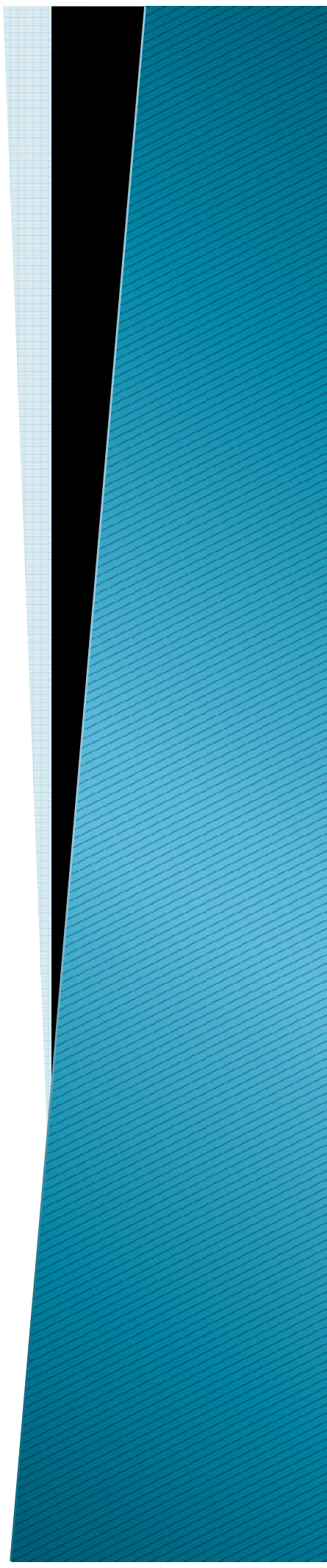


8. Gaia

Maiztasun-erantzuna



8. Gaia

Maiztasun-erantzuna

1. Sarrera
2. Bode-diagramak
3. Maiztasun-erantzuneko ezaugarriak
4. Egonkortasun erlatiboa

8. Gaia

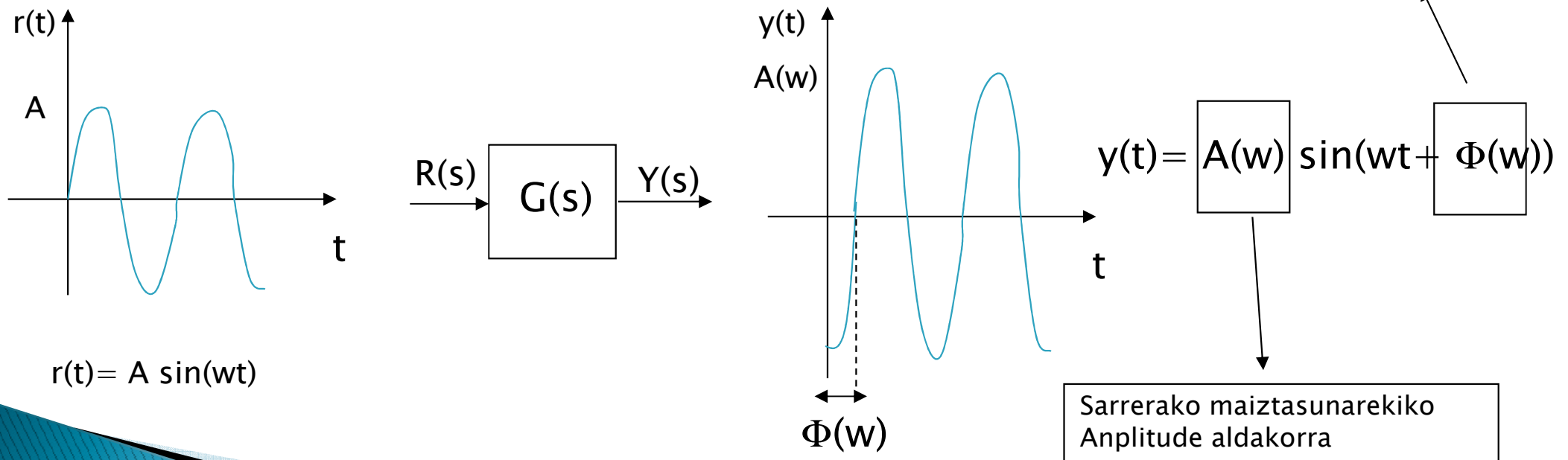
Maiztasun-erantzuna

Sarrera

Sistema linealen maiztasun-erantzuna aztertzeko, ondokoa suposatzen da :

- 1) Sarrerako seinalea sinusoidala: $r(t) = A \sin(\omega t)$
- 2) Maiztasunaren aldaketa $\omega = (0, +\infty)$
- 3) Erregimen iraunkorra

Beraz aurrekoa kontutan izanda:



8. Gaia

Maiztasun-erantzuna

Sarrera

Maiztasun-erantzunaren abantailak:

- 1) Seinale-sorgailu batekin modu erraz batean lortzen da sistema linealen irteera
- 2) Irteera eta sarrera seinaleen arteko erlazioa aldagai-aldaketa sinple baten bitartez egiten da: $s \rightarrow j\omega$

Aldiz, Maiztasunaren eta denboraren arteko erlazioa lortzea eta aplikatzea zail xamarra da.

$$r(t) = A \sin(\omega t) \xrightarrow{R(s)} \boxed{G(s)} \xrightarrow{Y(s)} y_{rp}(t) = A |G(j\omega)| \sin(\omega t + \Phi(\omega))$$

Hau da, maiztasun-planoko transferentzi funtzioa s , $j\omega$ -gatik ordezkatzuz lortzen da.

