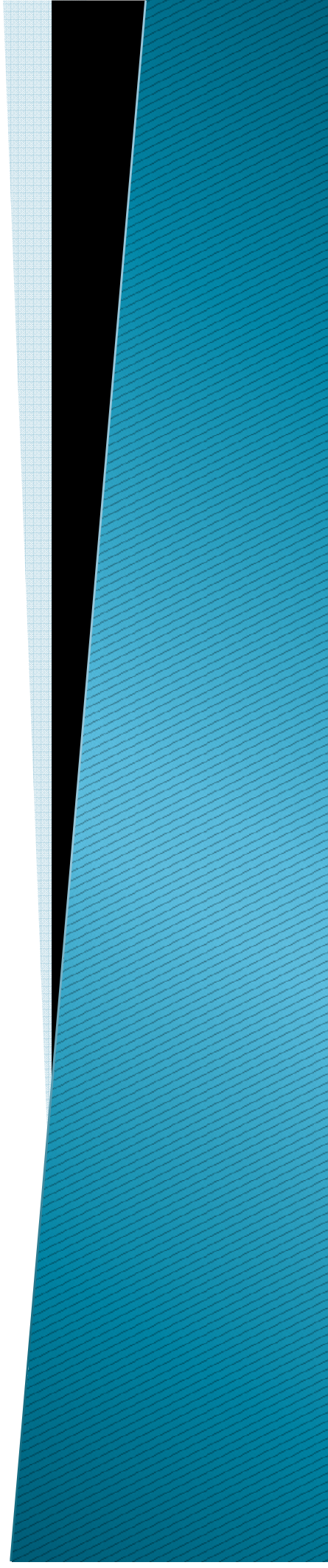


4. Gaia

Erregimen iragankorra eta iraunkorra



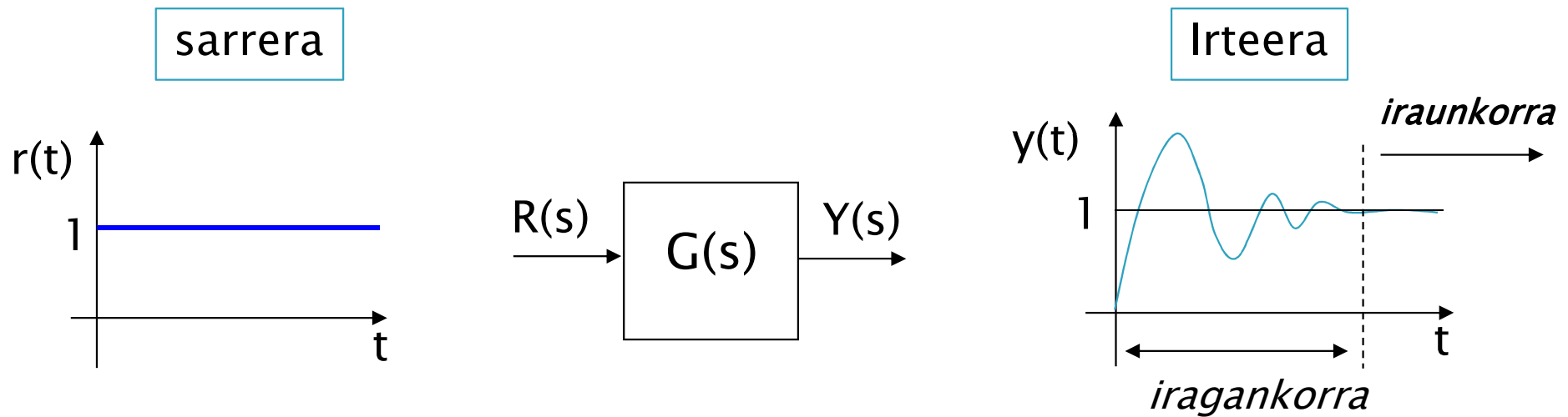
4 Gaia

Erregimen iragankorra eta iraunkorra

1. Sarrera: erregimen iragankorra eta iraunkorra
2. Sarrera-seinale motak
3. Erregimen iragankorra
4. Erregimen iraunkorra

4 Gaia. Erregimen iragankorra eta iraunkorra

Sarrera



Erantzun iragankorra: Irteeraren balioa hasierako egoeratik azkeneko egoera arte (egoera egonkorra lortu eta gero).

Erantzun egonkorra: Irteeraren balioa denboran $t \rightarrow \infty$.

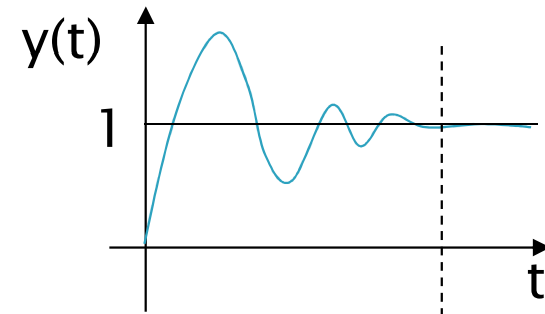
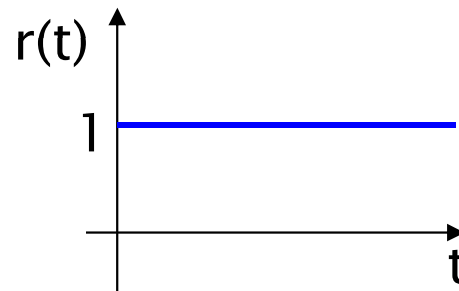
Kontrol sistema baten bi irteerak kontuan izan behar dira.

