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Maiztasun-erantzuna

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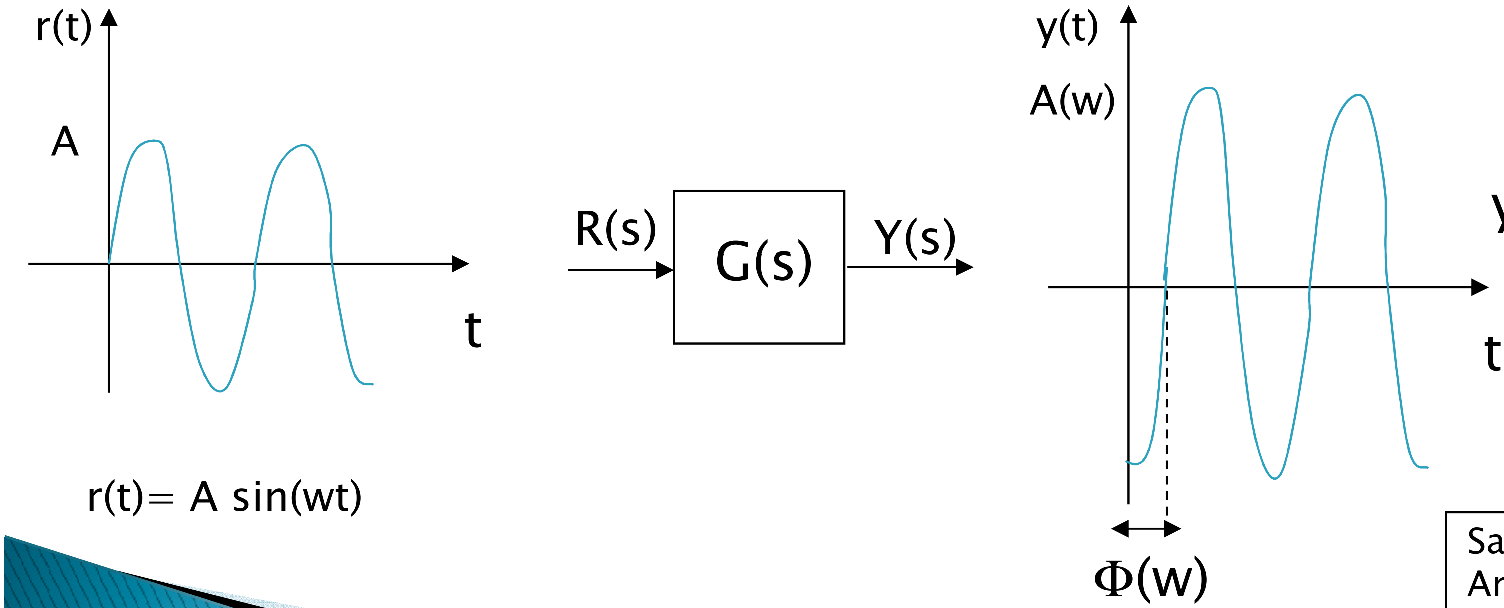
Maiztasun-erantzuna
Sarrera

Sistema linealen maiztasun-erantzuna aztertzeko, ondokoa suposatzen da :

1) Sarrerako seinalea sinusoidala: $r(t)= A \sin (wt)$
2) Maiztasunaren aldaketa $w=(0,+\infty)$
3) Erregimen iraunkorra

Beraz aurrekoa kontutan izanda:



$r(t)= A \sin(wt)$

$$y(t)= \boxed{A(w)} \sin(wt+ \boxed{\Phi(w)})$$

Sarrerako maiztasunaren menpeko desfasea

Sarrerako maiztasunarekiko Anplitude aldakorra

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Maiztasun-erantzuna
Sarrera

Maiztasun-erantzunaren abantailak:

1) Seinale-sorgailu batekin modu erraz batean lortzen da sistema linealen irteera
2) Irteera eta sarrera seinaleen arteko erlazioa aldagai-aldaketa sinple baten bitartez egiten da: $s \rightarrow jw$

Aldiz, Maiztasunaren eta denboraren arteko erlazioa lortzea eta aplikatzea zail xamarra da.

$$r(t) = A\sin(wt) \xrightarrow{R(s)} \boxed{G(s)} \xrightarrow{Y(s)} y_{rp}(t) = A\left|G(jw)\right|\sin(wt + \Phi(w))$$

Hau da, maiztasun-planoko transferentzi funtzioa *s* , *jw*-gatik ordezkatuz lortzen da.

Adibidea: 1° ordenako sistema:

$$G(s) = \frac{k}{T\,s+1} \xrightarrow{s \rightarrow jw} G(jw) = \frac{k}{jwT+1}$$

Modulua

$$\left|G(jw)\right| = \frac{k}{\sqrt{(T\,w)^2+1}}$$

Fasea

$$\Phi(w) = -arctg\,(wT)$$

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Maiztasun-erantzuna
Bode-diagramak