Adibidea: 1^o ordenako sistema:

$$G(s) = \frac{k}{Ts + 1} \xrightarrow{s \rightarrow j\omega} G(j\omega) = \frac{k}{j\omega T + 1}$$

Modulua

$$|G(j\omega)| = \frac{k}{\sqrt{(T\omega)^2 + 1}}$$

Fasea

$$\Phi(\omega) = -\arctg(\omega T)$$

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Maiztasun-erantzuna

Bode-diagramak

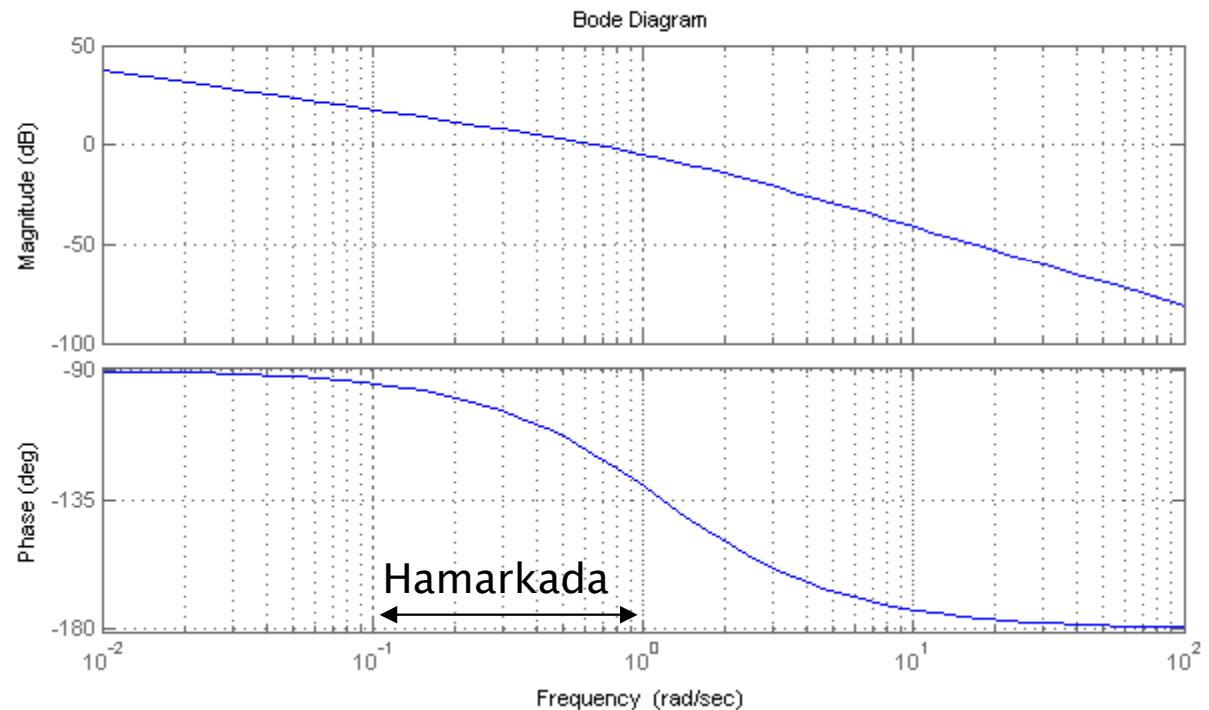
Denbora-planotik Maiztasun-planorako aldaketa sinplea da, baina bere **adierazpen grafikoa** oso konplexua izan daiteke sistema goi ordenakoa bada. Horregatik, Transferentzi funtzioaren **moduluaren** kasuan eskala logartimikoa erabiltzen da eta dezibelioaren definizioa agertuz:

$$G_{db} = |G(j\omega)|_{db} = 20 \log |G(j\omega)| \quad [\text{dB: dezibel}]$$

Hamarkada ere definitzen da bi maiztasunen arteko erlazioa ondokoa denean: $f_2/f_1 = 10$. Beraz, 2 maiztasun n hamarkadara daude euren erlazioa ondokoa denean: $f_2/f_1 = 10^n$

Moduluaren Bode-diagrama

Fasearen Bode-diagrama



8. Gaia

Maiztasun-erantzuna

Bode-diagramak

Adibidea: Ondoko Lazo irekiko Transferentzi funtzioaren bode-diagramak irudikatu:

$$G_{LA}(s) = \frac{10(s+3)}{s(s+2)(s^2+s+2)}$$

$$G_{LA}(s) = \frac{10(s+3)}{s(s+2)(s^2+s+2)} = \frac{10(\frac{1}{3}s+1)}{\frac{1}{3}s(s+2)(s^2+s+2)} = \frac{10\frac{1}{2}(\frac{1}{3}s+1)}{\frac{1}{3}s(\frac{1}{2}s+1)(s^2+s+2)} = \frac{10\frac{1}{2}\frac{1}{2}(\frac{1}{3}s+1)}{\frac{1}{3}s(\frac{1}{2}s+1)(\frac{1}{2}s^2+\frac{1}{2}s+1)}$$

$$G_{LA}(s) = 7.5 \frac{(\frac{1}{3}s+1)}{s(\frac{1}{2}s+1)(\frac{1}{2}s^2+\frac{1}{2}s+1)} \xrightarrow{s \rightarrow j\omega} G_{LA}(j\omega) = 7.5 \frac{(\frac{1}{3}j\omega+1)}{j\omega(\frac{1}{2}j\omega+1)(\frac{1}{2}j\omega^2+\frac{1}{2}j\omega+1)}$$

Gogoratu

$$20 \log(K) = -20 \log \frac{1}{K}$$

$$20 \log(10^n K) = 20 \log K + 20n$$

$$20 \log(10K) = 20 \log K + 20$$

$$G_{db} = |G(j\omega)|_{db} = 20 \log |G(j\omega)|$$



$$G_{db}(j\omega) = 20 \log \left(7.5 \frac{\left| \frac{1}{3}j\omega+1 \right|}{\left| j\omega \right| \left| \frac{1}{2}j\omega+1 \right| \left| \frac{1}{2}j\omega^2+\frac{1}{2}j\omega+1 \right|} \right)$$