4 Gaia. Erregimen iragankorra eta iraunkorra

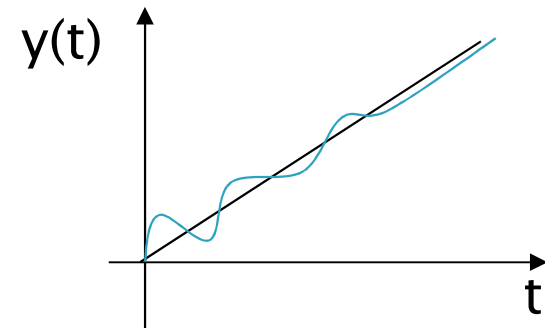
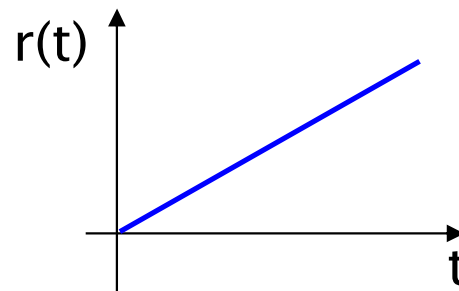
Sarrera-seinale motak

Kontrol-sistemen analisisian eta diseinuan, sarrera-seinale bereziak erabiltzen dira eta sistemen erantzuna aztertzen da sarrera horientzako.

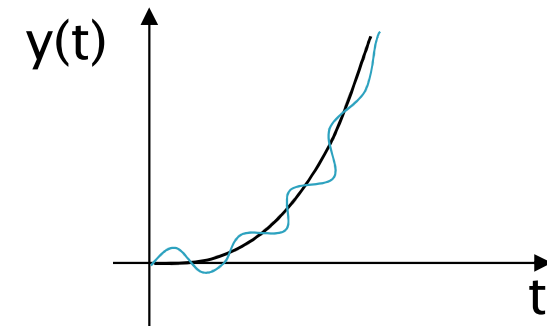
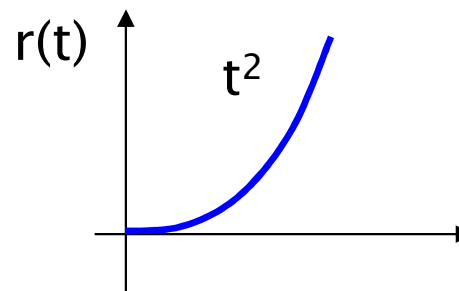
Sarrera maila



Sarrera malda



Inputso sarrera



4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iragankorra

Sistema baten erantzuna aztertzeko, lehenik sarrera bat aplikatu behar zaio (Maila, malda, parabolikoa ..) eta bigarren **zein ordenako sistema** den zehaztu behar da. Polo kopuruaren arabera **3 sistema-mota** desberdintzen dira:

1) 1. ordenako sistemak. Polo kopurua $n=1$

$$G(s) = \frac{k}{T s + 1}$$

2) 2. Ordenako sistemak. Polo kopurua $n=2$

$$G(s) = \frac{k w_n^2}{s^2 + 2 \zeta w_n s + w_n^2}$$

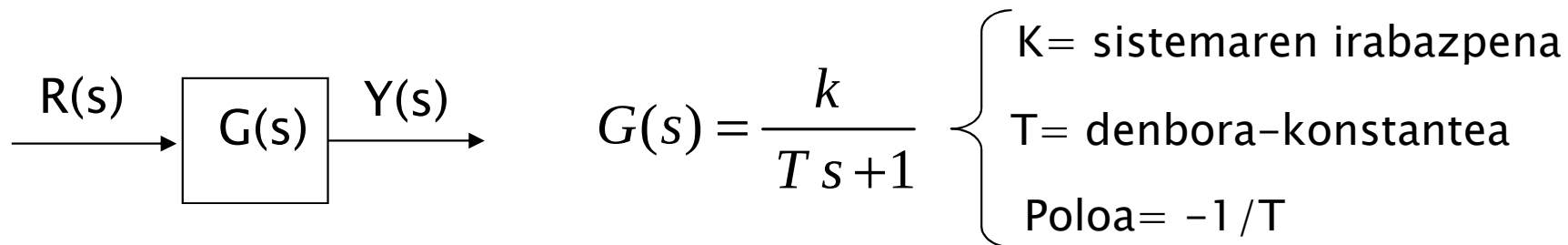
3) Goi ordenako sistemak
Polo kopurua $n > 2$, eta $n > m$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}$$

4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iragankorra

1. ORDENAKO SISTEMA



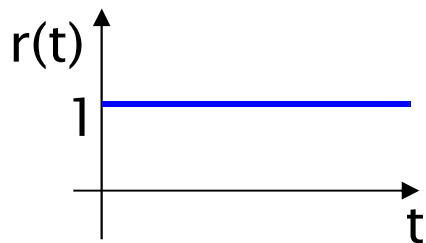
Sarrera maila funtzioa bada $R(s) = 1/s$, $y(t)$ erantzuna :

$$Y(s) = R(s) \frac{k}{Ts + 1} = \frac{1}{s} \frac{k}{Ts + 1} \xrightarrow[\text{trasnformatua}]{\text{Laplace-ren}}$$

$$y(t) = k \left(1 - e^{-\frac{t}{T}} \right)$$

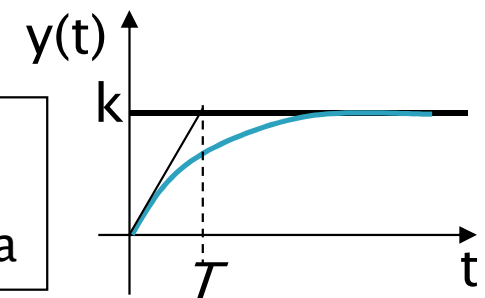
Erantzun iragankorra

$T > 0$ bada:



Ondorioak:

- 1) K sistemaren azken balioa ($t = \infty$), $T > 0$
- 2) T handitzen bada sistema motelagoa da



4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iragankorra

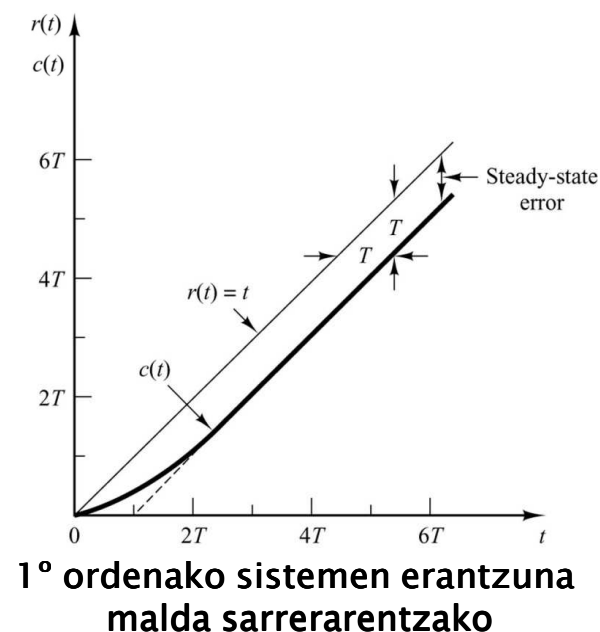
1. ORDENAKO SISTEMA

Ariketa:

1º ordenako sistemen erantzuna malda sarrerarentzako
Inpultso unitatearen L.T 1/s², denez, sistemaren erantzuna:

$$C(s) = \frac{1}{Ts + 1} \frac{1}{s^2}, \text{ Frakzio sinpleetan banatuz:}$$

$$C(s) = -\frac{T}{s} + \frac{1}{s^2} + \frac{T^2}{Ts + 1} \quad c(t) = -T + t + Te^{-t/T}, \quad t \geq 0$$



Ikus daitekeenez, $t \rightarrow \infty$ errorea egonkortzen da T balioan ($c(\infty) = \infty - T$),
hau da, $T \downarrow \Rightarrow$ errorea $\downarrow$

4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iragankorra

2. ORDENAKO SISTEMA

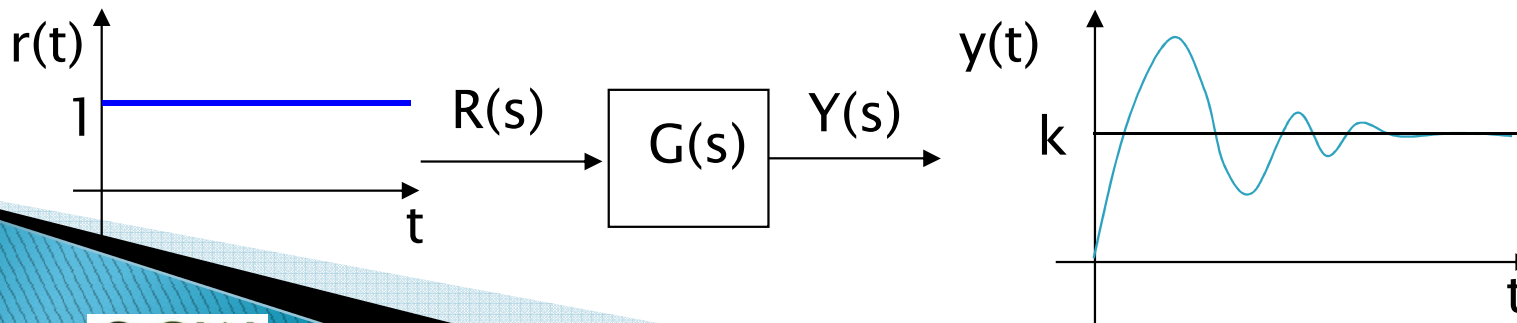
$$G(s) = \frac{k w_n^2}{s^2 + 2\delta w_n s + w_n^2}$$

K = sistemaren irabazpena
 δ = moteltze koefizientea
 W_n = Maiztasun naturala
 poloak $s = -\delta w_n \pm w_n \sqrt{1 - \delta^2} j$

Sarrera maila funtzioa $R(s) = 1/s$ eta $0 < \delta < 1$, $y(t)$ erantzuna:

$$Y(s) = \frac{1}{s} \frac{k w_n^2}{s^2 + 2\delta w_n s + w_n^2} \Rightarrow y(t) = k - k \frac{e^{-\delta w_n t}}{\sqrt{1 - \delta^2}} \sin\left(w_n \sqrt{1 - \delta^2} t + \arctg \frac{\sqrt{1 - \delta^2}}{\delta}\right)$$