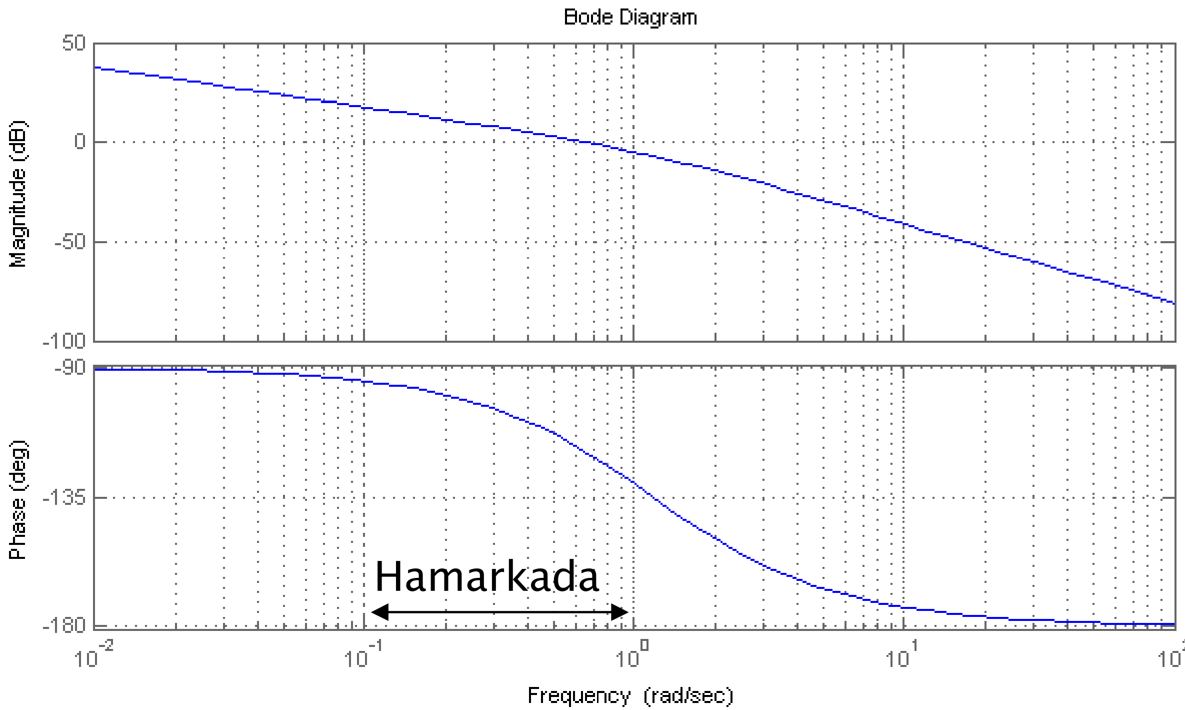
Denbora-planotik Maiztasun-planorako aldaketa sinplea da, baina bere **adierazpen grafikoa** oso konplexua izan daiteke sistema goi ordenakoa bada. Horregatik, Transferentzi funtzioaren **moduluaren** kasuan eskala logartimikoa erabiltzen da eta dezibelioaren definizoa agertuz:

$$G_{db} = \left| G(jw) \right|_{db} = 20 \log \left| G(jw) \right| \qquad \text{[dB: dezibel]}$$

Hamarkada ere definitzen da bi maiztasunen arteko erlazioa ondokoa denean: $f_2/f_1=10$. Beraz, 2 maiztasun **n** hamarkadara daude euren erlazioa ondokoa denean: $f_2/f_1=10^n$

Moduluaren Bode-diagrama

Fasearen Bode-diagrama



<u>Adibidea:</u> Ondoko Lazo irekiko Transferentzi funtzioaren bode-diagramak irudikatu:

$$G_{LA}(s)=\frac{10(s+3)}{s(s+2)(s^2+s+2)}$$

$$G_{LA}(s)=\frac{10(s+3)}{s(s+2)(s^2+s+2)}=\frac{10(\frac{1}{3}s+1)}{\frac{1}{3}s(s+2)(s^2+s+2)}=\frac{10\frac{1}{2}(\frac{1}{3}s+1)}{\frac{1}{3}s(\frac{1}{2}s+1)(s^2+s+2)}=\frac{10\frac{1}{2}\frac{1}{2}(\frac{1}{3}s+1)}{\frac{1}{3}s(\frac{1}{2}s+1)(\frac{1}{2}s^2+\frac{1}{2}s+1)}$$

$$G_{LA}(s)=7.5\frac{(\frac{1}{3}s+1)}{s(\frac{1}{2}s+1)(\frac{1}{2}s^2+\frac{1}{2}s+1)}\xrightarrow{s\to jw}G_{LA}(jw)=7.5\frac{(\frac{1}{3}jw+1)}{jw(\frac{1}{2}jw+1)(\frac{1}{2}jw^2+\frac{1}{2}jw+1)}$$

$$G_{db}=\left|G(jw)\right|_{db}=20\log\left|G(jw)\right|$$

Gogoratu

$$20\log(K)=-20\log\frac{1}{K}$$
$$20\log(10^n K)=20\log K+20n$$
$$20\log(10K)=20\log K+20$$

$$G_{dB}(jw)=20\log\left(7.5\frac{\left|(\frac{1}{3}jw+1)\right|}{\left|jw\right|\left|(\frac{1}{2}jw+1)\right|\left|(\frac{1}{2}jw^2+\frac{1}{2}jw+1)\right|}\right)$$

$$G_{\mathbf{dB}}(jw) = 20\log\left(7.5\frac{\left|(\frac{1}{3}jw+1)\right|}{\left|jw\right|\left|(\frac{1}{2}jw+1)\right|\left|(\frac{1}{2}jw^2+\frac{1}{2}jw+1)\right|}\right)$$