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Maiztasun-erantzuna

Bode-diagramak

$$G_{dB}(j\omega) = 20 \log \left(7.5 \frac{\left| \left(\frac{1}{3} j\omega + 1 \right) \right|}{|j\omega| \left| \left(\frac{1}{2} j\omega + 1 \right) \right| \left| \left(\frac{1}{2} j\omega^2 + \frac{1}{2} j\omega + 1 \right) \right|} \right)$$

Modulua

$$G_{dB}(j\omega) = 20 \log 7.5 + 20 \log \left| 1 + \frac{1}{3} j\omega \right| - 20 \log \omega - 20 \log \left| 1 + \frac{1}{2} j\omega \right| - 20 \log \left| 1 + \frac{1}{2} j\omega + \frac{1}{2} (j\omega)^2 \right|$$

Fasea

$$\Phi(\omega) = \text{Arg} \left| 1 + \frac{1}{3} j\omega \right| - \text{Arg} |j\omega| - \text{Arg} \left| 1 + \frac{1}{2} j\omega \right| - \text{Arg} \left| 1 + \frac{1}{2} j\omega + \frac{1}{2} (j\omega)^2 \right|$$

$$\Phi(\omega) = \arctg \left(\frac{\omega}{3} \right) - 90 - \arctg \left(\frac{\omega}{2} \right) - \arctg \left(\frac{\frac{1}{2} \omega}{1 - \frac{\omega^2}{2}} \right)$$

8. Gaia

Maiztasun-erantzuna

Bode-diagramak

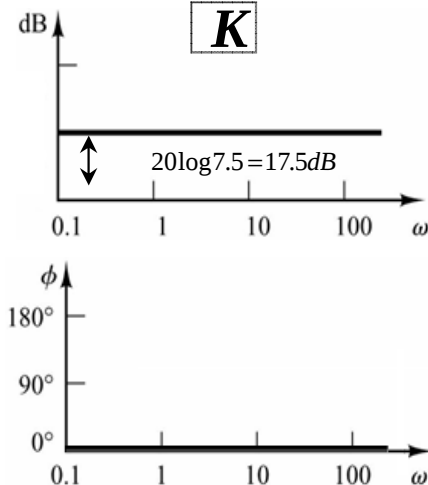
1) K irabazpenaren eragina:

Moduluaren diagrama K irabazpen baterako $20\log(K)$ (dB) zuzen horizontal bat gehitzen du, positiboa $K > 1$ denean eta negatiboa $K < 1$. K aldatzeak moduluaren grafikoa osoa gorantz edo beherantz mugitzea dakar. Baina fasean ez dago aldaketarik (zero gehitzen delako)

$$G_{dB}(j\omega) = \underbrace{20\log 7.5}_{17.5 \text{ dB}} + 20\log\left|1 + \frac{1}{3}j\omega\right| - \underbrace{20\log \omega}_{-20 \text{ dB}} - 20\log\left|1 + \frac{1}{2}j\omega\right| - 20\log\left|1 + \frac{1}{2}j\omega + \frac{1}{2}(j\omega)^2\right|$$

17.5 dB

K



2) $S^{\pm 1} = (j\omega)^{\pm 1}$ terminoen eragina (poloak eta zeroak jatorrian): Hamarkada (ω tik 10w) bakoitzeko ± 20 dB -ren aldaketa

$$20\log(j\omega^{\pm n}) = \pm 20n \log \omega$$

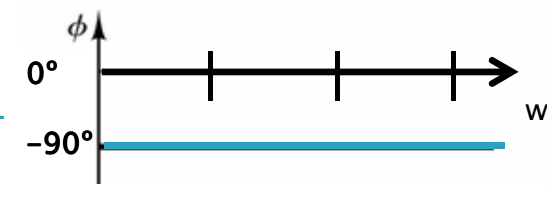
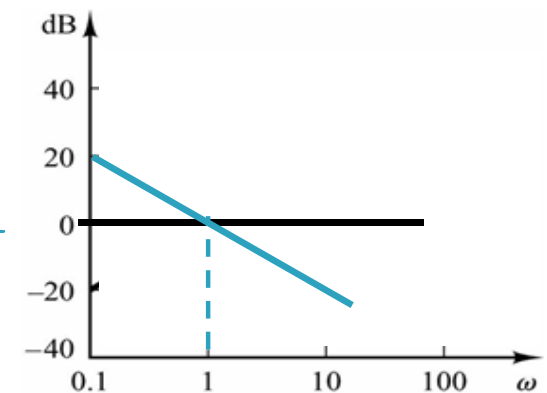
$$\omega = 1$$

$$20\log(j1^{\pm n}) = \pm 20n \log 1 = 0 \text{ dB}$$

$$\omega = 10$$

$$20\log(j10^{\pm n}) = \pm 20n \log 10 = \pm 20n \text{ dB}$$

$$\text{Arg}(j\omega^{\pm n}) = \pm n \arctan \frac{\omega}{0} = \pm n 90^\circ$$



8. Gaia Maiztasun-erantzuna Bode-diagramak

3) Lehenengo ordenako terminoen eragina: $(1 + Ts)^{\pm n} = (1 + Tj\omega)^{\pm n}$:

$$G_{dB}(j\omega) = 20 \log 7.5 + 20 \log \left| 1 + \frac{1}{3} j\omega \right| - 20 \log \omega - 20 \log \left| 1 + \frac{1}{2} j\omega \right| - 20 \log \left| 1 + \frac{1}{2} j\omega + \frac{1}{2} (j\omega)^2 \right|$$

$$20 \log(1 + Tj\omega)^{\pm n} = \pm 20n \log \sqrt{1 + \omega^2 T^2}$$

$$\omega \ll 1/T \Rightarrow \pm 20n \log \sqrt{1 + \omega^2 T^2} \approx \pm 20n \log 1 \approx 0 \text{ dB}$$

$$\omega = 1/T \Rightarrow \pm 20n \log \sqrt{1 + \omega^2 T^2} \approx \pm 20n \log \sqrt{2} \approx \pm 3n \text{ dB}$$

