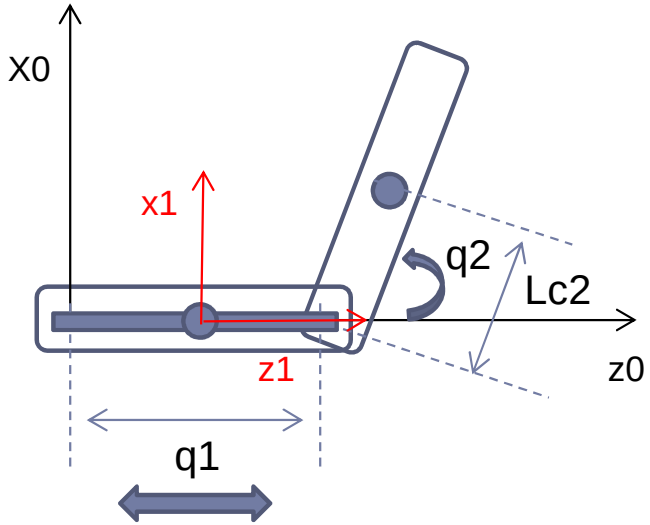
6. GAIA
ROBOTEN DINAMIKA ETA KONTROLA
ARIKETAK

ROBOTIKA

ARIKETAK

6.1 ariketa
Irudiko 2 askatasun-graduko robot planoaren Lagrangetarra kalkulatu. *Oharra: q1 prismatikoa eta q2 errotazionala*.



1 pausua: energia zinetikoaren kalkulua (K)

Kate-maila 1:

$$K_1 = \frac{1}{2} m_1 v_1^T v_1$$

Lehenengo kate-mailaren TMH a kalkulatzen dugu:

	θi	di	ai	αi
1	0	q1/2	0	0º

$$ {}^{0}_{1}A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \longrightarrow x = 0 \\ \longrightarrow y = 0 \\ \longrightarrow z = q1/2 \end{array} $$

Jon Legarreta/ Raquel Martinez

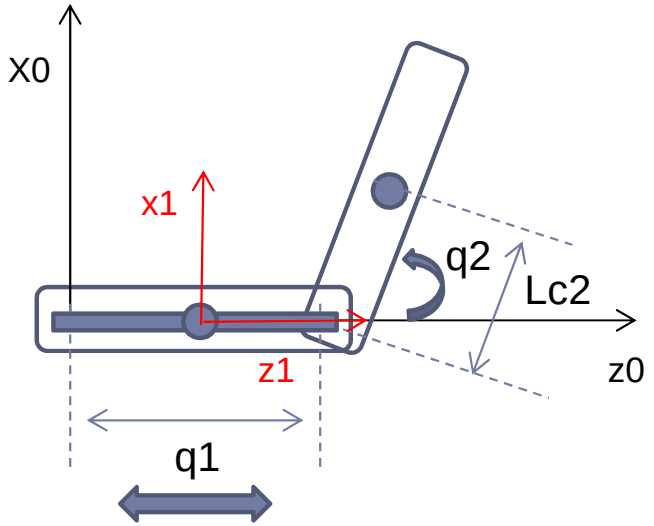
ARIKETAK

Jacobtarra erabiliz:

$$J = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$$

1 kate-mailaren abiadura lineala

$$v_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{\dot{q}1}{2} \end{bmatrix}$$



$$v_1^T v_1 = \begin{bmatrix} 0 & 0 & \frac{\dot{q}1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{\dot{q}_1}{2} \end{bmatrix} = \frac{\dot{q}_1^{\,2}}{4}$$

1 kate-mailaren energia zinetikoa :

$$\boxed{K_1 = \frac{1}{2} m_1 \frac{\dot{q}_1^{\,2}}{4}}$$

1 kate-mailaren energia potentziala :