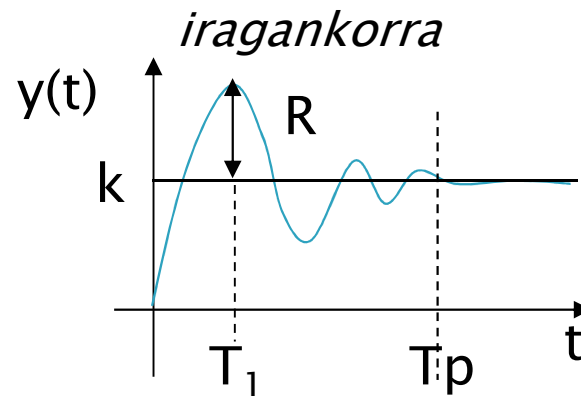
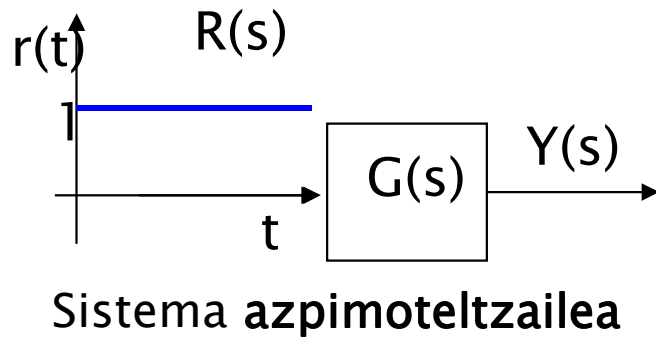
Erantzun iragankorra



4 Gaia. Erregimen iragankorra eta iraunkorra

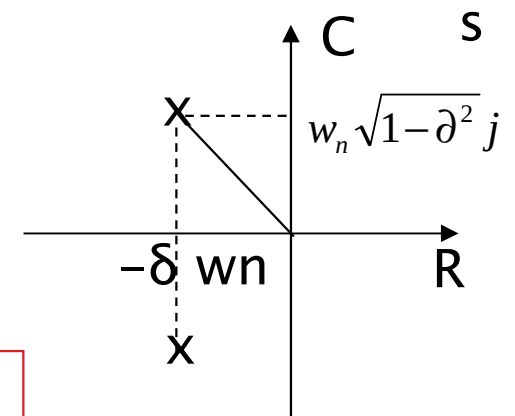
Erregimen iragankorra

2. ORDENAKO SISTEMA



Poloak:

$$s = -\delta \omega_n \pm \omega_n \sqrt{1 - \delta^2} j$$



$R =$ Gaineza

$$R(\%) = e^{-\frac{\pi \delta}{\sqrt{1 - \delta^2}}} \times 100 = \frac{y_{\max}(t) - y_{\text{final}}(t)}{y_{\text{final}}(t)} \times 100$$

$T_1 =$ Gailur-denbora

$$t_1 = \frac{\pi}{\omega_n \sqrt{1 - \delta^2}}$$

$T_p =$ Ezarpen-denbora

$$T_p = \begin{cases} t_{p(5\%)} = \frac{3}{\omega_n \delta} \\ t_{p(2\%)} = \frac{4}{\omega_n \delta} \end{cases}$$

4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iragankorra

2 ordenako sistemen erantzuna sarrera maila unitatea denean

3 kasu aztertuko dira moteltze koefizientearen arabera:

Sistema azpimoteltzailea ($0 < \delta < 1$)

sistemak 2 polo irudikari konjugatuak ditu:

$$s_{1,2} = -\delta \omega_n \pm j \omega_n \sqrt{1-\delta^2} = -\delta \omega_n \pm j \omega_d$$

non $\omega_d =$ maiztasun natural moteltzailea. Beraz, sistemaren erantzuna ondokoa da:

$$Y(s) = \frac{\omega_n^2}{(s^2 + 2\delta\omega_n s + \omega_n^2)s} \quad (0 < \delta < 1) \text{ denean:} \quad y(t) = 1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin\left(\omega_d t + \tan^{-1} \frac{\sqrt{1-\delta^2}}{\delta}\right)$$

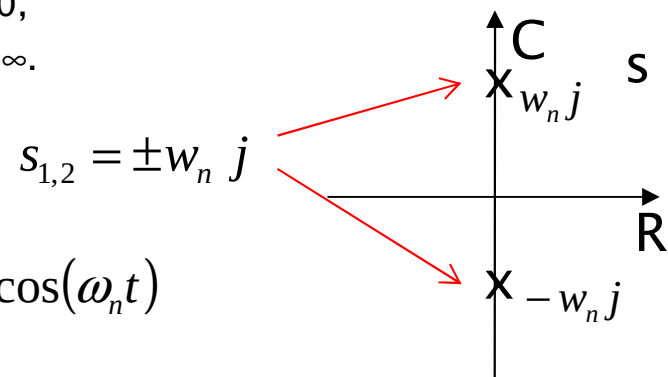
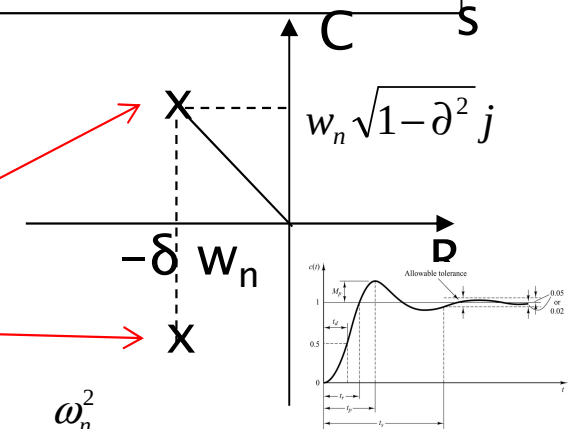
Irteera oszilakorra da ω_d maiztasunarekin. Baita ere, ikus daiteke errore seinalea

$$e(t) = r(t) - y(t) = \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin\left(\omega_d t + \tan^{-1} \frac{\sqrt{1-\delta^2}}{\delta}\right) \rightarrow 0, \quad t \rightarrow \infty.$$

Sistema kritikoki egonkorra ($\delta=0$)

kasu partikularra, polo irudikari hutsak :

$$y(t) = 1 - \sin\left(\omega_d t + \frac{\pi}{2}\right) = 1 - \cos(\omega_d t) = 1 - \cos(\omega_n t)$$