Modulua

$$G_{dB}(jw) = 20\log 7.5 + 20\log\left|1+\frac{1}{3}wj\right| - 20\log w - 20\log\left|1+\frac{1}{2}wj\right| - 20\log\left|1+\frac{1}{2}\;jw+\frac{1}{2}(jw)^2\right|$$

Fasea

$$\Phi(w) = Arg\left|1+\frac{1}{3}wj\right| - Arg\left|jw\right| - Arg\left|1+\frac{1}{2}wj\right| - Arg\left|1+\frac{1}{2}\;jw+\frac{1}{2}(jw)^2\right|$$

$$\Phi(w) = arctg\left(\frac{w}{3}\right) - 90 - arctg\left(\frac{w}{2}\right) - arctg\left(\frac{\frac{1}{2}\;w}{1-\frac{w^2}{2}}\right)$$

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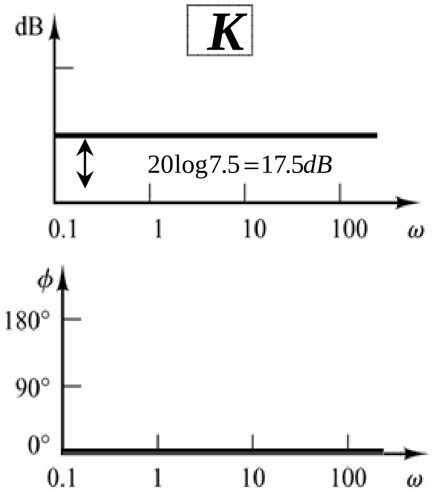
Maiztasun-erantzuna
Bode-diagramak

1) K irabazpenaren eragina:

Moduluaren diagrama K irabazpen baterako 20log(K) (dB) zuzen horizontal bat gehitzen du, positiboa k>1 denean eta negatiboa K<1. K aldatzeak moduluaren grafikoa osoa gorantz edo beherantz mugitzea dakar. Baina fasean ez dago aldaketarik (zero gehitzen delako)

$$G_{dB}(jw)=\underbrace{\boxed{20\log 7.5}}_{17.5\text{ dB}}+20\log\left|1+\frac{1}{3}wj\right|\underbrace{\boxed{-20\log w}}-20\log\left|1+\frac{1}{2}wj\right|-20\log\left|1+\frac{1}{2}\,jw+\frac{1}{2}(jw)^2\right|$$



2) $S^{\pm1}=(jw)^{\pm1}$ terminoen eragina (poloak eta zeroak jatorrian): Hamarkada (w tik10w) bakoitzeko ± 20dB -ren aldaketa

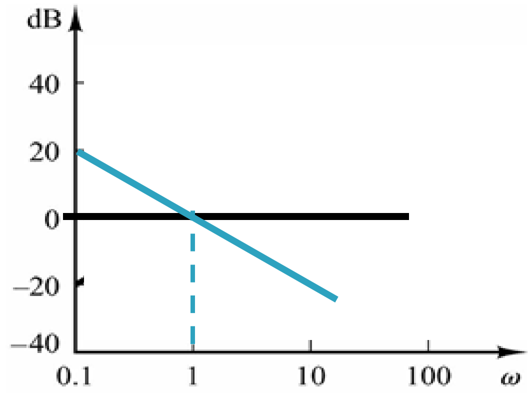
$$20\log\left(\left|jw^{\pm n}\right|\right)=\pm 20n\log w$$

$$w=1$$

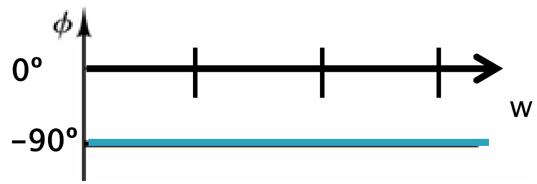
$$20\log\left(\left|j1^{\pm n}\right|\right)=\pm 20n\log 1=0\,dB$$

$$w=10$$

$$20\log\left(\left|j10^{\pm n}\right|\right)=\pm 20n\log 10=\pm 20n\,dB$$



$$Arg\left(jw^{\pm n}\right)=\pm n\,arct\frac{w}{0}=\pm n\,90^{o}$$



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Maiztasun-erantzuna
Bode-diagramak

3) Lehenengo ordenako terminoen eragina: (1+Ts) ±n= (1+Tjw) ±n :

$$G_{dB}(jw) = 20\log 7.5 \boxed{+20\log\left|1+\frac{1}{3}wj\right|} - 20\log w \boxed{-20\log\left|1+\frac{1}{2}wj\right|} - 20\log\left|1+\frac{1}{2}\ jw+\frac{1}{2}(jw)^2\right|$$

$$20\log(1+Tjw)^{\pm n} = \pm 20n\log\sqrt{1+\omega^2T^2}$$

$$w << 1/T \Rightarrow \pm 20n\log\sqrt{1+\omega^2T^2} \approx \pm 20n\log 1 \approx 0dB$$

$$w = 1/T \Rightarrow \pm 20n\log\sqrt{1+\omega^2T^2} \approx \pm 20n\log\sqrt{2} \approx \pm 3n\,dB$$