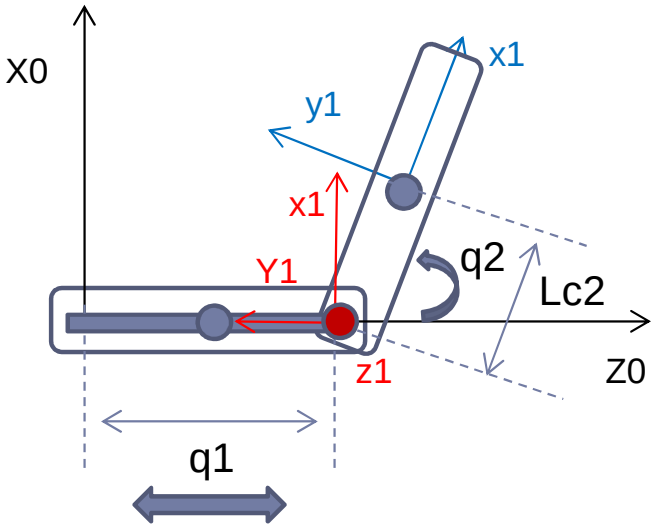
$$\boxed{U_1 = m_1 g\, h = 0}$$

Jon Legarreta/ Raquel Martinez

ARIKETAK

1 pausua: energia zinetikoaren kalkulua (K)

Kate-maila 2: $K_2 = \frac{1}{2} m_2 v_2^T v_2 + \frac{1}{2} \omega_2^T I_2 \omega_2$



2 kate-mailaren TMH a kalkulatzen dugu:

	θi	di	ai	αi
1	0	q1	0	-90º
2	q2-90	0	Lc2	0

$$ {}^0_2A = {}^0_1A\,{}^1_2A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & q1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C(q2-90) & 0 & 0 & L_{c2}C(q2-90) \\ S(q2-90) & 0 & 1 & L_{c2}S(q2-90) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_2A = {}^0_1A\,{}^1_2A = \begin{bmatrix} C(q2-90) & 0 & 0 & L_{c2}C(q2-90) \\ 0 & -1 & 0 & 0 \\ -S(q2-90) & 0 & -1 & -L_{c2}S(q2-90)+q1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$x = L_{c2}C(q2-90) = L_{c2}Sq2$$

$$y = 0$$

$$z = -L_{c2}S(q2-90) + q1 = L_{c2}Cq2 + q1$$

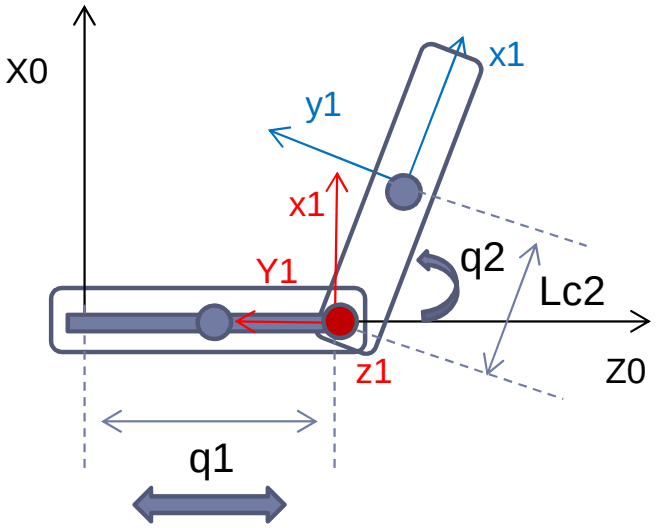
Jon Legarreta/ Raquel Martinez

1 pausua: energia zinetikoaren kalkulua (K)

Kate-maila 2: $K_2 = \frac{1}{2} m_2 v_2^T v_2 + \frac{1}{2} \omega_2^T I_2 \omega_2$

$$x = L_{c2} Sq2$$
$$y = 0$$
$$z = L_{c2} Cq2 + q1$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial q_3} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial q_3} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial q_3} \end{bmatrix} = \begin{bmatrix} 0 & L_{c2} Cq2 & 0 \\ 0 & 0 & 0 \\ 1 & -L_{c2} Sq2 & 0 \end{bmatrix}$$

