

AUTOEBALUAZIOA: EMAITZAK

TEORIA

1) Robot-beso baten ezaugarri ESTATIKOAK aipatu. (0.5 puntu)

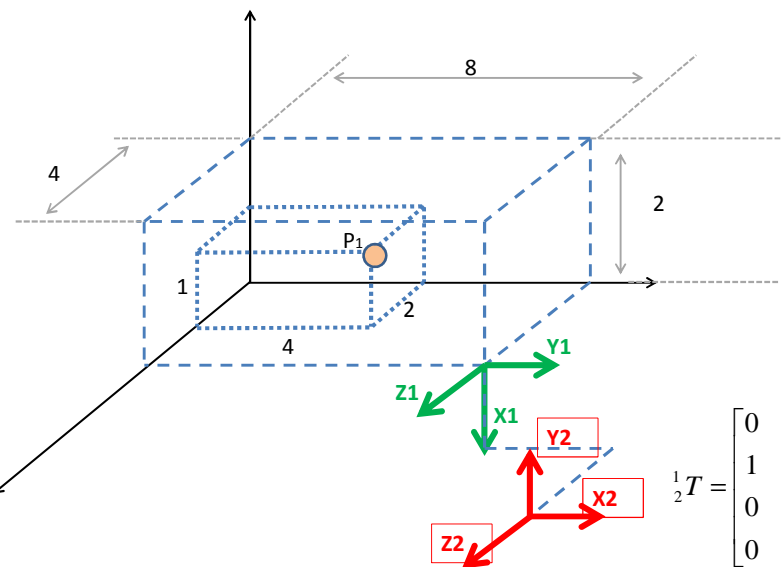
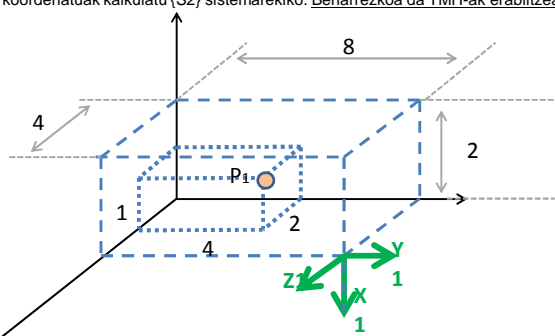
- 1) Askatasun-gradua
- 2) Lan-bolumena
- 3) Irigarritasuna
- 4) Maniobragarritasuna
- 5) Mugikortasuna

2) **ENKODERRA**, barruko edo kanpoko sentsore bat da? Zer neurtzen du? (0.5 puntu)

Barneko sentsore bat da eta posizioa neurtzen du

3) Irudian {S1} sistemaren eta P1 puntuaren lokalizazioa adierazten da. Jakinik {S1} eta {S2} sistemek erlazionatzen dituen TMH ondokoa dela {S2} sistema irudikatu eta P1 puntuaren koordinatuak kalkulatu {S2} sistemarekiko. Beharrezkoa da TMH-ak erabiltzea. (1.5 puntu)

$${}^1_2T = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$${}^1_2T = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 1) Traslazioa (1,2,3)
- 2) Biraketa R(Z, 90º)

P1 puntuaren koordinatuak 1 sistemarekiko : ${}^1P1 = [-1, -4, -2]$

P1 puntuaren koordinatuak 2 sistemarekiko:

$${}^2P1 \Rightarrow {}^2P1 = {}_1^2T \cdot {}^1P1$$

Aderantziko matrizea kalkulatu behar da

$${}^1_2T = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2_1T = \begin{bmatrix} 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2P1 = {}_1^2T \cdot {}^1P1 = \begin{bmatrix} 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -4 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ -2 \\ 1 \end{bmatrix}$$

P1 puntuaren koordinatuak 2 sistemarekiko:

$${}^2P1 = [-6, 2, -5]$$

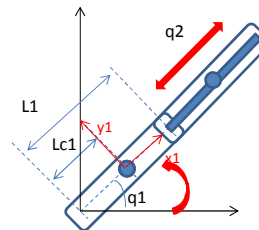
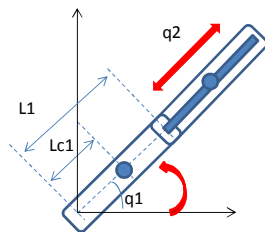
Jon Legarreta / Raquel Martinez



1 ARIKETA

Irudiko robota 2 askatasun-gradu ditu, lehenengoa errotazionala eta bigarrena PRISMATIKOA. Energi balantzean oinarrituz, LAGRANGETARRA kalkulatu.