4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iragankorra

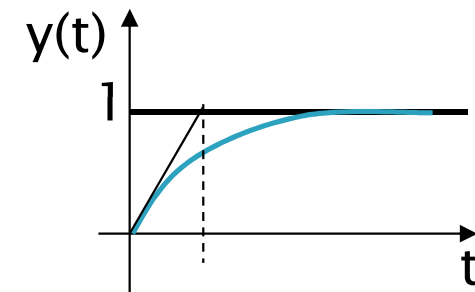
Kritikoki moteltzailea ($\delta=1$)

Polo erreal bikoitza $s_{1,2} = -\omega_n$
 Sistemaren erantzuna, bere alderantzizko transformatua

$$Y(s) = \frac{\omega_n^2}{(s + \omega_n)^2 s}$$

(tauletan 18) ondokoa da:

$$y(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)$$



Sistema Gainmoteltzailea ($\delta > 1$)

Sistemak bi polo erreal negatiboak ditu, baina desberdinak: $s_{1,2} = -\delta \omega_n \pm \omega_n \sqrt{\delta^2 - 1}$

Sistemaren erantzuna: $Y(s) = \frac{\omega_n^2}{(s + \delta \omega_n + \omega_n \sqrt{\delta^2 - 1})(s + \delta \omega_n - \omega_n \sqrt{\delta^2 - 1})s}$
 alderantzizko transformatua (taula 17):

$$y(t) = 1 + \frac{1}{2\sqrt{\delta^2 - 1}(\delta + \sqrt{\delta^2 - 1})} e^{-(\delta + \sqrt{\delta^2 - 1})\omega_n t} - \frac{1}{2\sqrt{\delta^2 - 1}(\delta - \sqrt{\delta^2 - 1})} e^{-(\delta - \sqrt{\delta^2 - 1})\omega_n t}$$

$t \rightarrow \infty$ denean zerorantz doazen, bi esponentzialen batuketa izanik.
 esponentzial horietako bat beti zerorantz arinago joango da eta 1º ordenako sistema bat bezala har daiteke

4 Gaia. Erregimen iragankorra eta iraunkorra

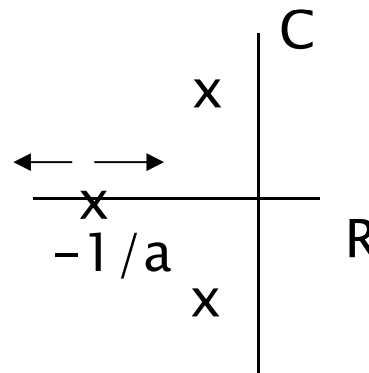
Erregimen iragankorra

2. ORDENAKO SISTEMAK ZERO ETA POLO GEHIGARRIEKIN

Maila unitatea $R(s)=1/s$ y $0<\delta<1$:

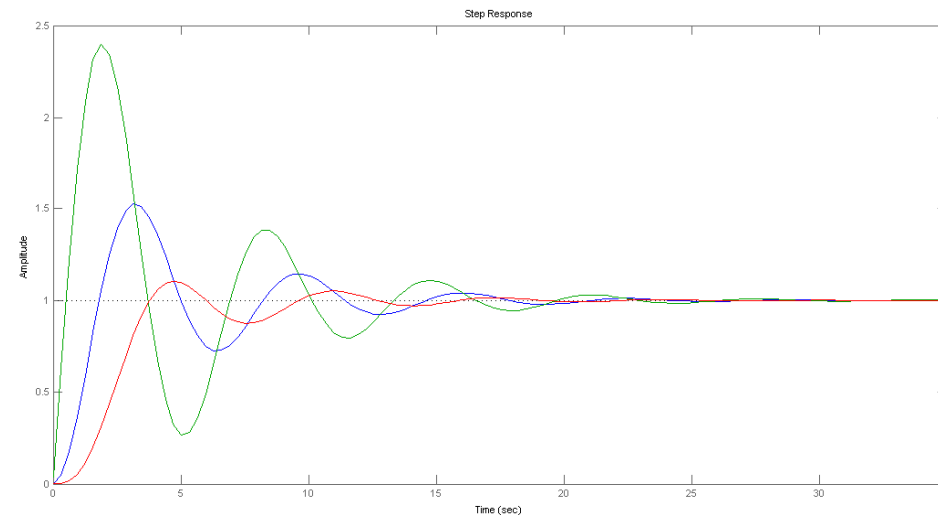
Zero gehigarria

$$G(s) = \frac{k w_n^2 (1 + as)}{s^2 + 2\delta w_n s + w_n^2}$$



Polo gehigarria

$$G(s) = \frac{k w_n^2}{(s^2 + 2\delta w_n s + w_n^2)(1 + as)}$$



4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iragankorra

GOI ORDENAKO SISTEMAK

Goi ordenako sistemen erantzun iragankorra zein iraunkorra aztertzeko, batez ere bi sinplifikazio egitea beharrezkoa da. Sinplifikazio hauen ondoren sistema 1. edo 2. orednako sistema batean bihurtuz.