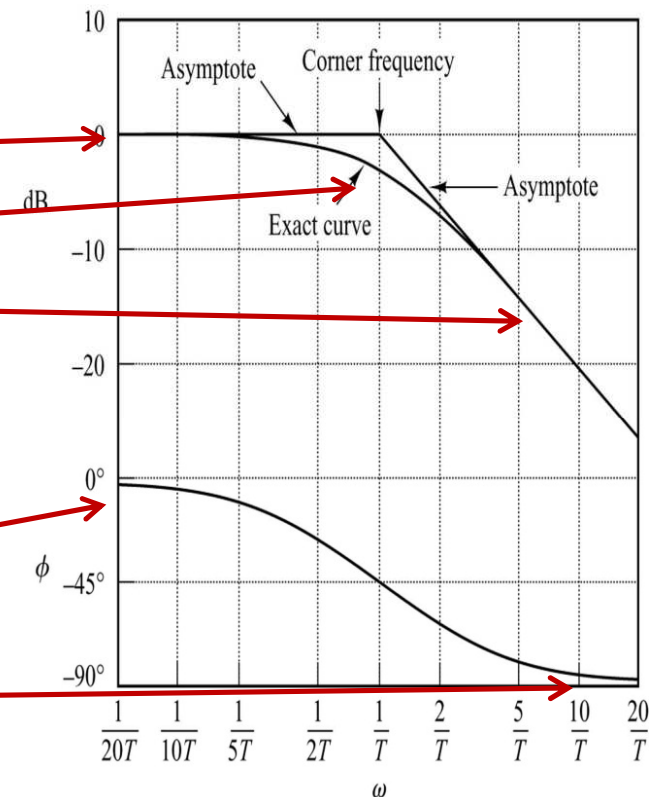
$$\omega \gg 1/T \Rightarrow \pm 20n \log \sqrt{1 + \omega^2 T^2} \approx \pm 20n \log \omega \approx \pm 20n \text{ dB}$$

FASEA:

$$\text{Arg}((1 + Tj\omega)^{\pm n}) = \pm n \arctg \frac{T\omega}{1}$$

$$\omega \ll 1/T \Rightarrow \text{Arg}((1 + Tj\omega)^{\pm n}) \approx \pm n \arctg \frac{0}{1} \approx 0^\circ$$

$$\omega \gg 1/T = \infty \Rightarrow \text{Arg}((1 + Tj\omega)^{\pm n}) \approx \pm n \arctg \infty \approx \pm n 90^\circ$$



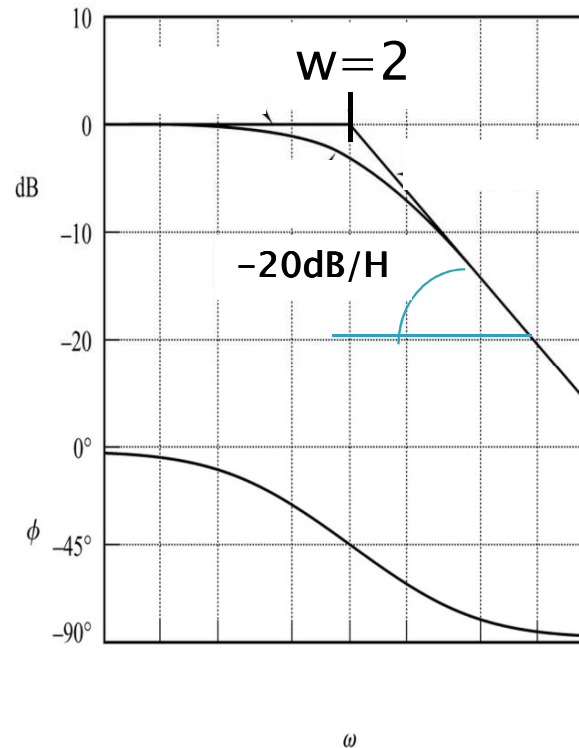
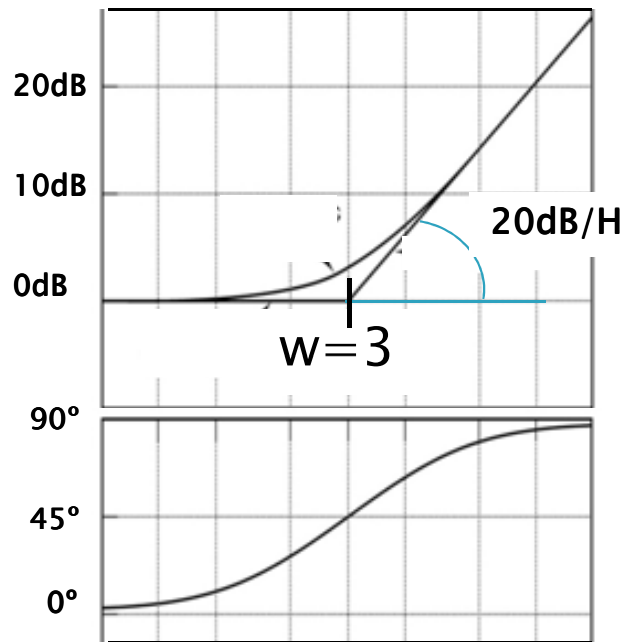
8. Gaia

Maiztasun-erantzuna

Bode-diagramak

3) Lehenengo ordenako terminoen eragina: $(1 + Ts)^{\pm n} = (1 + Tj\omega)^{\pm n}$:

$$G_{dB}(j\omega) = 20 \log 7.5 + \underbrace{20 \log \left| 1 + \frac{1}{3} j\omega \right|}_{\text{Bode plot 1}} - 20 \log \omega - \underbrace{20 \log \left| 1 + \frac{1}{2} j\omega \right|}_{\text{Bode plot 2}} - 20 \log \left| 1 + \frac{1}{2} j\omega + \frac{1}{2} (j\omega)^2 \right|$$