D-Hen erreferentzi sistemen esleipena_1. kate maila



Jon Legarreta / Raquel Martinez



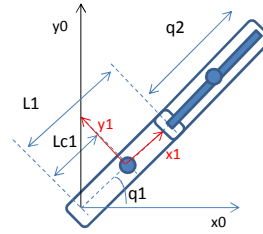
1 pausua: energia zinetikoaren kalkulua (K)

Kate-maila 1: $K_1 = \frac{1}{2} m_1 \mathbf{v}_1^T \mathbf{v}_1 + \frac{1}{2} \boldsymbol{\omega}_1^T \mathbf{I}_1 \boldsymbol{\omega}_1$

Lehenengo kate-mailaren TMH a kalkulatzeko dugu:

$${}^0_1 A = \begin{bmatrix} Cq_1 & 1 & -Sq_1 & 0 & L_{c1}Cq_1 & 1 \\ Sq_1 & 1 & Cq_1 & 0 & L_{c1}Sq_1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$\rightarrow x = L_{c1}Cq_1$
 $\rightarrow y = L_{c1}Sq_1$
 $\rightarrow z = 0$



θ_i	d_i	a_i	α_i
1	q_1	0	L_{c1}
			0°

Jacobtarra erabiliz:

1 kate-mailaren abiadura lineala

$$J = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -L_{c1}Sq_1 & 0 & 0 \\ L_{c1}Cq_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \mathbf{v}_1 = \begin{bmatrix} -L_{c1}Sq_1 & 0 & 0 \\ L_{c1}Cq_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -L_{c1}Sq_1 \dot{q}_1 \\ L_{c1}Cq_1 \dot{q}_1 \\ 0 \end{bmatrix}$$

1 pausua: energia zinetikoaren kalkulua (K)

Energia zinetikoa $K_1 = \frac{1}{2} m_1 \mathbf{v}_1^T \mathbf{v}_1 + \frac{1}{2} \boldsymbol{\omega}_1^T \mathbf{I}_1 \boldsymbol{\omega}_1$

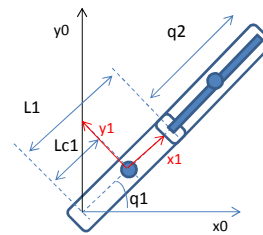
$$\mathbf{v}_1^T \mathbf{v}_1 = \begin{bmatrix} -L_{c1}Sq_1 \dot{q}_1 & L_{c1}Cq_1 \dot{q}_1 & 0 \\ -L_{c1}Sq_1 \dot{q}_1 & L_{c1}Cq_1 \dot{q}_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = L_{c1}^2 \dot{q}_1^2$$

1 kate-mailaren abiadura **angeluarra** :

$$\vec{\omega}_1 = \dot{q}_1 \vec{z}_0$$

1 kate-mailaren energia zinetikoa :

$$K_1 = \frac{1}{2} m_1 L_{c1}^2 \dot{q}_1^2 + \frac{1}{2} \mathbf{I}_1 \dot{q}_1^2$$



1 pausua: energia zinetikoaren kalkulua (K)

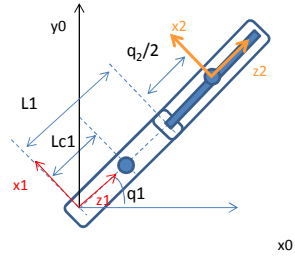
2 kate-maila :

$$K_2 = \frac{1}{2} m_2 \mathbf{V}_2^T \mathbf{V}_2 + \frac{1}{2} \omega_2^T \mathbf{I}_2 \omega_2$$