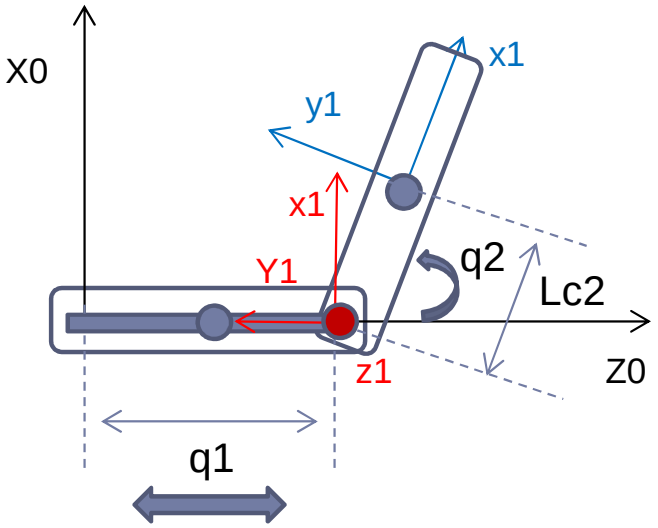


$$v_2 = \begin{bmatrix} 0 & L_{c2} Cq2 & 0 \\ 0 & 0 & 0 \\ 1 & -L_{c2} Sq2 & 0 \end{bmatrix} \begin{bmatrix} \dot{q1} \\ \dot{q2} \\ 0 \end{bmatrix} = \begin{bmatrix} L_{c2} Cq2\, \dot{q2} \\ 0 \\ \dot{q1} - L_{c2} Sq2\, \dot{q2} \end{bmatrix}$$

$$v_2^T v_2 = \begin{bmatrix} L_{c2} Cq2\, \dot{q2} & 0 & \dot{q1} - L_{c2} Sq2\, \dot{q2} \end{bmatrix} \begin{bmatrix} L_{c2} Cq2\, \dot{q2} \\ 0 \\ \dot{q1} - L_{c2} Sq2\, \dot{q2} \end{bmatrix} = \dot{q1}^2 + L_{c2}^2\, \dot{q2}^2 - 2\, \dot{q1} \dot{q2}\, L_{c2} Sq2$$

Jon Legarreta/ Raquel Martinez

2 kate-mailaren abiadura **angeluarra** : $\vec{w}_2 = \dot{q}_2 \vec{z}_0$



Terminoak batuz, 2 kate-mailaren energia zinetikoa:

$$K_2 = \frac{1}{2} m_2 \left[\dot{q1}^2 + L_{c2}{}^2 \dot{q2}^2 - 2\dot{q1}\dot{q2} L_{c2} Sq2 \right] + \frac{1}{2} I_2 \dot{q}_2{}^2$$

2 kate-mailaren energia potentziala: $U_1 = m_2 g\, L_{c2} Sq_2$

3. Pasua **Lagrangetarraren** kalkulua

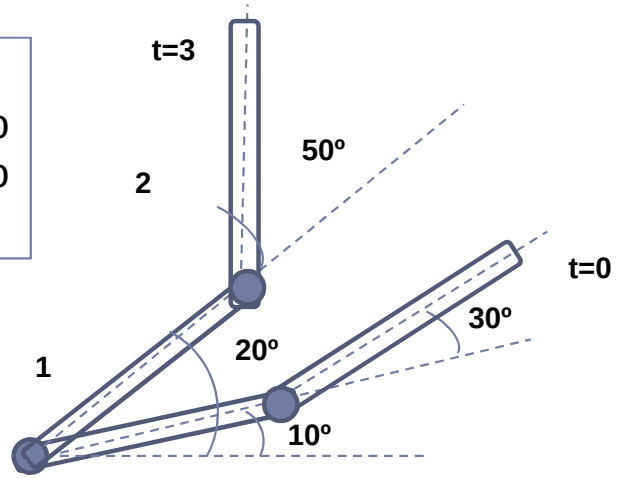
$$L(q,\dot{q}) = \frac{1}{2}\dot{q}^T D(q)\dot{q} - U(q) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}(q)\cdot\dot{q}_i\cdot\dot{q}_j - U(q)$$

