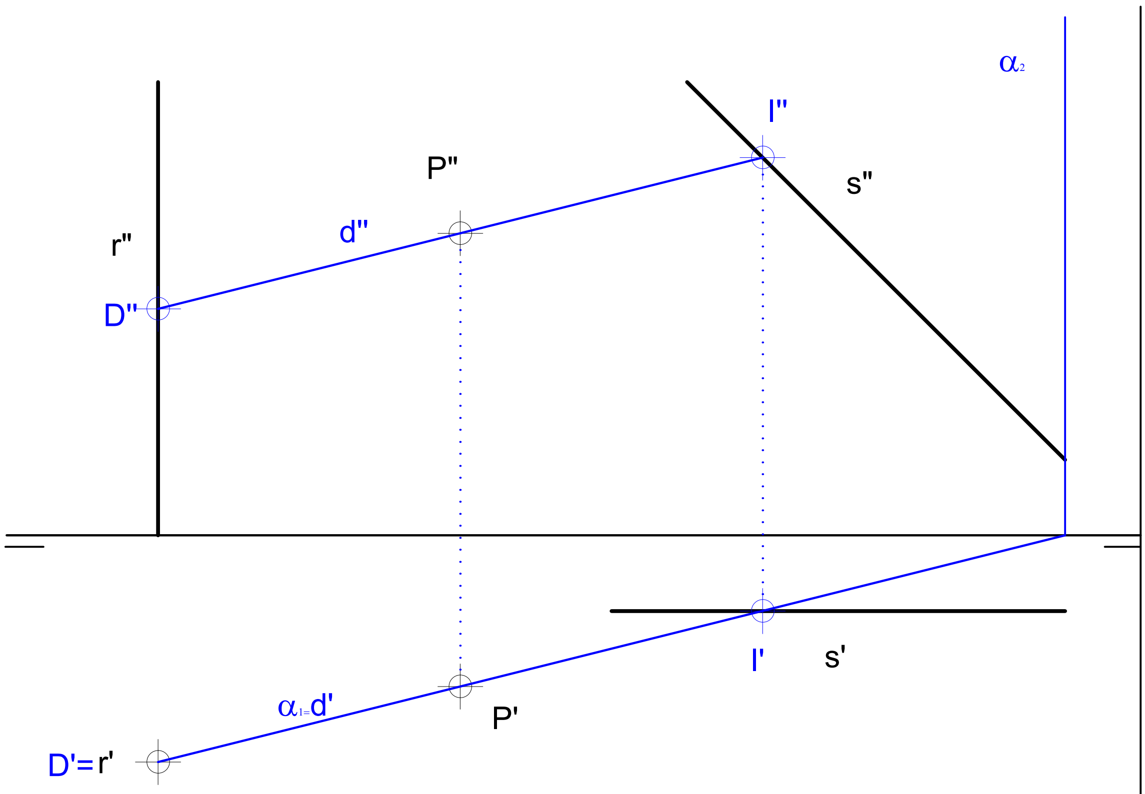
$$(M|M') = \left(\begin{matrix} -2 & 8 & 7 \\ 4 & -4 & -3 \end{matrix}\middle| \begin{matrix} 19 \\ 4 \end{matrix}\right)$$

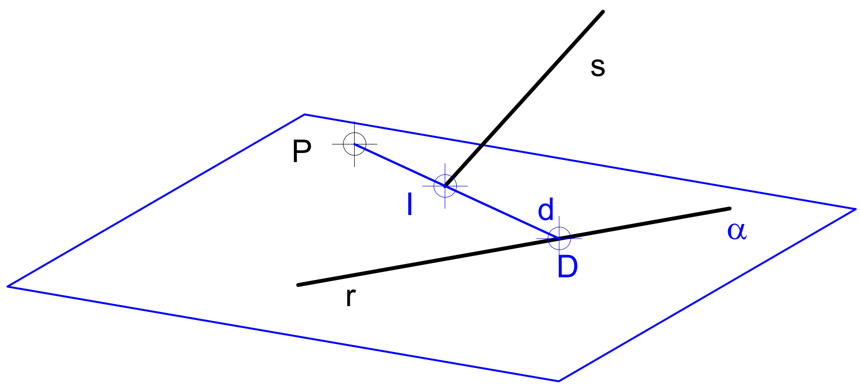
The rank of the matrix M is 2, moreover $\begin{vmatrix} -2 & 8 \\ 4 & -4 \end{vmatrix} \neq 0 \Rightarrow rank(M) = rank\left(M'\right) = 2 < 3 =$ number of unknowns $\Rightarrow$ The system is compatible indeterminate, being the intersection of the planes the line which implicit equation is: $\begin{cases} -2x + 8y + 7z - 19 = 0 \\ 4x - 4y - 3z - 4 = 0 \end{cases}$.

