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AIR POLLUTION

PROBLEMS (II)

SOLUTIONS

Problem 6.-

The diameter of the medium-efficiency standard cyclone (D_c) can be derived from the following equation:

$$Q = \frac{u}{w\,H} = \frac{u}{0.2D_c\;0.5D_c}$$

Where Q = flow rate

u= gas inlet velocity

w= width of cyclone inlet

h= height of the cyclone inlet

Taking into account the dimensions of a standard cyclone (w= 0.2D_c and h= 0.5D_c), flow rate is:

$$Q = \frac{u}{w\,H} = \frac{u}{0.2D_c\;0.5D_c}$$

Since flow rate is 1,000 $m^3 \cdot h^{-1}$,

$$\frac{1000\ m^3}{h} \cdot \frac{1\ h}{3600\ s} = \frac{\frac{10\ m}{s}}{0.2\ D_c\ 0.5\ D_c}$$

$$D_c = 0.5270\ m$$

The 50% collection efficiency (dp_c), that is, the diameter of particles collected with 50% efficiency, is:

$$dp_c = \sqrt{\frac{9\ \mu\ w}{2\ \pi\ N_e\ u\ (\rho_p - \rho_{gas})}}$$

Where μ= is the viscosity of the gas

w= width of cyclone inlet

Ne= number of revolutions

u= gas inlet velocity

ρ_p= density particles

ρ_{gas}= density gas

The number of revolutions (N_e) is

$$N_e = \frac{t_r\ u}{\pi\ D_c}$$

Where t_r= gas residence time

The residence time in the cyclone is given by

$$t_r = \frac{V_i}{Q} = \frac{0.2683\ m^3}{\frac{1000\ m^3}{h} \cdot \frac{1\ h}{3600\ s}} = 0.966\ s$$

The internal volume of the cyclone (V_i) is the cylinder's volume plus the cone's volume:

$$V_i = \pi \left(\frac{D_c}{2}\right)^2 \cdot 1.5\ D_c + \frac{1}{3}\pi\left(\frac{D_c}{2}\right)^2 \cdot 2.5\ D_c = 0.2683\ m^3$$

Thus,

$$t_r = \frac{0.2683\ m^3}{\frac{1000\ m^3}{h} \cdot \frac{1\ h}{3600\ s}} = 0.966\ s$$

$$N_e = \frac{0.966\ s \cdot \frac{10\ m}{s}}{\pi\ 0.5270\ m} = 5.83\ rev \cong 6\ rev$$

So,

$$dp_c = \sqrt{\frac{9\ \frac{1.8 \cdot 10^{-5}\ N\ s}{m^2}\ 0.1054\ m}{2\ \pi\ 6\ \frac{10\ m}{s}\left(\frac{1200\ kg}{m^3} - \frac{1.25\ kg}{m^3}\right)}} = 6.14 \cdot 10^{-6}\ m\ \frac{10^6\ \mu m}{1\ m} = \mathbf{6.14\ \mu m}$$

Problem 7.-

The gas inlet velocity (u) is

$$u = Q \cdot w \cdot h$$

Where Q =flow rate
w =width of inlet
h= height of inlet

Considering the dimensions of a standard cyclone and a flow rate of 4.0 $m^3 \cdot s^{-1}$, the velocity can be approximated by:

$$u = Q\ 0.2\ D_c \cdot 0.5\ D_c = \frac{4\ m^3}{s}\ 0.2\ D_c \cdot 0.5\ D_c = \frac{160\ m}{s}$$

The 50% collection efficiency (dp_c) is given by:

$$dp_c = \sqrt{\frac{9\ \mu\ w}{2\ \pi\ N_e\ u\ (\rho_p - \rho_{gas})}}$$

Where μ= is the viscosity of the gas
N_e= number of revolutions
ρ_p= density particles
ρ_{gas}= density gas

The number of revolutions (N_e)

$$N_e = \frac{t_r\ u}{\pi\ D_c}$$

The residence time (t_r) is

$$t_r = \frac{V_i}{Q}$$

The internal volume of the cyclone (V_i) is the sum of the cylinder's volume and the cone's volume:

$$V_i = \pi\left(\frac{D_c}{2}\right)^2 \cdot\ 1.5\ D_c + \frac{1}{3}\ \pi\left(\frac{D_c}{2}\right)^2 \cdot\ 2.5\ D_c = 0.2290\ m^3$$