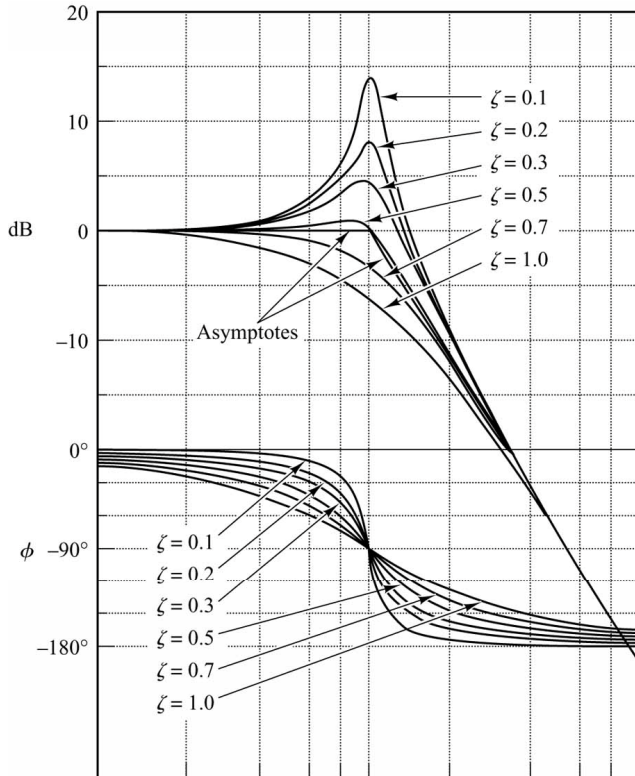
8. Gaia

Maiztasun-erantzuna

Bode-diagramak

4) Bigarren ordenako terminoen eragina: $(1 + as + bs^2)^{\pm n}$

$$G_{dB}(jw) = 20 \log 7.5 + 20 \log \left| 1 + \frac{1}{3} wj \right| - 20 \log w - 20 \log \left| 1 + \frac{1}{2} wj \right| - 20 \log \left| 1 + \frac{1}{2} jw + \frac{1}{2} (jw)^2 \right|$$



$$(1 + as + bs^2)^{\pm n} \Rightarrow (w_n^2 + 2\zeta w_n s + s^2)^{\pm n} = \left(1 + \frac{2\zeta}{w_n} s + \frac{1}{w_n^2} s^2\right)^{\pm n} \Rightarrow \left(1 + \frac{2\zeta}{w_n} jw + \frac{1}{w_n^2} jw^2\right)^{\pm n}$$

$$20 \log \left(1 + \frac{2\zeta}{w_n} jw + \frac{1}{w_n^2} jw^2\right)^{\pm n} = \pm 20n \log \sqrt{\left(1 - \frac{w^2}{w_n^2}\right)^2 + \left(\frac{2\zeta}{w_n} w\right)^2}$$

$$w_n = \sqrt{2}$$

$$w \ll w_n \Rightarrow \approx \pm 20n \log 1 = 0 \text{ dB}$$

$$w \gg w_n \Rightarrow \approx \pm 20n \log \left(\frac{w}{w_n}\right)^2 \approx \pm 40n \log w \text{ dB}$$

$$w = w_n \Rightarrow \approx \pm 20n \log 2\zeta$$

$$\text{Arg}\left(\left(1 + \frac{2\zeta}{w_n} jw + \frac{1}{w_n^2} jw^2\right)^{\pm n}\right) = \pm n \arctg \frac{\frac{2\zeta}{w_n} w}{1 - \frac{w^2}{w_n^2}}$$

$$w \ll w_n \Rightarrow \approx \pm n \arctg \frac{0}{1} \approx 0^\circ$$

$$w \gg w_n = \infty \Rightarrow \approx \pm n \arctg \frac{0}{-1} \approx \pm n 180^\circ$$