EXERCISE 8

Let be r the line that passes through the points $(13,3,3)$ *and* $(13,3,0)$*, and* s *the one that passes through* $(6,1,6)$ *and* $(1,1,1)$*. Determine the lines that containing the point* $P=(9,2,4)$ *intersect the lines* r *and* s*.*

Draw all the lines that contain the point P and intersect the lines r and s.





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EXERCISE 8

Let be r the line that passes through the points $(13,3,3)$ *and* $(13,3,0)$, *and* s *the one that passes through* $(6,1,6)$ *and* $(1,1,1)$. *Determine the lines that containing the point* $P = (9,2,4)$ *intersect the lines* r *and* s.

<u>*Solution:*</u>

First of all we will obtain the plane π that passing through the point $P(9,2,4)$ contains the line r. To do this, we consider the following not parallel two vectors:

- Direction vector of the line r: $\vec{v}_r = (13,3,3) - (13,3,0) = (0,0,3)$.
- Vector $\vec{v}_{p1} = (13,3,3) - (9,2,4) = (4,1,-1)$

Therefore, the plane π is:

$$\pi: \begin{cases} x = 9 + 4\mu \\ y = 2 + \mu \\ z = 4 + 3\lambda - \mu \end{cases} \Rightarrow \begin{vmatrix} x-9 & 0 & 4 \\ y-2 & 0 & 1 \\ z-4 & 3 & -1 \end{vmatrix} = 0 \Rightarrow \pi: -3x + 12y + 3 = 0$$

Similarly, we obtain the plane π' that passing through the point $P(9,2,4)$ contains the line s. We consider the following not parallel two vectors:

- Direction vector of the line s: $\vec{v}_s = (6,1,6) - (1,1,1) = (5,0,5)$.
- Vector $\vec{v}_{p2} = (6,1,6) - (9,2,4) = (-3,-1,2)$.

So, the plane π' is given by:

$$\pi': \begin{cases} x = 9 + 5\lambda - 3\mu \\ y = 2 - \mu \\ z = 4 + 5\lambda + 2\mu \end{cases} \Rightarrow \begin{vmatrix} x-9 & 5 & -3 \\ y-2 & 0 & -1 \\ z-4 & 5 & 2 \end{vmatrix} = 0 \Rightarrow \pi': x - 5y - z + 5 = 0$$

The lines that containing the point P intersect the lines r and s are the intersection between the planes π and π' ($\pi \cap \pi'$):

$$\begin{cases} -3x + 12y + 3 = 0 \\ x - 5y - z + 5 = 0 \end{cases} \Rightarrow \begin{cases} -3x + 12y = -3 \\ x - 5y = z - 5 \end{cases}$$

The system is compatible indeterminate, since: $A = \begin{pmatrix} -3 & 12 \\ 1 & -5 \end{pmatrix} \Rightarrow |A| = 3$

The solution of the system is the following line:

$$x = \frac{\begin{vmatrix} -3 & 12 \\ z-5 & -5 \end{vmatrix}}{3} = 25 - 4z$$

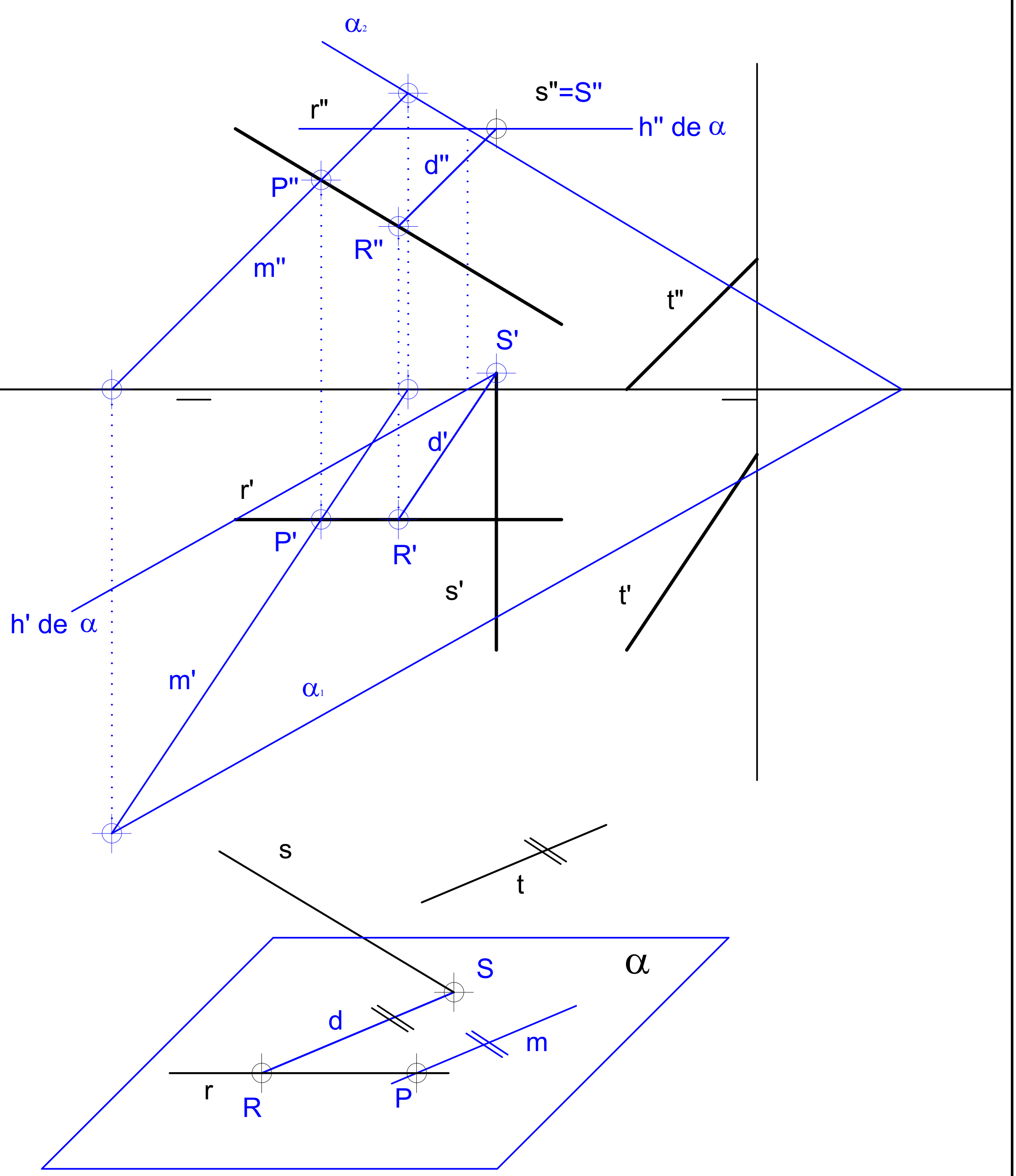
$$y = \frac{\begin{vmatrix} -3 & -3 \\ 1 & z-5 \end{vmatrix}}{3} = -z + 6$$

$$r: \begin{cases} x = 25 - 4z \\ y = -z + 6 \\ z = z \end{cases}$$

EXERCISE 9

Determine the lines that intersecting the lines $r: \begin{cases} 3x - 5y = 4 \\ y = 2 \end{cases}$ *and* $s: \begin{cases} x = 4 \\ z = 4 \end{cases}$ *are parallel to the line* $t: \frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-2}$.

Draw the lines that intersect the lines r and s and are parallel to the line t.



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EXERCISE 9

Determine the lines that intersecting the lines r: $\begin{cases} 3x-5y=4 \\ y=2 \end{cases}$ *and* s: $\begin{cases} x=4 \\ z=4 \end{cases}$ *are parallel to the line* t: $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-2}$.

<u>Solution:</u>

The requested lines are parallel to the line t, so the direction vector of them must be parallel to the direction vector of the line t, which is $\vec{v_t}=(2,3,-2)$. Therefore, the direction vector of the requested line is $(2,3,-2)$.

On the other hand, using the implicit equations of line r, we compute its parametric equations and a generic point included in it:

r: $\begin{cases} 3x-5y=4 \\ y=2 \end{cases} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14/3 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow A = \left(\frac{14}{3}, 2, \lambda\right)$ is a generic point included in r.

In the same way, we calculate a generic point included in s:

s: $\begin{cases} x=4 \\ z=4 \end{cases} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix} \Rightarrow B = (4, \mu, 4)$ is a generic point included in s.

The requested lines intersect the lines r and s, if they pass through a point included in r and through another point included in s. So, we compute the lines that pass through the generic points A and B:

$$\frac{x-4}{4-14/3} = \frac{y-\mu}{\mu-2} = \frac{z-4}{4-\lambda} \Rightarrow \frac{x-4}{-2/3} = \frac{y-\mu}{\mu-2} = \frac{z-4}{4-\lambda}$$

The direction vector of this line is $\left(-\frac{2}{3}, \mu-2, 4-\lambda\right)$, and this vector must be parallel to the vector $(2,3,-2)$, so:

$\frac{-2/3}{2} = \frac{\mu-2}{3} = \frac{4-\lambda}{-2} \Rightarrow \lambda = \frac{10}{3}$ and $\mu = 1$

Substituting the obtained values in the equation of the line, we obtain that:

$$\frac{x-4}{-2/3} = \frac{y-1}{1-2} = \frac{z-4}{4-\frac{10}{3}} \Rightarrow \frac{x-4}{-2/3} = \frac{y-1}{-1} = \frac{z-4}{2/3} = \delta$$

Then, the parametric equations of the requested lines are:

$$\begin{cases} x = 4 - \frac{2}{3}\delta \\ y = 1 - \delta \\ z = 4 + \frac{2}{3}\delta \end{cases}$$