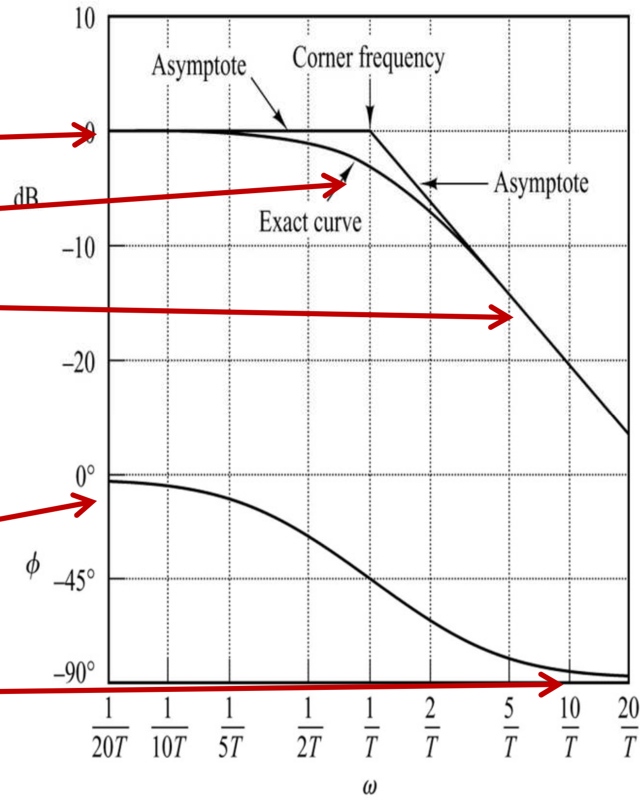
$$w >> 1/T \Rightarrow \pm 20n\log\sqrt{1+\omega^2T^2} \approx \pm 20n\log w \approx \pm 20n\,dB$$

FASEA:

$$Arg((1+Tjw)^{\pm n}) = \pm n\,arctg\frac{Tw}{1}$$

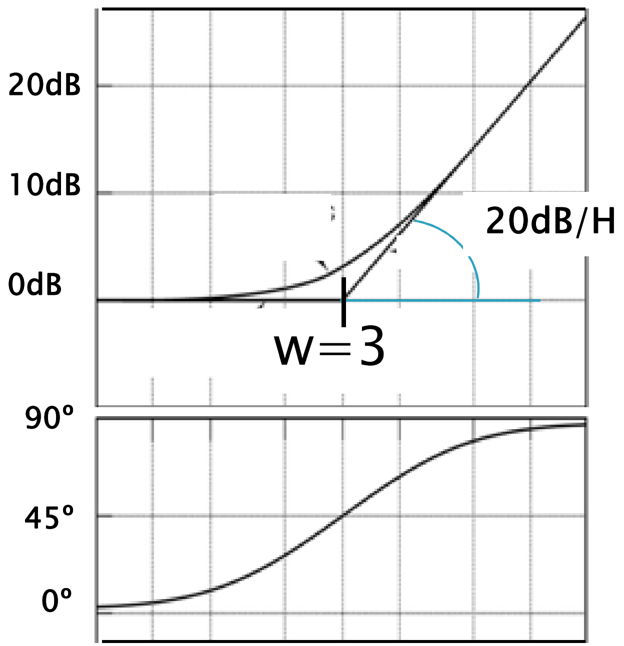
$$w <<< 1/T \Rightarrow Arg((1+Tjw)^{\pm n}) \approx \pm n\,arctg\frac{0}{1} \approx 0^\circ$$

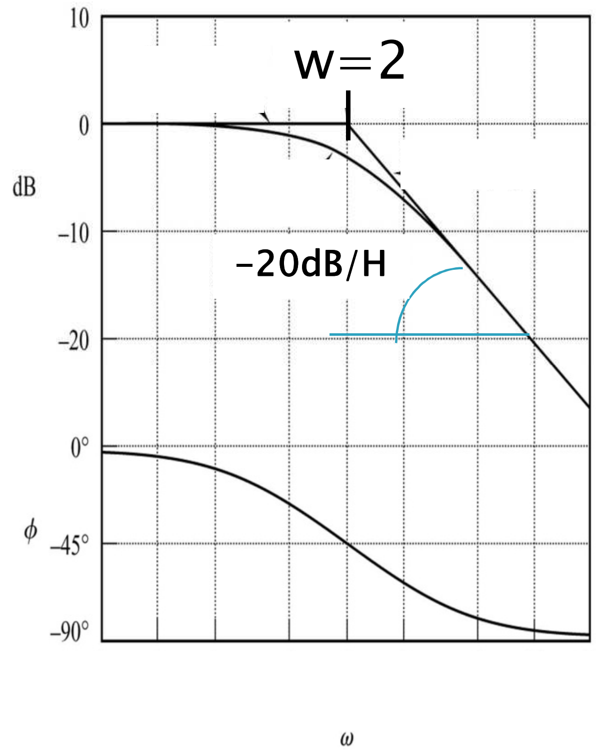
$$w >>> 1/T = \infty \Rightarrow Arg((1+Tjw)^{\pm n}) \approx \pm n\,arctg\infty \approx \pm n90^\circ$$



3) Lehenengo ordenako terminoen eragina: $(1+Ts)^{\pm n} = (1+Tjw)^{\pm n}$:

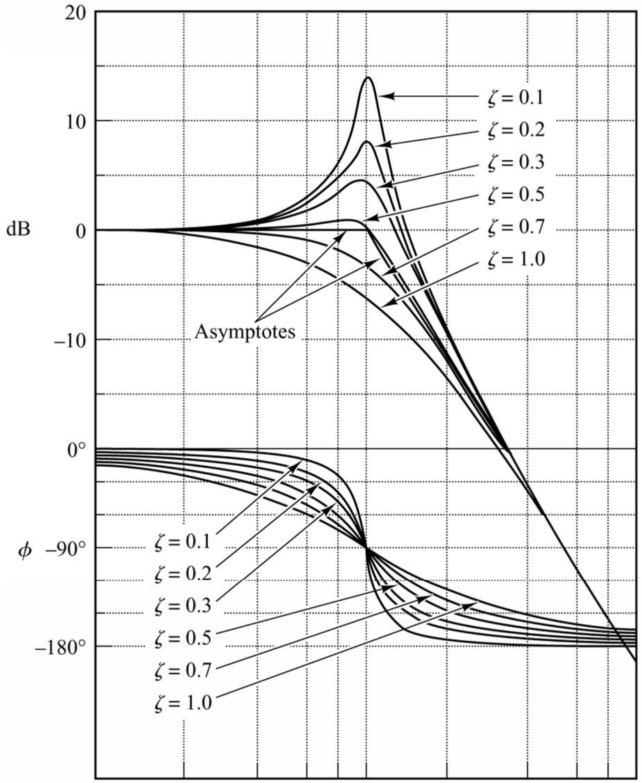
$$G_{dB}(jw) = 20\log 7.5 \underbrace{\boxed{+20\log\left|1+\frac{1}{3}wj\right|}} - 20\log w \underbrace{\boxed{-20\log\left|1+\frac{1}{2}wj\right|}} - 20\log\left|1+\frac{1}{2}\ jw+\frac{1}{2}(jw)^2\right|$$





4)Bigarren ordenako terminoen eragina: $(1+as+bs^2)^{\pm n}$

$$G_{dB}(jw) = 20\log 7.5 + 20\log\left|1+\frac{1}{3}wj\right| - 20\log w - 20\log\left|1+\frac{1}{2}wj\right| \boxed{-20\log\left|1+\frac{1}{2}\ jw+\frac{1}{2}(jw)^2\right|}$$



$$(1+as+bs^2)^{\pm n} \Rightarrow (w_n{}^2+2\varsigma w_n s+s^2)^{\pm n} = (1+\frac{2\varsigma}{w_n}\ s+\frac{1}{w_n{}^2}s^2)^{\pm n} \Rightarrow (1+\frac{2\varsigma}{w_n}\ jw+\frac{1}{w_n{}^2}\ jw^2)^{\pm n}$$

$$\boxed{w_n = \sqrt{2}}$$

$$20\log(1+\frac{2\varsigma}{w_n}\ jw+\frac{1}{w_n{}^2}\ jw^2)^{\pm n} = \pm 20n\log\sqrt{\left(1-\frac{w^2}{w_n{}^2}\right)^2+\left(\frac{2\varsigma}{w_n}\ w\right)^2}$$

$$w <<< w_n \Rightarrow \approx \pm 20n\log 1 = \boxed{0\,dB}$$

$$w >>> w_n \Rightarrow \approx \pm 20n\log\left(\frac{w}{w_n}\right)^2 \approx \boxed{\pm 40n\log w\,dB}$$

$$w = w_n \Rightarrow \approx \pm 20n\log 2\varsigma$$

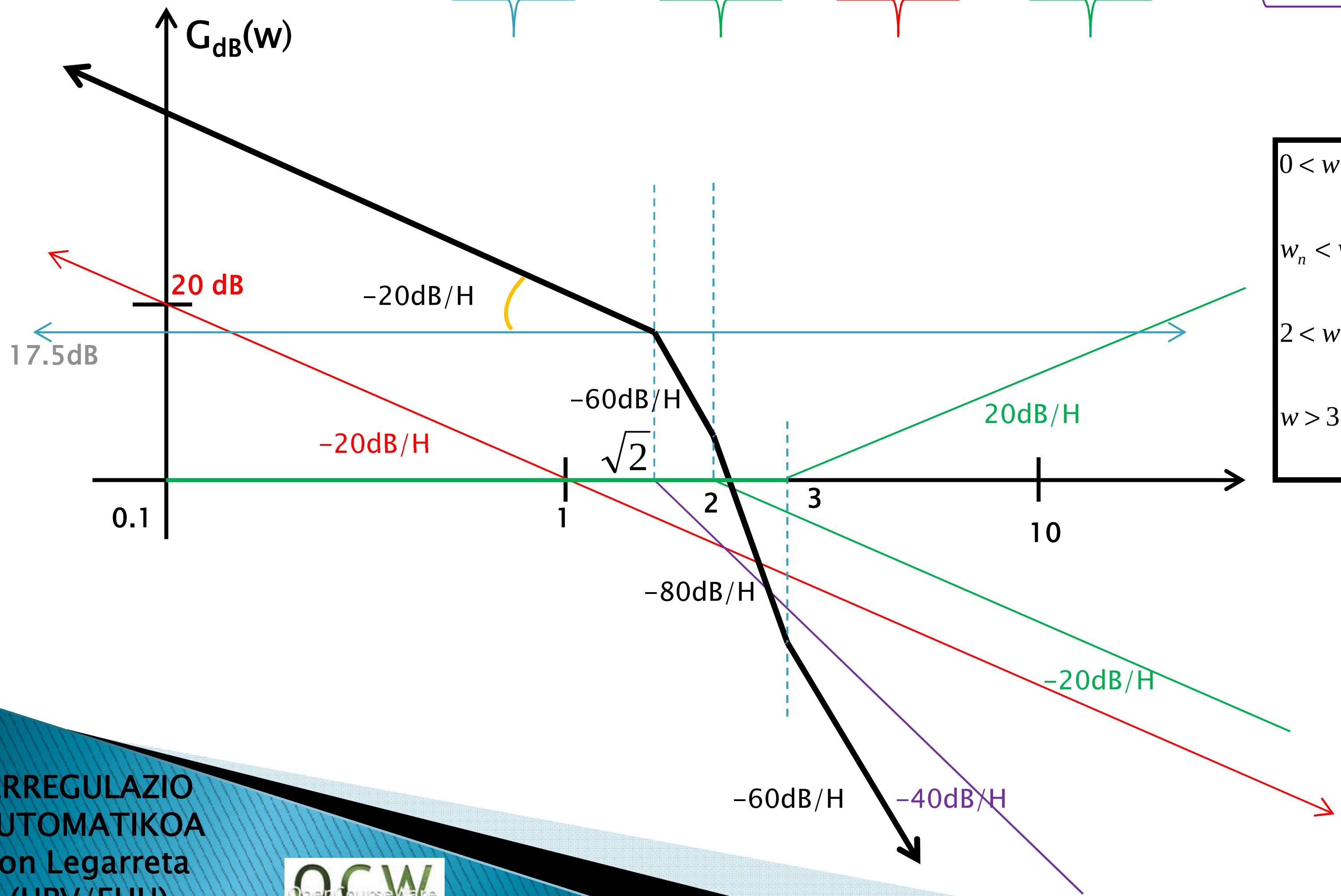
$$Arg((1+\frac{2\varsigma}{w_n}\ jw+\frac{1}{w_n{}^2}\ jw^2)^{\pm n}) = \pm n\,arctg\frac{\frac{2\varsigma}{w_n}\ w}{1-\frac{w^2}{w_n{}^2}}$$