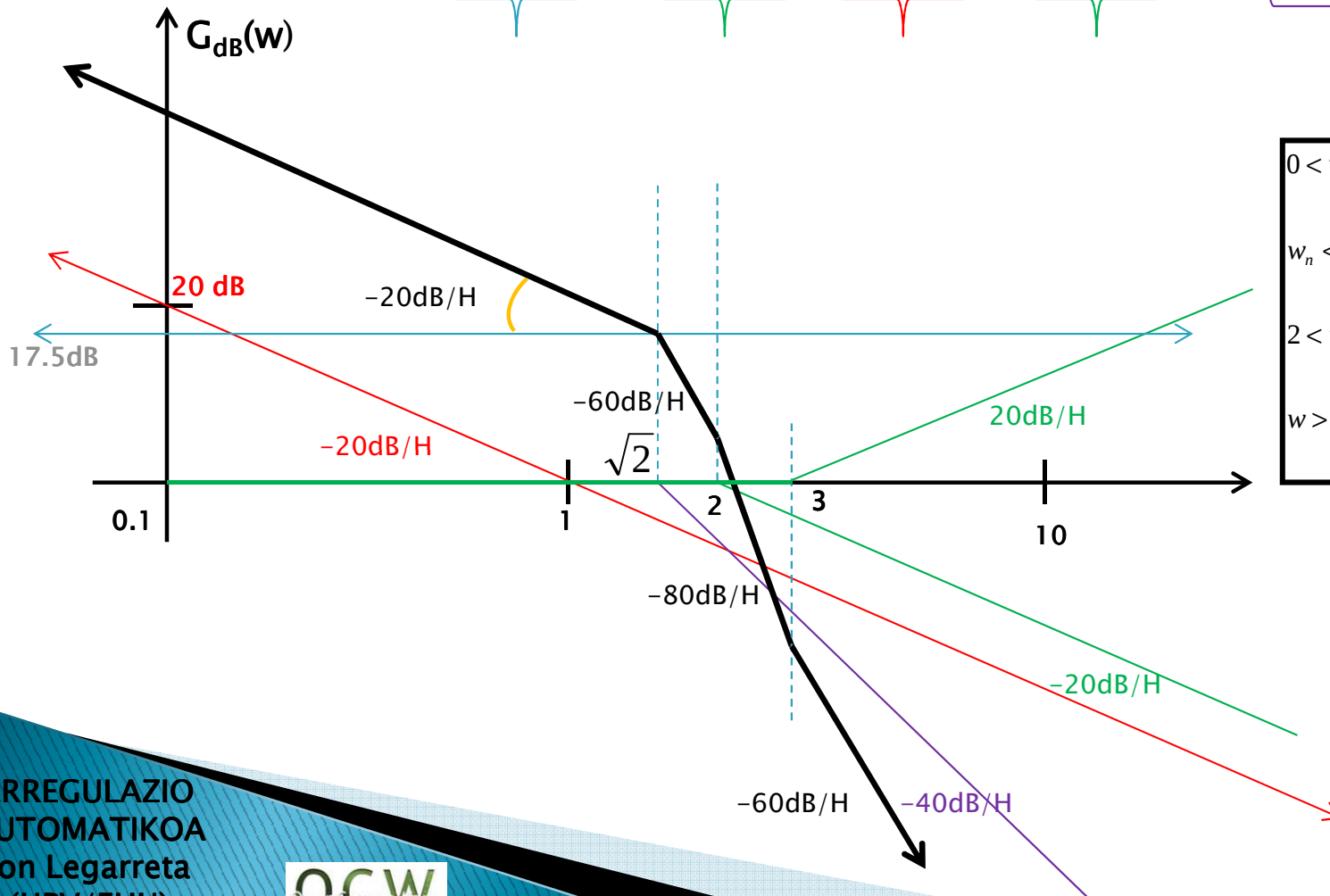
8. Gaia

Maiztasun-erantzuna

Bode-diagramak

5) Termino guztien eragina batu egiten da

$$G_{dB}(j\omega) = \underbrace{20 \log 7.5}_{\text{blue}} + \underbrace{20 \log \left| 1 + \frac{1}{3} j\omega \right|}_{\text{green}} - \underbrace{20 \log \omega}_{\text{red}} - \underbrace{20 \log \left| 1 + \frac{1}{2} j\omega \right|}_{\text{green}} - \underbrace{20 \log \left| 1 + \frac{1}{2} j\omega + \frac{1}{2} (j\omega)^2 \right|}_{\text{purple}}$$



$0 < \omega < \omega_n = \sqrt{2} \Rightarrow -20 \text{ dB} / H$
 $\omega_n < \omega < 2 \Rightarrow -20 \text{ dB} / H - 40 \text{ dB} / H = -60 \text{ dB} / H$
 $2 < \omega < 3 \Rightarrow -60 \text{ dB} / H - 20 \text{ dB} / H = -80 \text{ dB} / H$
 $\omega > 3 \Rightarrow -80 \text{ dB} / H + 20 \text{ dB} / H = -60 \text{ dB} / H$

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Maiztasun-erantzuna

Bode-diagramak

6) Fasearen diagraman maiztasunari balioak emanez egiten da:

$$\Phi(\omega) = \arctg\left(\frac{\omega}{3}\right) - 90 - \arctg\left(\frac{\omega}{2}\right) - \arctg\left(\frac{\frac{1}{2}\omega}{1 - \frac{\omega^2}{2}}\right)$$

$$\Phi(0) = \arctg\left(\frac{0}{3}\right) - 90 - \arctg\left(\frac{0}{2}\right) - \arctg\left(\frac{\frac{1}{2}0}{1 - \frac{0^2}{2}}\right) = -90^\circ$$

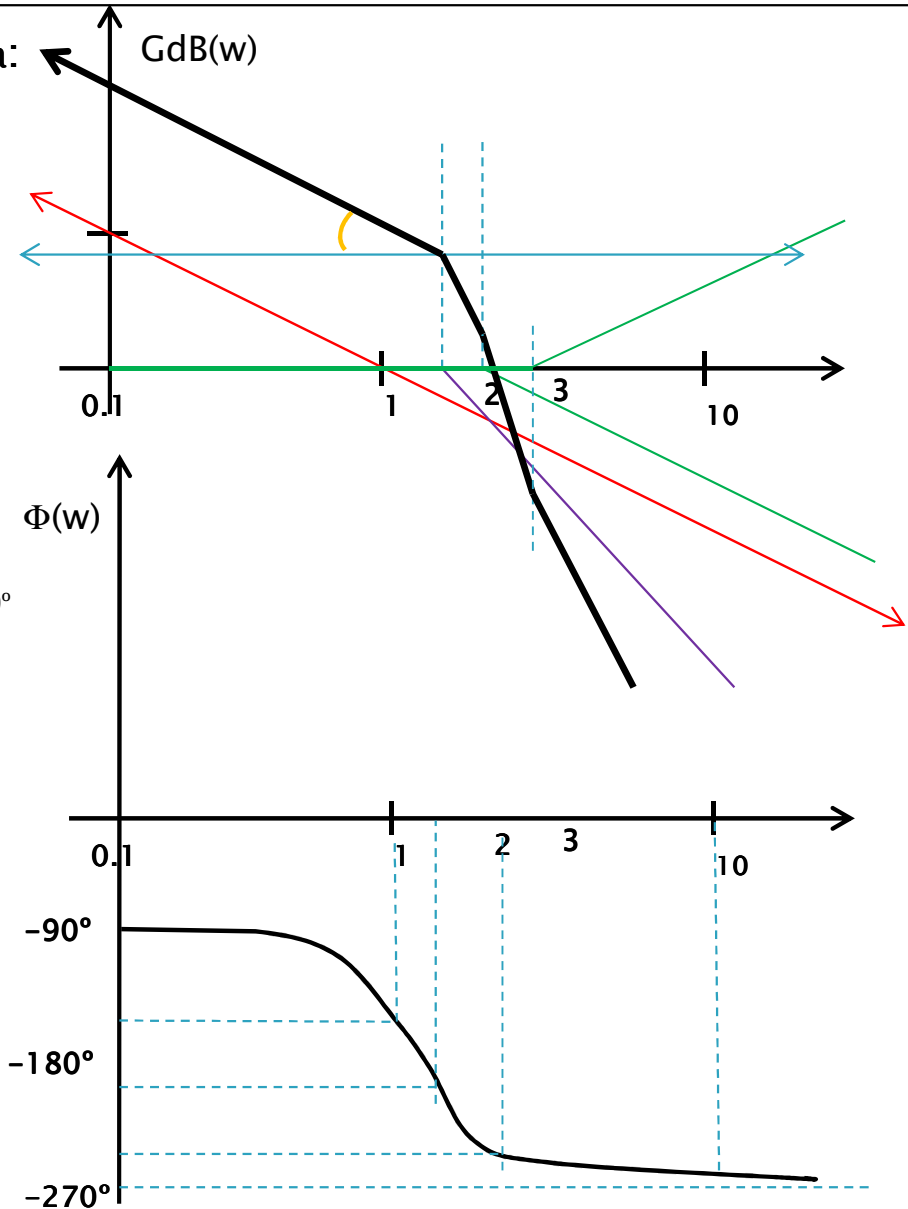
$$\Phi(1) = \arctg\left(\frac{1}{3}\right) - 90 - \arctg\left(\frac{1}{2}\right) - \arctg\left(\frac{\frac{1}{2}}{1 - \frac{1}{2}}\right) = -143.12^\circ$$

