2. GAIA

LAPLACEREN TRANSFORMATUA

2.1 ARIKETA

Azken balioaren teorema erabiliz, ondoko funtzioen azken balioa kalkulatu:

1) $F_1(s) = \dfrac{1}{s(s+1)}$

Emaitza:

s F1(s) poloa negatiboa denez, azken balioaren teorema aplikatu daiteke

$$f_1(\infty) = \underset{t\to\infty}{Lim}\, f_1(t) = \underset{s\to 0}{Lim}\, s\, F_1(s) = \underset{s\to 0}{Lim}\, s\, \frac{1}{s(s+1)} = 1$$

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2) $F_2(s) = \dfrac{2s^2+3s+50}{(s+10)(s+5)(s+1)}$

Emaitza:

s F1(s) ren poloak negatiboak direnez, azken balioaren teorema aplikatu daiteke

$$f_2(\infty) = \underset{t\to\infty}{Lim}\, f_2(t) = \underset{s\to 0}{Lim}\, s\, F_2(s) = \underset{s\to 0}{Lim}\, s\, \frac{2s^2+3s+50}{(s+10)(s+5)(s+1)} = 0$$

2.2 ARIKETA

Hurrengo funtzioen Laplaceren alderantzizko transformatua kalkulatu:

1) $F(s)=\dfrac{5(s+2)}{s^2(s+1)(s+3)}$

Emaitza:

$$F(s)=\frac{5(s+2)}{s^2(s+1)(s+3)}=\frac{a}{s}+\frac{b}{s^2}+\frac{c}{s+1}+\frac{d}{s+3}$$ Koefizienteak kalkulatuz:

$$b=s^2\left.\frac{5(s+2)}{s^2(s+1)(s+3)}\right|_{s=0}=\frac{10}{3},\qquad c=(s+1)\left.\frac{5(s+2)}{s^2(s+1)(s+3)}\right|_{s=-1}=\frac{5}{2}$$

$$d=(s+3)\left.\frac{5(s+2)}{s^2(s+1)(s+3)}\right|_{s=-3}=\frac{5}{18}$$, zenbakitzaileak berdinduz, lortzen dugu: a=-25/9

Taulak erabiliz:

$$f(t)=L^{-1}(F(s))=L^{-1}\left(-\frac{25}{9s}\right)+L^{-1}\left(\frac{10}{3s^2}\right)+L^{-1}\left(\frac{5}{2(s+1)}\right)+L^{-1}\left(\frac{5}{18(s+3)}\right)$$

Azkenean, emaitza:

$$f(t)=-\frac{25}{9}+\frac{10}{3}t+\frac{5}{2}e^{-t}+\frac{5}{18}e^{-3t}$$

2) $F(s)=\dfrac{s+1}{s\ (s^2+s+1)}$

Emaitza:

$$F(s)=\frac{s+1}{s\ (s^2+s+1)}=\frac{a}{s}+\frac{bs+c}{s^2+s+1},$$ Koefizienteak kalkulatuz:

$$a=s\frac{(s+1)}{s(s^2+s+1)}\bigg|_{s=0}=1$$, zenbakitzaileak berdinduz b+1=0 →b=-1, c+1=1→c=0

Orduan

$$\boldsymbol{F(s)}=\frac{1}{\boldsymbol{s}}+\frac{-\boldsymbol{s}}{\boldsymbol{s}^2+\boldsymbol{s}+1}=\frac{1}{\boldsymbol{s}}-\frac{(\boldsymbol{s}+\frac{1}{2})}{(\boldsymbol{s}+\frac{1}{2})^2+\left(\sqrt{\frac{3}{4}}\right)^2}+\frac{\frac{1}{2}}{\sqrt{\frac{3}{4}}}\frac{\sqrt{\frac{3}{4}}}{(\boldsymbol{s}+\frac{1}{2})^2+\left(\sqrt{\frac{3}{4}}\right)^2}$$