Adibidea:

Demagun ondoko transferentzi funtzioa:

Kalkulatu bere sistema baliokidea

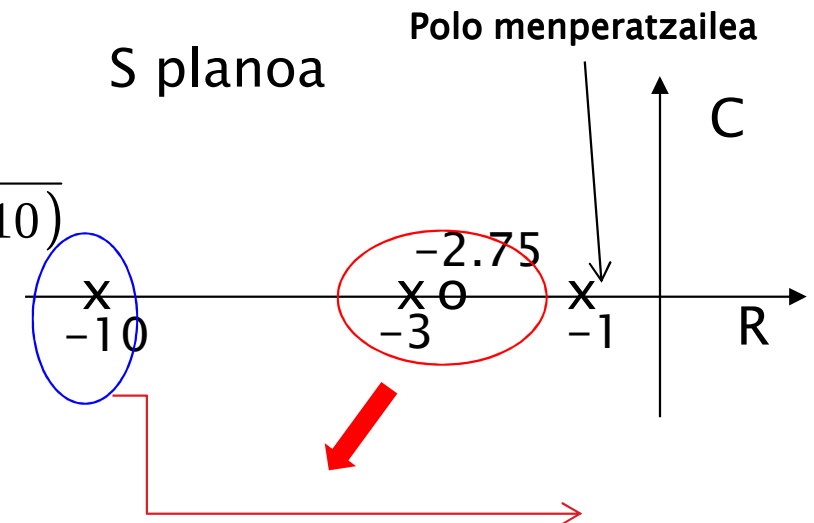
(sinplifikatua)

$$G(s) = \frac{(s + 2.75)}{(s + 1)(s + 3)(s + 10)}$$

Egin daitezkeen sinplifikazioak:

- 1) -3 poloa -2.75 zeroarekin ezabatu
- 2) -10 puntuan dagoen poloa mesprezatu, polo menperatzaitetik (-1) oso urrun dagoelako

Ondorioz, sistema baliokidea 1. ordenako sistema bat da



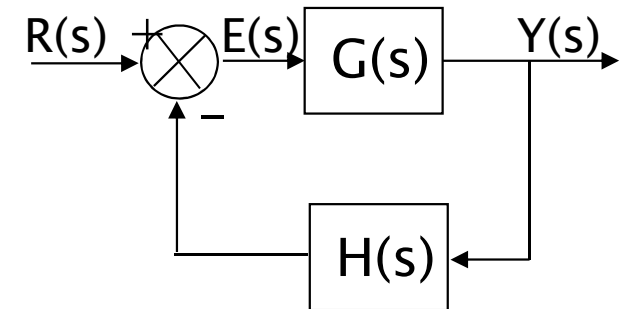
$$y(t) = 0.092 - 0.097e^{-t} - 0.006e^{-3t} - 0.011e^{-10t}$$

$$G(s) = \frac{0.092}{(s + 1)}$$

4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iraunkorra

Demagun ondoko sistema berrelikatua. Sistemaren erantzun iraunkorra aztertzeko, sistemari sarrera bat aplikatzen zaionean agertzen diren erroreen ($E(s)$) azterketa beharrezkoa da.



$$\left. \begin{array}{l} Y(s) = G(s)E(s) \\ E(s) = R(s) - H(s)Y(s) \end{array} \right\} G_{LC}(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$G_{LC}(s)$ Lazo itxiko T.F.

$G(s)H(s)$ lazo irekiko T.F.

$E(s)$ erroreaken T. F.

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

Erregimen iraunkorreko errorea definitzen da:

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} s \frac{R(s)}{1 + G(s)H(s)}$$

4 Gaia. Erregimen iragankorra eta iraunkorra

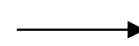
Erregimen iraunkorra

Errore geldikor koefizienteak:

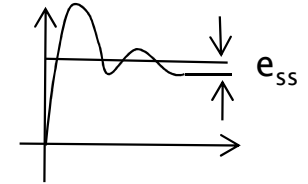
Posiziozkoa k_p :

$$R(s) = 1/s$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)} = \frac{1}{1 + K_p}$$



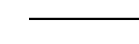
$$k_p = \lim_{s \rightarrow 0} G(s)H(s)$$



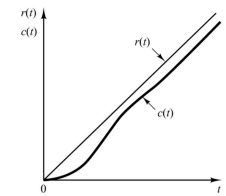
Abiadurazkoa K_v :

$$R(s) = 1/s^2$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{1/s}{1 + G(s)H(s)} = \frac{1}{K_v}$$



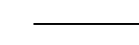
$$k_v = \lim_{s \rightarrow 0} s G(s)H(s)$$



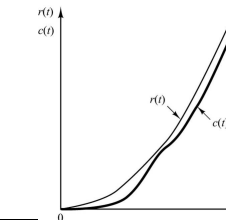
Azeleraziozkoa K_a :

$$R(s) = 1/s^3$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{1/s^2}{1 + G(s)H(s)} = \frac{1}{K_a}$$



$$k_a = \lim_{s \rightarrow 0} s^2 G(s)H(s)$$



Sistema-mota:

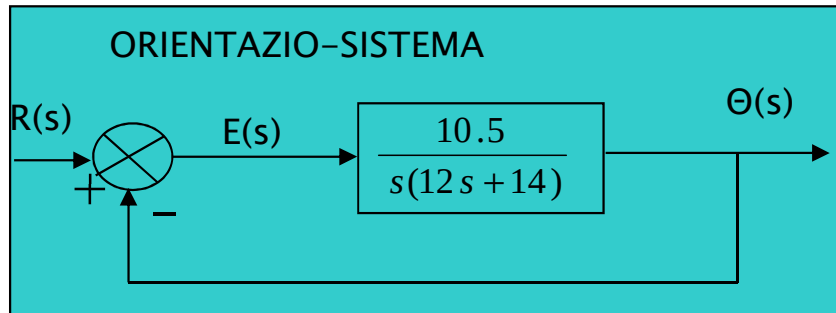
j =jatorriko polo-kopurua

$$G(s)H(s) = \frac{k(c_1s+1)...(c_ms+1)}{s^j(p_1s+1)...(p_{n-j}s+1)}$$

$j \backslash R(s)$	$1/s$	$1/s^2$	$1/s^3$
$j = 0$	$\frac{1}{1 + K_p}$	∞	∞
$j = 1$	0	$\frac{1}{K_v}$	∞
$j = 2$	0	0	$\frac{1}{K_a}$
$j \geq 3$	0	0	0