Bigarren kate-mailaren TMH a kalkulatzeko dugu:

$${}^0_2 A = {}^0_1 A {}^1_2 A = \begin{bmatrix} -S q_1 & 1 & 0 & C q_1 & 1 & 0 \\ C q_1 & 1 & 0 & S q_1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + q_2/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_2 A = {}^0_1 A {}^1_2 A = \begin{bmatrix} -S q_1 & 1 & 0 & C q_1 & C q_1(L_1 + q_2/2) \\ C q_1 & 1 & 0 & S q_1 & S q_1(L_1 + q_2/2) \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} =x \\ =y \\ =z \end{matrix}$$



	θ_i	d_i	a_i	α_i
1	Q_1+90°	0	0	90°
2	0	$L_1 + q_2/2$	0	0°

1 pausua: energia zinetikoaren kalkulua (K)

Kate-maila 2:

$$K_2 = \frac{1}{2} m_2 \mathbf{V}_2^T \mathbf{V}_2 + \frac{1}{2} \omega_2^T \mathbf{I}_2 \omega_2$$

2 kate-mailaren TMH a kalkulatzeko dugu:

	θ_i	d_i	a_i	α_i
1	Q_1+90°	0	0	90°
2	0	$L_1 + q_2/2$	0	0°

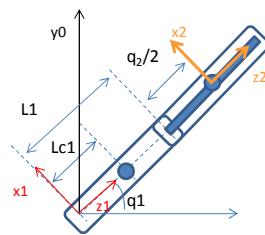
$$x = C q_1(L_1 + q_2/2)$$

$$y = S q_1(L_1 + q_2/2)$$

$$z = 0$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3} \end{bmatrix} = \begin{bmatrix} -S q_1(L_1 + q_2/2) & C q_1/2 & 0 \\ C q_1(L_1 + q_2/2) & S q_1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{v}_2 = \begin{bmatrix} -S q_1(L_1 + q_2/2) & C q_1/2 & 0 \\ C q_1(L_1 + q_2/2) & S q_1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dot{q}_1[S q_1(L_1 + q_2/2)] + \dot{q}_2 C q_1/2 \\ -\dot{q}_1[C q_1(L_1 + q_2/2)] + \dot{q}_2 S q_1/2 \\ 0 \end{bmatrix}$$



1 pausua: energia zinetikoaren kalkulua (K)

Kate-maila 2:
$$K_2 = \frac{1}{2} m_2 \mathbf{v}_2^T \mathbf{v}_2 + \frac{1}{2} \omega_2^T \mathbf{I}_2 \omega_2$$

Lehenengo kate-mailaren TMH a kalkulatuzen dugu:

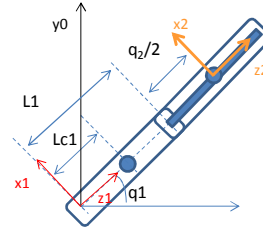
$$\mathbf{v}_2^T \mathbf{v}_2 = \begin{bmatrix} \dot{q}_1 [S q_1 (L_1 + q_2/2)] + \dot{q}_2 C q_1/2 & -\dot{q}_1 [C q_1 (L_1 + q_2/2)] + \dot{q}_2 S q_1/2 & 0 \\ -\dot{q}_1 [C q_1 (L_1 + q_2/2)] + \dot{q}_2 S q_1/2 & \dot{q}_1^2 [S q_1 (L_1 + q_2/2)] + \dot{q}_2^2 C q_1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{v}_2^T \mathbf{v}_2 = \dot{q}_1^2 (L_1 + q_2/2)^2 + \dot{q}_2^2 - 2 \dot{q}_1 \dot{q}_2 (L_1 + q_2/2) S(q_1/2)$$