$$w <<< w_n \Rightarrow\approx \pm n\,arctg\frac{0}{1} \approx 0^{o}$$

$$w >>> w_n = \infty \Rightarrow\approx \pm n\,arctg\frac{0}{-1} \approx \pm n180^{o}$$

5) Termino guztien eragina batu egiten da

$$G_{dB}(jw) = 20\log 7.5 + 20\log\left|1+\frac{1}{3}wj\right| - 20\log w - 20\log\left|1+\frac{1}{2}wj\right| - 20\log\left|1+\frac{1}{2}\,jw+\frac{1}{2}(jw)^2\right|$$



$$0 < w < w_n = \sqrt{2} \Rightarrow -20dB/H$$

$$w_n < w < 2 \Rightarrow -20\,dB/H - 40\,dB/H = -60\,dB/H$$

$$2 < w < 3 \Rightarrow -60\,dB/H - 20\,dB/H = -80\,dB/H$$

$$w > 3 \Rightarrow -80\,dB/H + 20\,dB/H = -60\,dB/H$$

6) Fasearen diagraman maiztasunari balioak emanez egiten da:

$$\boxed{\Phi(w) = arctg\left(\frac{w}{3}\right) - 90 - arctg\left(\frac{w}{2}\right) - arctg\left(arctg\,\frac{\frac{1}{2}w}{1-\frac{w^2}{2}}\right)}$$

$$\Phi(0) = arctg\left(\frac{0}{3}\right) - 90 - arctg\left(\frac{0}{2}\right) - arctg\left(\frac{\frac{1}{2}0}{1-\frac{0^2}{2}}\right) = -90^{o}$$

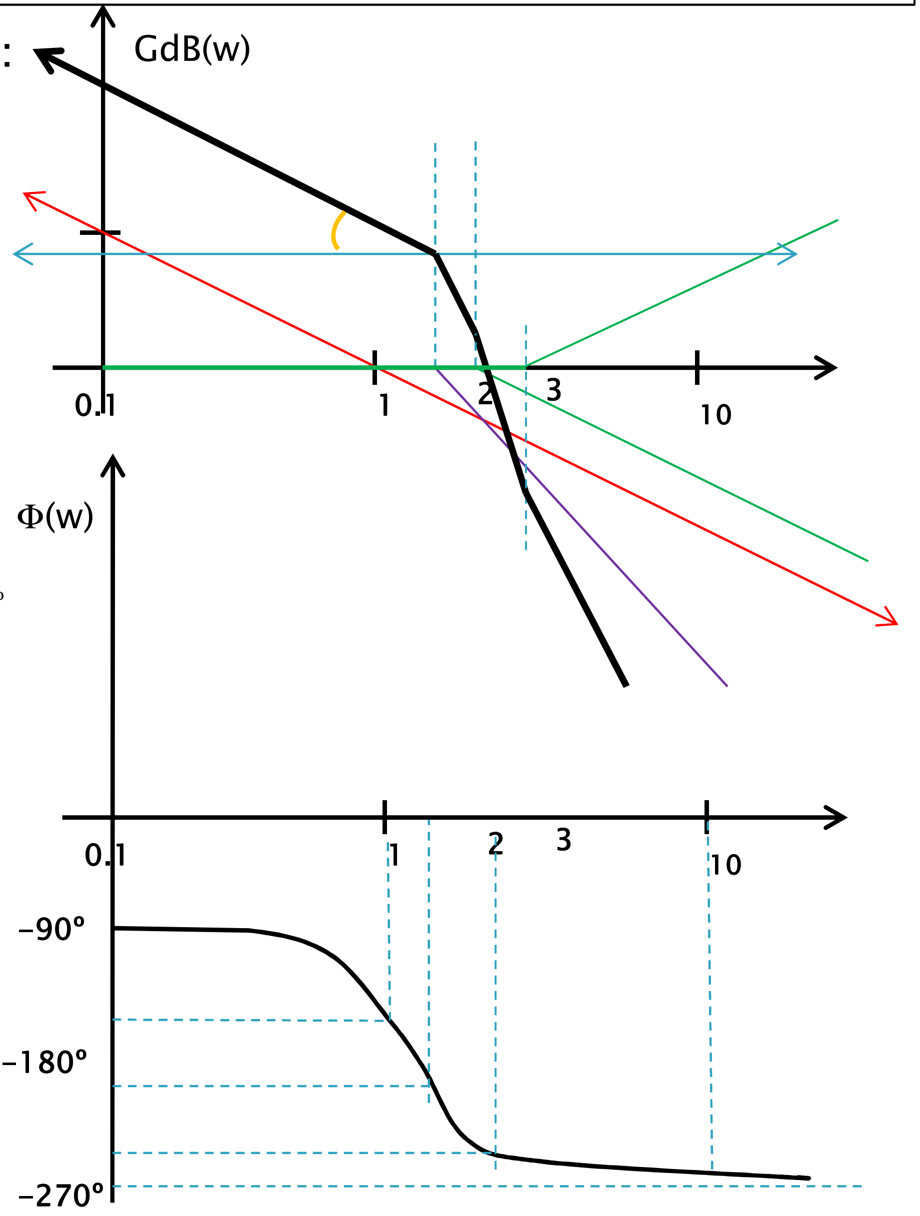
$$\Phi(1) = arctg\left(\frac{1}{3}\right) - 90 - arctg\left(\frac{1}{2}\right) - arctg\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) = -143.12^{o}$$