$$L = \frac{1}{2} m_1 \frac{\dot{q_1}^2}{4} + \frac{1}{2} m_2 \left[\dot{q1}^2 + L_{c2}{}^2 \dot{q2}^2 - 2\dot{q1}\dot{q2} L_{c2} Sq2 \right] + \frac{1}{2} I_2 \dot{q}_2{}^2 - m_2 g\, L_{c2} Sq_2$$

Jon Legarreta/ Raquel Martinez

ARIKETAK

6.2 ariketa
Demagun 2 biraketa artikulazioko robota. Irudian hasierako eta amaierako posizioak adierazten dira 3 segundotan. Beharrezkoak diren interpolazio kubikoak kalkulatu.



1. kate-maila

$a = q_{ini} = 10 \qquad b = 0$

$$c = \frac{3(q_{fin} - q_{ini})}{t_{fin}^2} = \frac{3(20-10)}{3^2} = 3.33$$

$$d = \frac{-2(q_{fin} - q_{ini})}{t_{fin}^3} = \frac{-2(20-10)}{3^3} = -0.74$$

$$\boxed{q(t) = 10 + 3.33t^2 - 0.74t^3}$$

2. Kate-maila

$a = q_{ini} = 30 \qquad b = 0$

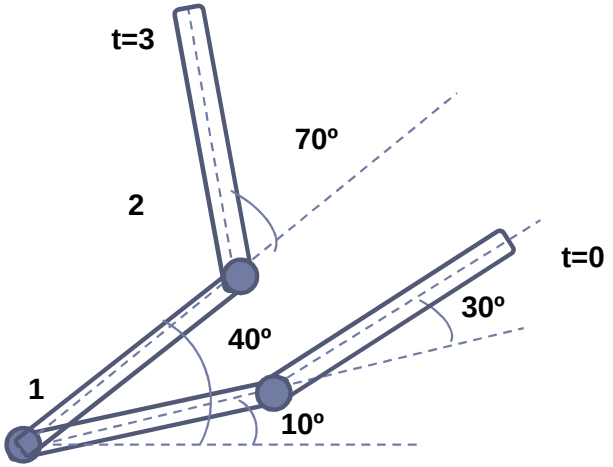
$$c = \frac{3(q_{fin} - q_{ini})}{t_{fin}^2} = \frac{3(50-30)}{3^2} = 6.67$$

$$d = \frac{-2(q_{fin} - q_{ini})}{t_{fin}^3} = \frac{-2(50-30)}{3^3} = -1.48$$

$$\boxed{q(t) = 30 + 6.67t^2 - 1.48t^3}$$

Jon Legarreta/ Raquel Martinez

6.3 ariketa
Demagun 2 biraketa-artikulazioko robota. Irudian hasierako eta amaierako posizioak adierazten dira 3 segundotan. Kalkulatu **doiketa parabolikoekin** beharrezkoak diren interpolazio **linealak**, 40º/s^2 azelerazioa baldin badaukate.