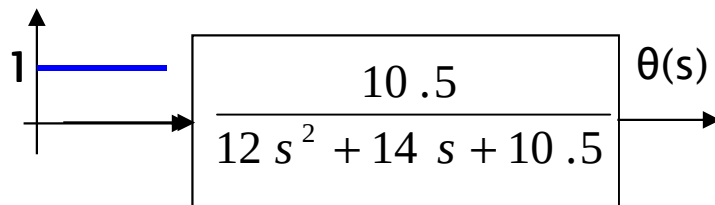
4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iraunkorra



Sarrera maila unitatea

$$R(s) = 1/s$$

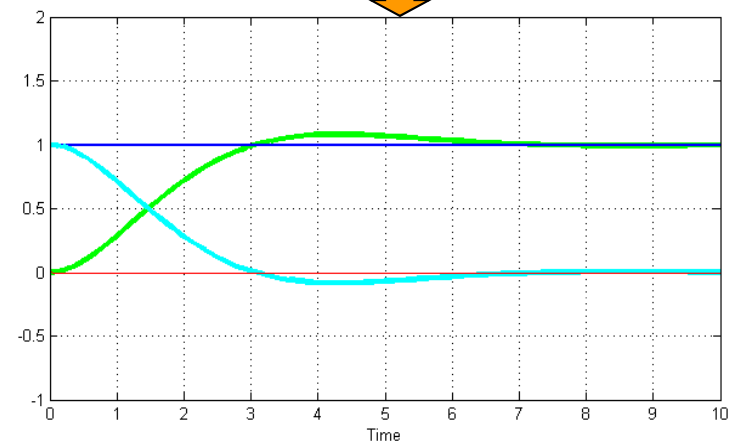
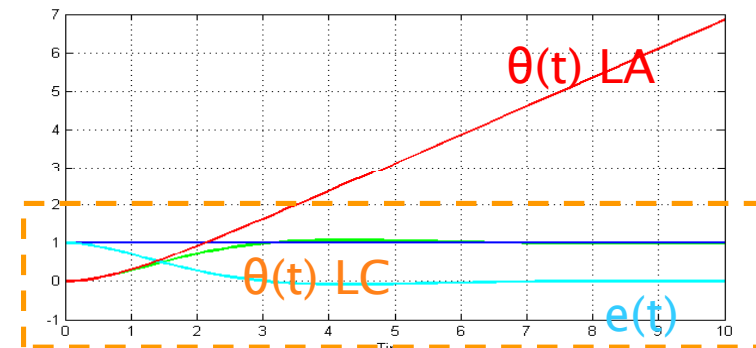


Azken balioa:

$$\lim_{t \rightarrow \infty} \vartheta(t) = \lim_{s \rightarrow 0} s \vartheta(s) = \lim_{s \rightarrow 0} s R(s) G_{LC}(s) = 1$$

ERROREA

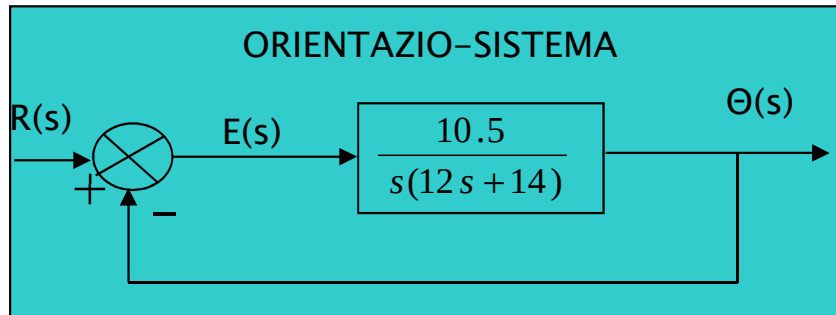
$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} s \frac{R(s)}{1 + G(s)H(s)} = 0$$



$$G_{LC}(s) = \frac{\vartheta(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{10.5}{12s^2 + 14s + 10.5}$$

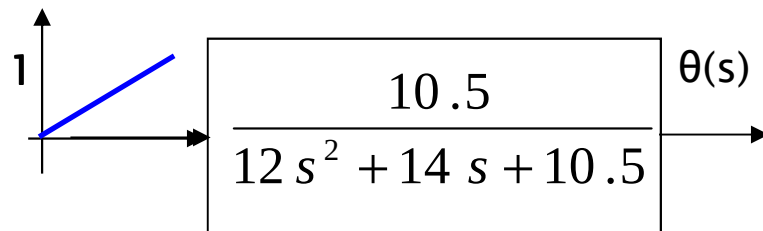
4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iraunkorra