Taulak erabiliz:

$$\boldsymbol{f(t)}=\boldsymbol{L}^{-1}(\boldsymbol{F(s)})=\boldsymbol{L}^{-1}\left(\frac{1}{\boldsymbol{s}}\right)-\boldsymbol{L}^{-1}\left(\frac{(\boldsymbol{s}+\frac{1}{2})}{(\boldsymbol{s}+\frac{1}{2})^2+\left(\sqrt{\frac{3}{4}}\right)^2}\right)+\frac{1}{\sqrt{3}}\boldsymbol{L}^{-1}\left(\frac{\sqrt{\frac{3}{4}}}{(\boldsymbol{s}+\frac{1}{2})^2+\left(\sqrt{\frac{3}{4}}\right)^2}\right)$$

Azkenean, emaitza:

$$\boldsymbol{f(t)}=1-\boldsymbol{e}^{-\frac{1}{2}\boldsymbol{t}}\ \cos\left(\sqrt{\frac{3}{4}}\ \boldsymbol{t}\right)+\frac{1}{\sqrt{3}}\boldsymbol{e}^{-\frac{1}{2}\boldsymbol{t}}\ \sin\left(\sqrt{\frac{3}{4}}\ \boldsymbol{t}\right)$$

2.3 ARIKETA

Ondoko ekuazio diferentzialak ebatzi:

1) $\ddot{x}(t) + 2\dot{x}(t) + 5x(t) = 3, \quad x(0) = \dot{x}(0) = 0$

Emaitza:

Ekuazioaren bi aldeetan Laplaceren transformatua aplikatuz eta hasierako baldintzak kontutan izanik:

$$L(\ddot{x}(t)) + L(2\dot{x}(t)) + L(5x(t)) = L(3) \rightarrow s^2\, X(s) + 2s\, X(s) + 5\, X(s) = \frac{3}{s} \rightarrow$$

$$\rightarrow X(s)[s^2 + 2s + 5] = \frac{3}{s} \rightarrow X(s) = \frac{3}{s(s^2 + 2s + 5)}$$

$$X(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{a}{s} + \frac{bs + c}{s^2 + 2s + 5}$$, koefizienteak kalkulatzen dira:

$$a = s\frac{3}{s(s^2 + 2s + 5)}\bigg|_{s=0} = \frac{3}{5}$$, zenbakitzaileak berdinduz: b=-3/5 y c= -6/5

$$X(s) = \frac{3}{5s} - \frac{3}{5}\frac{s + 2}{s^2 + 2s + 5} = \frac{3}{5s} - \frac{3}{5}\frac{(s+1)}{(s+1)^2 + 2^2} - \frac{3}{10}\frac{2}{(s+1)^2 + 2^2}$$

Taulak erabiliz:

$$x(t) = L^{-1}(X(s)) = L^{-1}\left(\frac{3}{5s}\right) - \frac{3}{5}L^{-1}\left(\frac{(s+1)}{(s+1)^2 + 2^2}\right) - \frac{3}{10}L^{-1}\left(\frac{2}{(s+1)^2 + 2^2}\right)$$

Azkenean, emaitza:

$$x(t) = \frac{3}{5} - \frac{3}{5}e^{-t}\cos(2\ t) - \frac{3}{10}e^{-t}\sin(2\ t)$$

2) $\ddot{x}(t)+3\dot{x}(t)+6x(t)=0, \quad x(0)=0,\ \dot{x}(0)=3$

Emaitza:

Ekuazioaren bi aldeetan Laplaceren transformatua aplikatuz eta hasierako baldintzak kontutan izanik:

$$L(\ddot{x}(t))+L(3\dot{x}(t))+L(6x(t))=0 \rightarrow s^2\,X(s)-3+3s\,X(s)+6\,X(s)=0 \rightarrow$$

$$\rightarrow X(s)[s^2+3s+6]=3 \rightarrow X(s)=\frac{3}{s^2+3s+6}=\frac{6}{\sqrt{15}}\frac{\sqrt{15}/2}{(s+3/2)^2+\left(\sqrt{15}/2\right)^2}$$

Taulak erabiliz:

$$x(t)=L^{-1}(X(s))=\frac{6}{\sqrt{15}}L^{-1}\left(\frac{\sqrt{15}/2}{(s+3/2)^2+\left(\sqrt{15}/2\right)^2}\right)$$

Azkenean, emaitza:

$$x(t)=\frac{6}{\sqrt{15}}e^{-3/2\,t}\sin\left(\sqrt{15}/2\ \ t\right)$$