1 kate-maila

Datuak:

$$t_p = 1.5 - \frac{\sqrt{40^2 1.5^2 - 40(40-10)}}{40} = 0.27\ s$$

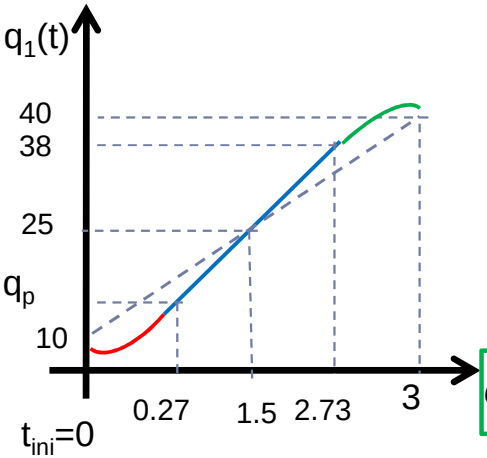
$$\beta = \ddot{q}\, t_p = 40 \cdot 0.27 = 10.8^o / s$$

$$a_{III} = q_{fin} + \frac{\ddot{q}_{III}}{2} t_{fin}^2 = 40 - \frac{40}{2} 9 = -140$$

$$q_p = \alpha = 10 + 20 t_p^{\ 2} = 11.458^o$$

$$t_{fin} = 3\,s$$
$$t_m = 1.5\,s$$
$$q_{ini} = 10^\circ$$
$$q_{fin} = 40^\circ$$
$$q_m = 25^\circ$$
$$\ddot{q} = 40^o / s^2$$
$$\ddot{q}_{III} = -40^o / s^2$$

$$c_{III} = \frac{\ddot{q}_{III}}{2} = -20$$
$$b_{III} = -\ddot{q}\, t_{fin} = 120$$



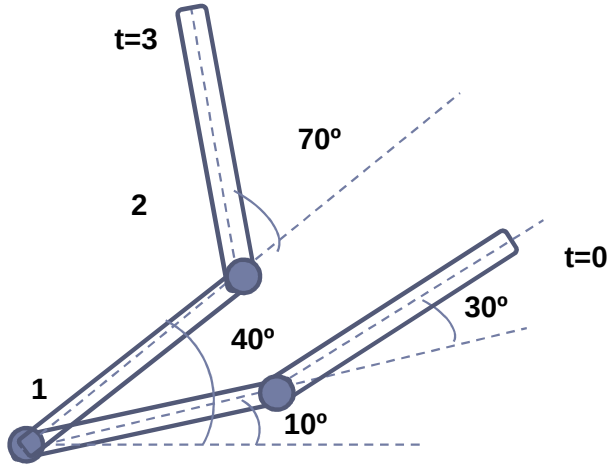
$$q_I(t) = q_{ini} + \frac{\ddot{q}}{2} t^2 = 10 + 20 t^2 \quad \rightarrow \quad t \in (0\ ,\ 0.27\ s)$$

$$q_{II}(t) = \alpha + \beta (t - t_p) = 11.458 + 10.8(t - 0.27) \quad \rightarrow \quad 0.27 < t < 2.73$$

$$q_{III}(t) = a_{III} + b_{III}\, t + c_{III}\, t^2 = -140 + 120 t - 20 t^2 \quad \rightarrow \quad 2.73 < t < 3$$

Jon Legarreta/ Raquel Martinez

6.3 ariketa
Demagun 2 biraketa-artikulazioko robota. Irudian hasierako eta amaierako posizioak adierazten dira 3 segundotan. Kalkulatu **doiketa parabolikoekin** beharrezkoak diren interpolazio **linealak**, 40º/s^2 azelerazioa baldin badaukate.



2 kate-maila

Datuak:

$$t_{fin} = 3\,s$$
$$t_m = 1.5\,s$$
$$q_{ini} = 30^\circ$$
$$q_{fin} = 70^\circ$$
$$q_m = 50^\circ$$
$$\ddot{q} = 40^o / s^2$$
$$\ddot{q}_{III} = -40^o / s^2$$