$$\Phi(\sqrt{2}) = arctg\left(\frac{\sqrt{2}}{3}\right) - 90 - arctg\left(\frac{\sqrt{2}}{2}\right) - arctg\left(\frac{\frac{1}{2}\sqrt{2}}{1-\frac{\sqrt{2}^2}{2}}\right) = -190^{o}$$

$$\Phi(2) = arctg\left(\frac{2}{3}\right) - 90 - arctg\left(\frac{2}{2}\right) - arctg\left(\frac{\frac{1}{2}2}{1-\frac{2^2}{2}}\right) = -236.31^{o}$$

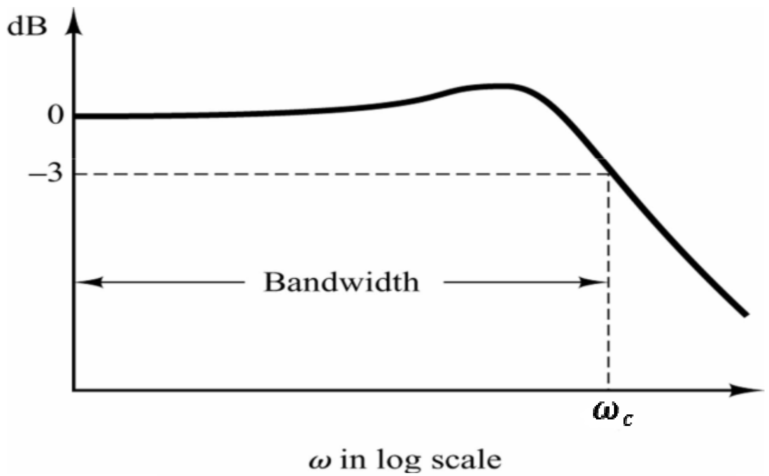
$$\Phi(10) = arctg\left(\frac{10}{3}\right) - 90 - arctg\left(\frac{10}{2}\right) - arctg\left(\frac{\frac{1}{2}10}{1-\frac{10^2}{2}}\right) = -269^{o}$$

$$\Phi(\infty) = arctg\left(\frac{\infty}{3}\right) - 90 - arctg\left(\frac{\infty}{2}\right) - arctg\left(\frac{\frac{1}{2}\infty}{1-\frac{\infty^2}{2}}\right) = -270^{o}$$

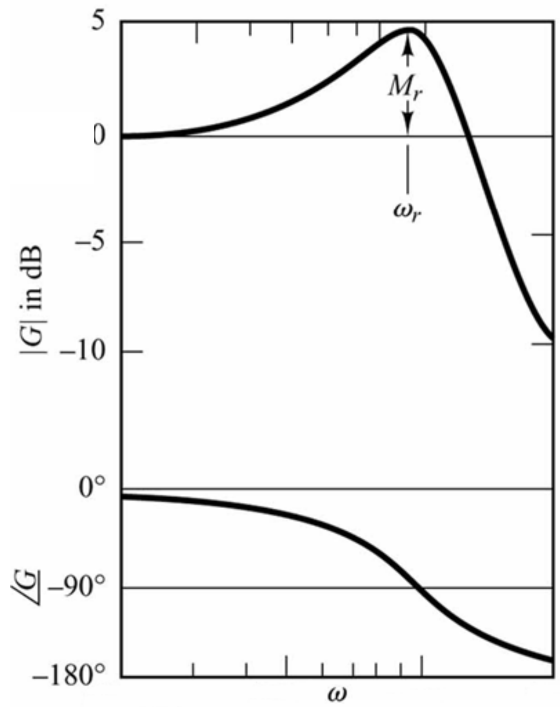


Maiztasun-erantzuneko ezaugarriak

Ebaketa maiztasuna(w_c), maiztasun honetan Lazo itxiko T. F-aren moduluak -3 dB galtzen dituenean (w=0 rekiko)