$$\Phi(\sqrt{2}) = \arctg\left(\frac{\sqrt{2}}{3}\right) - 90 - \arctg\left(\frac{\sqrt{2}}{2}\right) - \arctg\left(\frac{\frac{1}{2}\sqrt{2}}{1 - \frac{\sqrt{2}^2}{2}}\right) = -190^\circ$$

$$\Phi(2) = \arctg\left(\frac{2}{3}\right) - 90 - \arctg\left(\frac{2}{2}\right) - \arctg\left(\frac{\frac{1}{2}2}{1 - \frac{2^2}{2}}\right) = -236.31^\circ$$

$$\Phi(10) = \arctg\left(\frac{10}{3}\right) - 90 - \arctg\left(\frac{10}{2}\right) - \arctg\left(\frac{\frac{1}{2}10}{1 - \frac{10^2}{2}}\right) = -269^\circ$$

$$\Phi(\infty) = \arctg\left(\frac{\infty}{3}\right) - 90 - \arctg\left(\frac{\infty}{2}\right) - \arctg\left(\frac{\frac{1}{2}\infty}{1 - \frac{\infty^2}{2}}\right) = -270^\circ$$



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Maiztasun-erantzuna

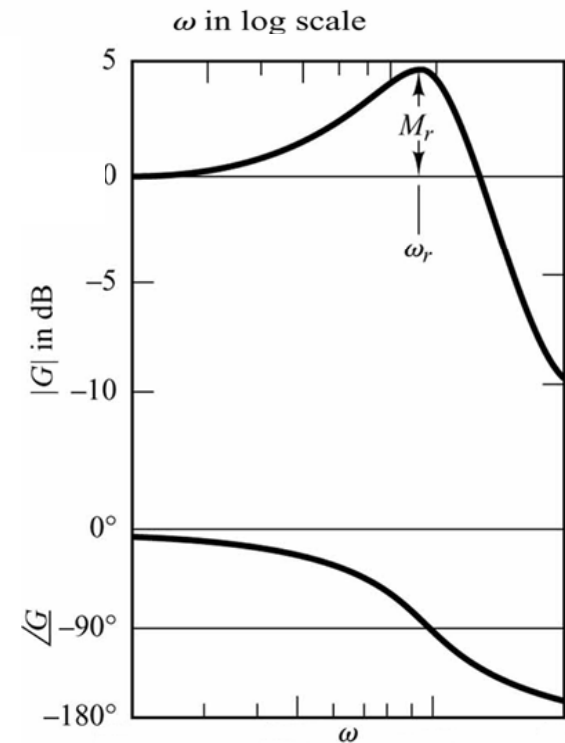
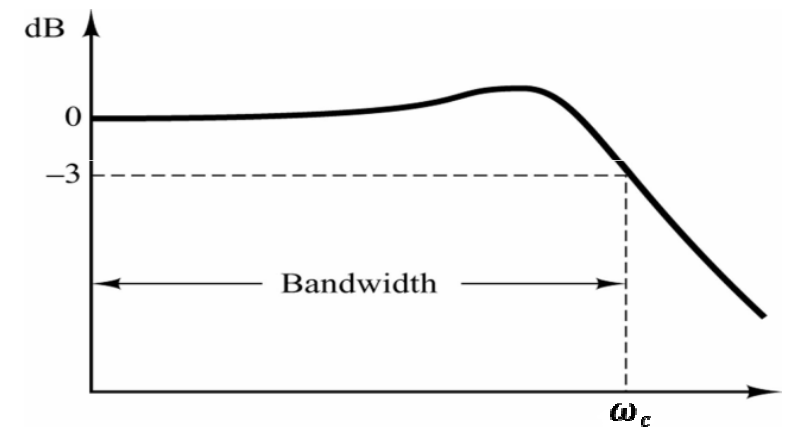
Maiztasun-erantzuneko ezaugarriak

Ebaketa maiztasuna (ω_c), maiztasun honetan Lazo itxiko T. F-aren moduluak -3 dB galtzen dituenean ($w=0$ rekiko)

Banda zabalera (BW), $w=0$ tik ω_c -rako maiztasun tarteak.
BW handiagoa, sistema arinagoa eta perturbazioen aurrean ezegonkorragoa

Ebaketa abiadura (ω_c). Moduluaren malda ω_c maiztasunean

Erresonantzi Maximoa (M_r). Moduluaren balio maximoa



8. Gaia

Maiztasun-erantzuna

Egonkortasun erlatiboa

Fase-tartea (MF):

Sistema bat ezegonkortasunera eramateko beharrezkoa den fase-atzerapena

$$MF = 180 + \Phi(\omega_{gc})$$

Irabazpen-tartea (MG):