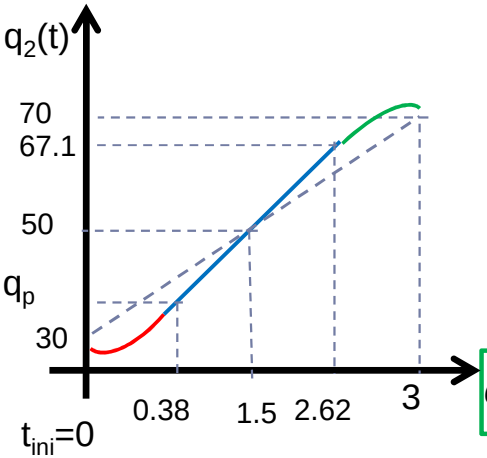
$$t_p = 1.5 - \frac{\sqrt{40^2 1.5^2 - 40(70-30)}}{40} = 0.38\,s$$

$$\beta = \ddot{q}\, t_p = 40 \cdot 0.38 = 15.28^o / s$$

$$a_{III} = q_{fin} + \frac{\ddot{q}_{III}}{2} t_{fin}^2 = 70 - \frac{40}{2} 9 = -110$$

$$q_p = \alpha = 30 + 20 t_p{}^2 = 32.89^o$$

$$c_{III} = \frac{\ddot{q}_{III}}{2} = -20$$
$$b_{III} = -\ddot{q}_{III}\, t_{fin} = 120$$



$$q_I(t) = q_{ini} + \frac{\ddot{q}}{2} t^2 = 30 + 20t^2 \quad \rightarrow \quad t \in (0,\ 0.38\ s)$$

$$q_{II}(t) = \alpha + \beta(t - t_p) = 32.89 + 15.28(t - 0.38) \quad \rightarrow \quad 0.38 < t < 2.62$$

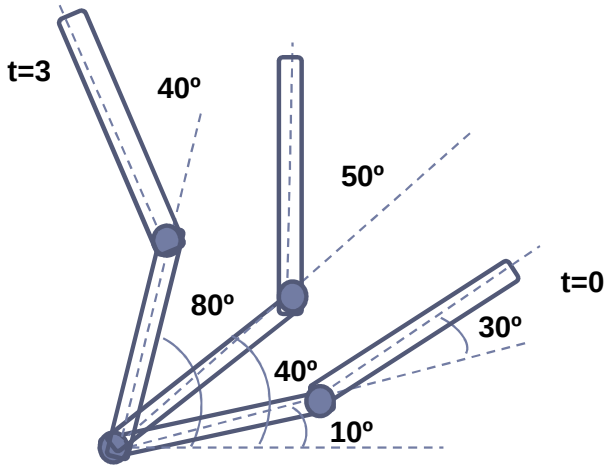
$$q_{III}(t) = a_{III} + b_{III}\, t + c_{III}\, t^2 = -110 + 120t - 20t^2 \quad \rightarrow \quad 2.62 < t < 3$$

Jon Legarreta/ Raquel Martinez

ARIKETAK

6.4 ariketa

Demagun 2 biraketa-artikulazioko robota. Irudian hasierako, bitarteko eta amaierako posizioak adierazten dira 2 segundotan. Bitarteko puntuetan kate-maila bien abiadura 5º/skoa izanik, kalkulatu beharrezkoak diren interpolazio kubikoak. Hasierako eta amaierako abiadurak zero direla kontsideratzen da.



1. kate-maila

I zatia

$$q_I(t) = a_I + b_I\, t + c_I\, t^2 + d_I\, t^3$$

$$a_I = q_{ini} = 10^o$$

$$b_I = \dot{q}_{ini} = 0$$

$$c_I = \frac{3(q_{fin} - q_{ini})}{t_{fin}^2} - \frac{2\dot{q}_{ini} + \dot{q}_{fin}}{t_{fin}} = \frac{3(40-10)}{1^2} - \frac{2\cdot 0 + 5}{1} = 85$$

$$d_I = \frac{-2(q_{fin} - q_{ini})}{t_{fin}^3} + \frac{\dot{q}_{fin} + \dot{q}_{ini}}{t_{fin}^2} = \frac{-2(40-10)}{1^3} + \frac{5+0}{1^2} = -55$$