Banda zabalera (BW), w=0 tik Wc-rako maiztasun tartea.
BW handiagoa, sistema arinagoa eta perturbazioen aurrean ezegonkorragoa



Ebaketa abiadura (Vc). Moduluaren malda wc maiztasunean

Erresonantzi Maximoa (Mr). Moduluaren balio maximoa

Fase-tartea (MF):
Sistema bat ezegonkortasunera eramateko beharrezkoa den fase-atzerapena

$$MF = 180 + \Phi(w_{gc})$$

Irabazpen-tartea (MG):
Sistema bat ezegonkortasunera eramateko beharrezkoa den irabazpenaren balioa.

$$MG = -20 \log \left| GH(w_{fc}) \right|$$

Irabazpen kritikoaren maiztasuna

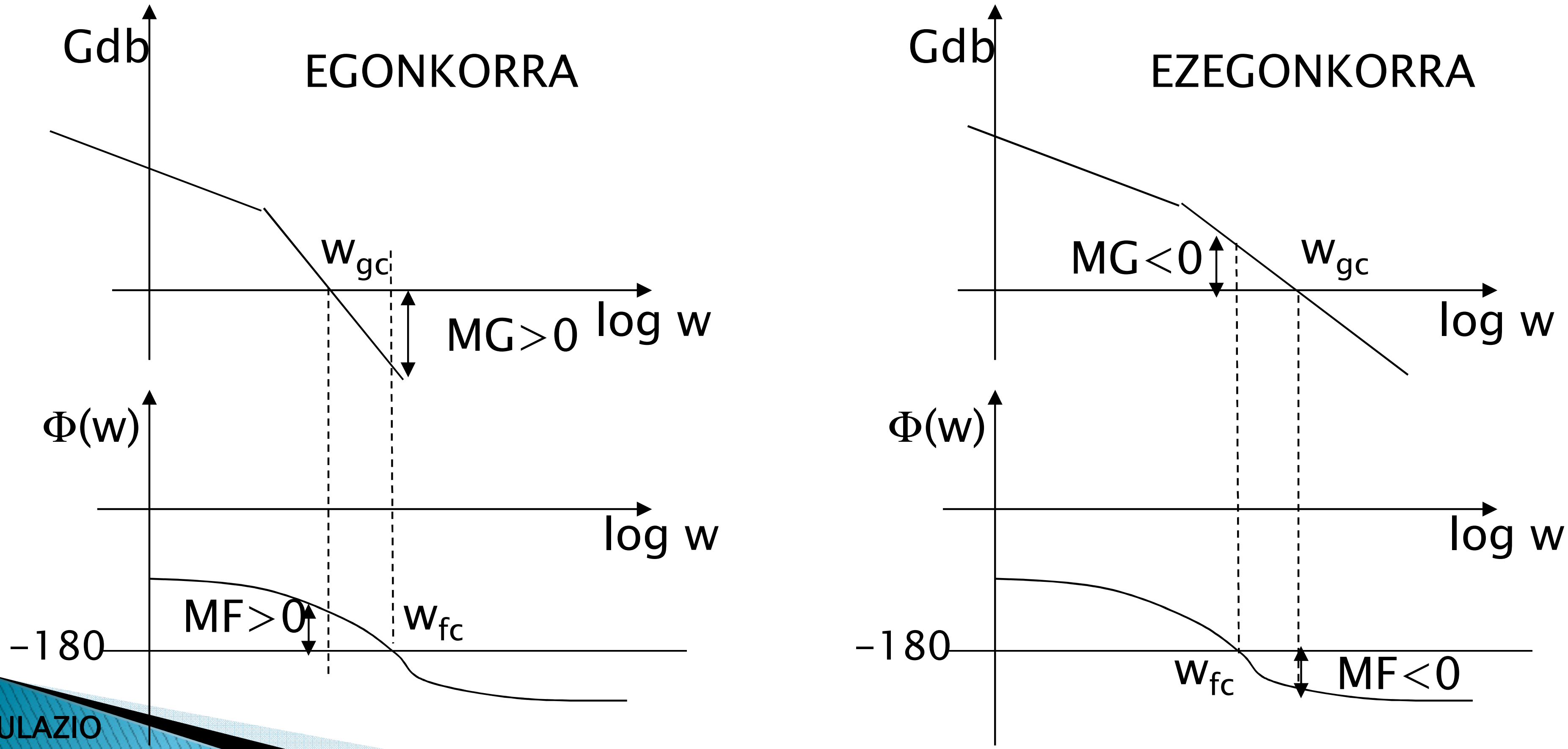
$$w_{gc} \rightarrow \left| GH(jw_{gc}) \right| = 1 \rightarrow G_{db} H_{db}(jw_{gc}) = 0db$$

Fase kritikoaren maiztasuna

$$w_{fc} \rightarrow \Phi(w_{fc}) = -180^{o}$$

Egonkortasun erlatiboa

Fase minimoko sistema bat, **egonkorra da Fase-tartea eta Irabazpen-tartea postiboak** direnean. Irudian adierazten dira egonkorra eta ezegonkorraren kasuak.



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