Sistema bat ezegonkortasunera eramateko beharrezkoa den irabazpenaren balioa.

$$MG = -20 \log |GH(\omega_{fc})|$$

Irabazpen kritikoaren maiztasuna

$$\omega_{gc} \rightarrow |GH(j\omega_{gc})| = 1 \rightarrow G_{db} H_{db}(j\omega_{gc}) = 0db$$

Fase kritikoaren maiztasuna

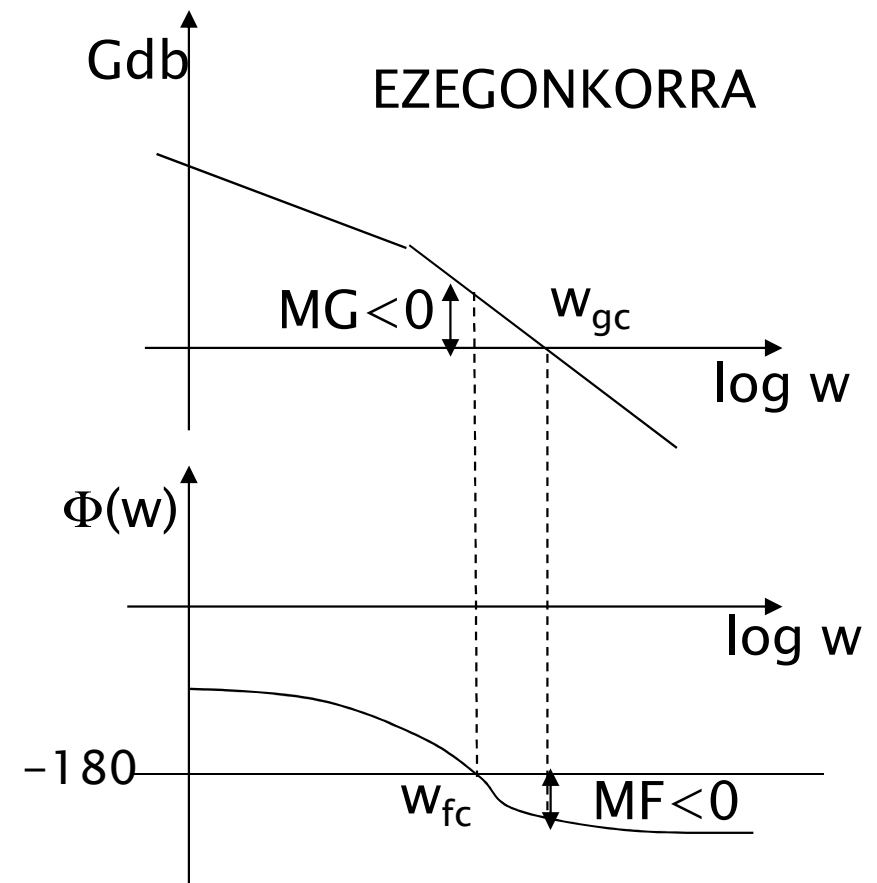
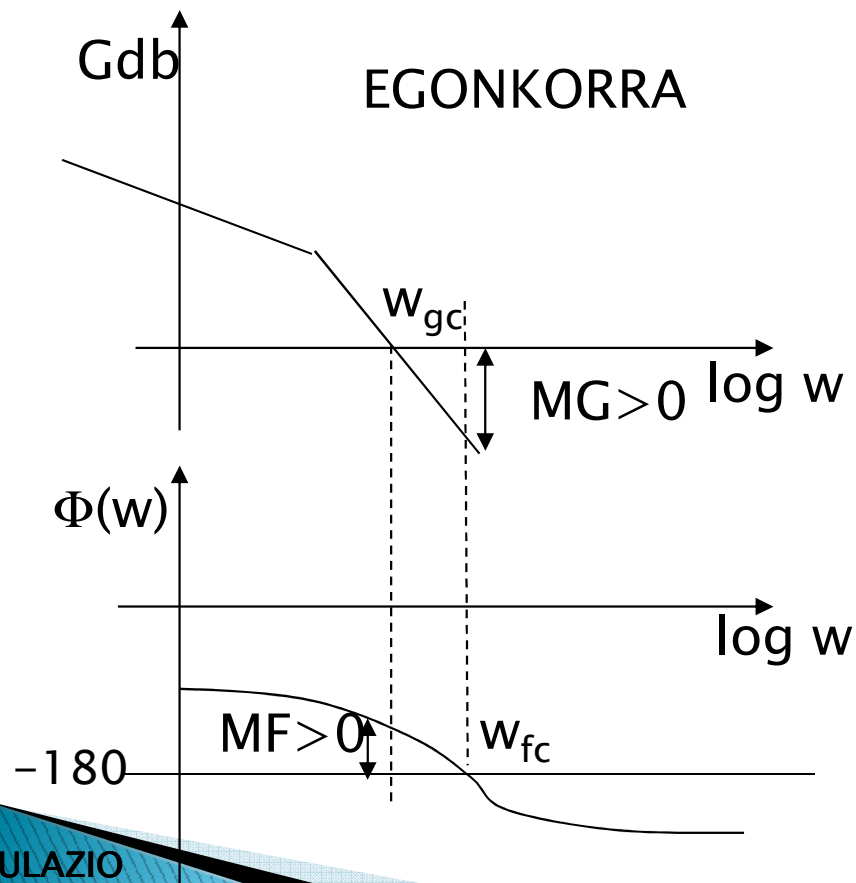
$$\omega_{fc} \rightarrow \Phi(\omega_{fc}) = -180^\circ$$

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Maiztasun-erantzuna

Egonkortasun erlatiboa

Fase minimoko sistema bat, egonkorra da Fase-tartea eta Irabazpen-tartea postiboak direnean. Irudian adierazten dira egonkorra eta ezegonkorren kasuak.



8. Gaia Maiztasun–erantzuna



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