II zatia

$$q_{II}(t) = a_{II} + b_{II}\, t + c_{II}\, t^2 + d_{II}\, t^3$$

$$a_{II} = q_{II\,ini} = 40^o$$

$$b_{II} = \dot{q}_{II\,ini} = 5$$

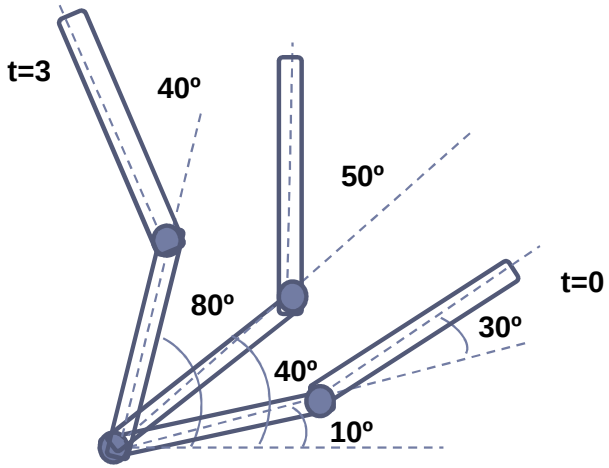
$$c_I = \frac{3(q_{fin} - q_{II\,ini})}{t_{fin}^2} - \frac{2\dot{q}_{II\,ini} + \dot{q}_{fin}}{t_{fin}} = \frac{3(80-40)}{1^2} - \frac{2\cdot 5 + 0}{1} = 110$$

$$d_I = \frac{-2(q_{fin} - q_{II\,ini})}{t_{fin}^3} + \frac{\dot{q}_{fin} + \dot{q}_{II\,ini}}{t_{fin}^2} = \frac{-2(80-40)}{1^3} + \frac{0+5}{1^2} = -75$$

ARIKETAK

6.4 ariketa

Demagun 2 biraketa-artikulazioko robota. Irudian hasierako, bitarteko eta amaierako posizioak adierazten dira 2 segundotan. Bitarteko puntuetan kate-maila bien abiadura 5º/skoa izanik, kalkulatu beharrezkoak diren interpolazio kubikoak. Hasierako eta amaierako abiadurak zero direla kontsideratzen da.



2. kate-maila

I zatia

$$q_I(t) = a_I + b_I\, t + c_I\, t^2 + d_I\, t^3$$

$$a_I = q_{ini} = 30^o$$

$$b_I = \dot{q}_{ini} = 0$$

$$c_I = \frac{3(q_{fin} - q_{ini})}{t_{fin}^2} - \frac{2\dot{q}_{ini} + \dot{q}_{fin}}{t_{fin}} = \frac{3(50-30)}{1^2} - \frac{2\cdot 0 + 5}{1} = 55$$

$$d_I = \frac{-2(q_{fin} - q_{ini})}{t_{fin}^3} + \frac{\dot{q}_{fin} + \dot{q}_{ini}}{t_{fin}^2} = \frac{-2(50-30)}{1^3} + \frac{5+0}{1^2} = -35$$

II zatia

$$q_{II}(t) = a_{II} + b_{II}\, t + c_{II}\, t^2 + d_{II}\, t^3$$

$$a_{II} = q_{II\,ini} = 50^o$$

$$b_{II} = \dot{q}_{II\,ini} = 5$$

$$c_I = \frac{3(q_{fin} - q_{II\,ini})}{t_{fin}^2} - \frac{2\dot{q}_{II\,ini} + \dot{q}_{fin}}{t_{fin}} = \frac{3(40-50)}{1^2} - \frac{2\cdot 5 + 0}{1} = -40$$

$$d_I = \frac{-2(q_{fin} - q_{II\,ini})}{t_{fin}^3} + \frac{\dot{q}_{fin} + \dot{q}_{II\,ini}}{t_{fin}^2} = \frac{-2(40-50)}{1^3} + \frac{0+5}{1^2} = 25$$

Jon Legarreta/ Raquel Martinez