2 kate-mailaren abiadura angeluarra:
$$\vec{\omega}_2 = \dot{q}_1 \vec{z}_0$$

Terminoak batuz, 2 kate-mailaren energia zinetikoa:

$$K_2 = \frac{1}{2} m_2 \left[\dot{q}_1^2 (L_1 + q_2/2)^2 + \dot{q}_2^2 - 2 \dot{q}_1 \dot{q}_2 (L_1 + q_2/2) S(q_1/2) \right] + \frac{1}{2} \mathbf{I}_2 \dot{q}_1^2$$



2. pausua: energia potentzialaren kalkulua (U)

Besoaren energia potentziala, robotaren kate-maila bakoitzaren energia potentzialen batuketara da, hau da:

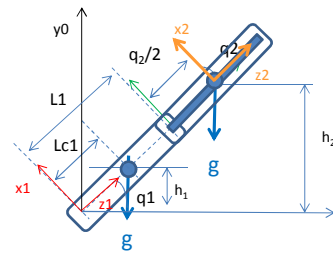
$$U = U(q)$$

$$U = \sum_{i=1}^n U_i \quad U_i = m_i \mathbf{g}^T \mathbf{p}_{ci}$$

m_i = i kate-mailaren masa
 \mathbf{g} = grabitate-bektorea
 \mathbf{p}_{ci} = i kate-mailaren masa-zentrua kokatzen duen bektorea, hasierako erreferentzi sistemarekiko

1 kate-maila:
$$U_1 = m_1 g \underbrace{L_{c1}}_{h1} \text{sen } q_1$$

2 kate maila:
$$U_2 = m_2 g \underbrace{(L_1 + q_2/2)}_{h2} \text{sen } q_1$$



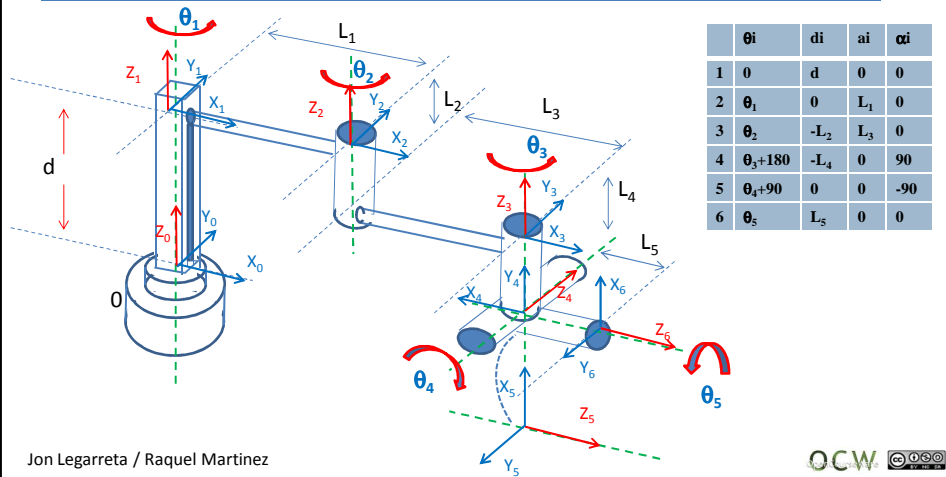
3. Pasua Lagrangetarraren kalkulua

$$L = \frac{1}{2} m_1 L_{c1}^2 \dot{q}_1^2 + \frac{1}{2} \mathbf{I}_1 \dot{q}_1^2 + \frac{1}{2} m_2 \left[\dot{q}_1^2 (L_1 + q_2/2)^2 + \dot{q}_2^2 - 2 \dot{q}_1 \dot{q}_2 (L_1 + q_2/2) S(q_1/2) \right] + \frac{1}{2} \mathbf{I}_2 \dot{q}_1^2 - m_1 g L_{c1} \text{sen } q_1 - m_2 g (L_1 + q_2/2) \text{sen } q_1$$

2 ARIKETA

6 askatasun graduko robotaren erreferentzi sistema izanik: (3.5 puntu)

- 1) D-Hen algoritmoa erabiliz, robotaren 6 artikulazioen erreferentzi sistemak esleitu (lehenengo artikulazioa prismaticoa eta besteak errotazionalak).
- 2) D-H algoritmoen parametroen taula lortu.



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