Sarrera malda unitatea

$$R(s) = 1/s^2$$

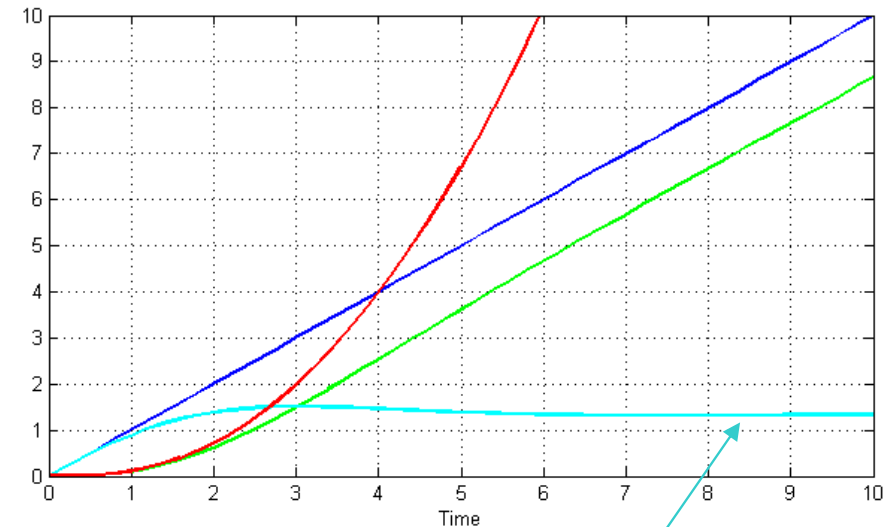


ERROREA:

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} s \frac{R(s)}{1 + G(s)H(s)} = \lim_{s \rightarrow 0} s \frac{1/s^2}{1 + \frac{10.5}{s(12s+14)}} = \frac{14}{10.5} = 1.33$$

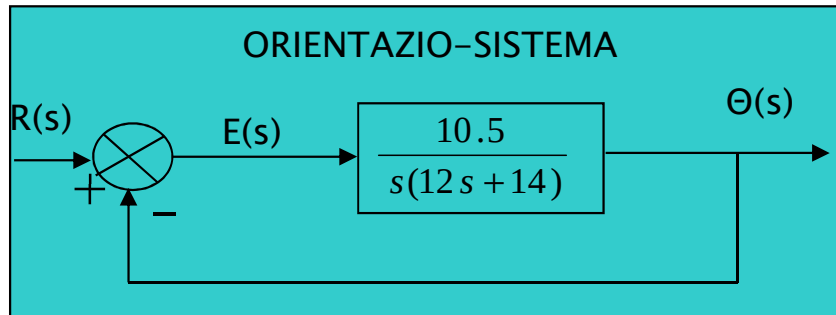
AZKEN BALIOA:

$$\lim_{t \rightarrow \infty} \vartheta(t) = \lim_{s \rightarrow 0} s \vartheta(s) = \lim_{s \rightarrow 0} s R(s) G_{LC}(s) = \lim_{s \rightarrow 0} s \frac{1}{s^2} \frac{10.5}{12s^2 + 14s + 10.5} = \infty$$



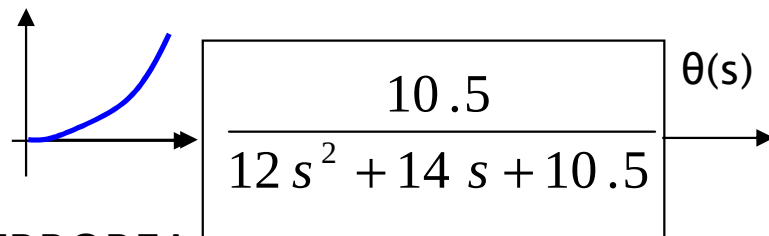
4 Gaia. Erregimen iragankorra eta iraunkorra

Erregimen iraunkorra



Sarrera parabolikoa

$$R(s) = 1/s^3$$

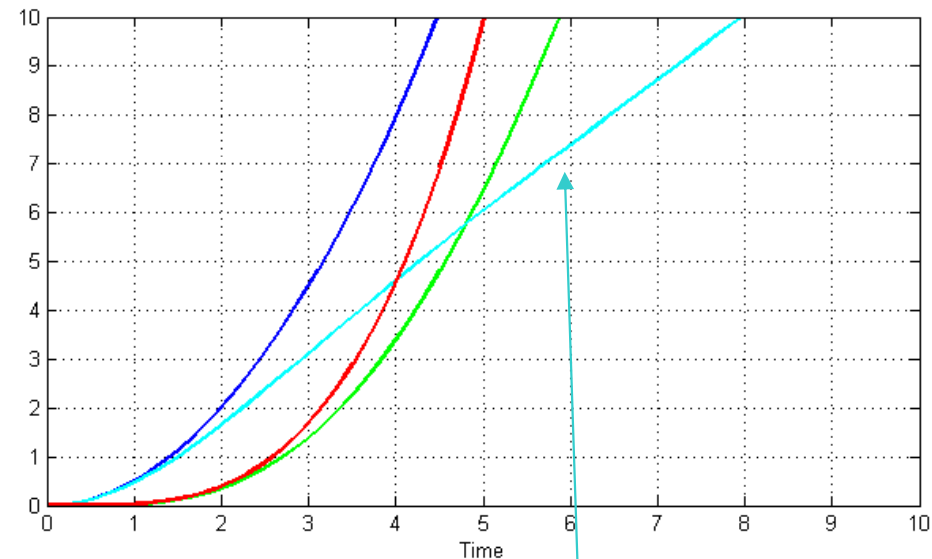


ERROREA:

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} s \frac{R(s)}{1 + G(s)H(s)} = \lim_{s \rightarrow 0} s \frac{\frac{1}{s^3}}{1 + \frac{10.5}{s(12s + 14)}} = \frac{14}{10.5} = \infty$$

AZKEN BALIOA:

$$\lim_{t \rightarrow \infty} \vartheta(t) = \lim_{s \rightarrow 0} s \vartheta(s) = \lim_{s \rightarrow 0} s R(s) G_{LC}(s) = \lim_{s \rightarrow 0} s \frac{1}{s^3} \frac{10.5}{12s^2 + 14s + 10.5} = \infty$$



4 Gaia. Erregimen iragankorra eta iraunkorra



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