So,

$$t_r = \frac{0.2290\ m^3}{\frac{4\ m^3}{s}} = 0.0573\ s$$

$$N_e = \frac{0.0573\ s \cdot \frac{160\ m}{s}}{\pi\ 0.5\ m} = 5.83\ rev\ \cong 6\ rev$$

Considering the following equation and the width of the cyclone (0.2Dc), the '50% cut diameter' (that is, the diameter of particles collected with 50% efficiency) is:

$$dp_c = \sqrt{\frac{9\ \frac{1.8\cdot 10^{-5}\ Pa}{s}\ 0.1\ m}{2\ \pi\ 6\ \frac{160\ m}{s}\left(\frac{800\ kg}{m^3} - \frac{1.2\ kg}{m^3}\right)}} = 1.8589\cdot 10^{-6}\ m\ \frac{10^6\ \mu m}{1\ m} = 1.8589\ \mu m$$

Finally, the efficiency of collection of 10 µm diameter particles is given by the relationship developed by Lapple:

$$\frac{dp}{dp_c} = \frac{10}{1.8589} = 0.9665\cdot 100 = \mathbf{96.65}\%$$

Problem 8.-

Considering that the pulverized coal-fired steam power plant burns 5 tons of coal per day, it releases 150 m^3 of gases per kilogram of coal and burning of a ton of coal entails the release of 5 kg of particles, the concentration of particles in the exhaust gases is:

$$\frac{5\ kg\ PM}{t\ coal}\cdot\frac{1\ t\ coal}{10^3\ kg}\cdot\frac{1\ kg\ coal}{150\ m^3\ gases} = \frac{3.33\ 10^{-5}\ kg\ PM}{m^3}$$

$$\frac{3.33\ 10^{-5}\ kg\ PM}{m^3}\cdot\frac{10^3\ kg}{1\ kg} = \mathbf{0.0333\ \frac{g\ PM}{m^3}}$$

A plate-type electrostatic precipitator (ESP) is going to be installed in order to attract and collect these particles. The collecting plates of the selected model are 4 m long and 1.5 m tall in the direction of flow. Therefore, area of each collecting plate, considering both sides, is:

$$A_{plate} = 4\ m \cdot 1.5\ m \cdot 2 = 12\ m^2$$

Assuming that the migration velocity of the particles emitted by this process would be about 5 $m \cdot min^{-1}$, the total collection area required to achieve an overall collection efficiency of 99%can be calculated applying Deutsch-Anderson equation:

$$\eta = 1 - e^{\left(\frac{-A\,w}{Q}\right)}$$

Where η = collection efficiency
A= total collection area
w = migration velocity
Q= volumetric gas flow

Rearranging:

$$A = \frac{-Q}{w}\ln(1 - \eta)$$

$$A = \frac{\frac{150\ m^3}{kg\ coal} \cdot \frac{5000\ kg\ coal}{day} \cdot \frac{day}{24\ h} \cdot \frac{1\ h}{60\ min}}{\frac{5\ m}{min}} \ln(1 - 0.99) = 479.705\ m^2$$

Thus, the number of plates required would be:

$$\frac{A}{A_{plate}} + 1 = \frac{479.705\ m^2}{12\ m^2} + 1 = 40.975\ \text{plates} \cong \mathbf{41\ plates}$$

Problem 9.-

A cast-iron manufacturing process releases daily 10 t of particles to the atmosphere. The average density of these particles is 2,300 $kg \cdot m^{-3}$. The business owners and engineers of this factory are considering two systems to clean this particle-laden gas: an electrofilter (EF) and a cyclone. In order to help managers to make decisions, lets calculate the global efficiency and the fractional efficiency for PM_{10} of these two collectors.

To calculate global efficiency (that is, the total mass collected), we determine the total mass of each size range entering into these two devices. For example, for PM with a size between 0 and 10 μm is:

$$\frac{10\ t}{day} \cdot \frac{10^3\ kg}{1\ t} \cdot \frac{20\ kg\ PM_{0-10}}{1000\ kg\ PM} = \frac{2000\ kg\ PM_{0-10}}{day}$$

Then, we calculate the mass of each fraction collected in the device (that is, the product of the total mass input and the efficiency of collection of each size range). For example, for PM with a size between 0 and 10 μm is:

$$\frac{2000\ \text{kg PM}_{0-10}}{\text{day}} \cdot \frac{90\ \text{kg PM}_{0-10}\ \text{collected}}{100\ \text{kg PM}_{0-10}} = \frac{1800\ \text{kg PM}_{0-10}\ \text{collected}}{\text{day}}$$

Next, we estimate the mass of each size range not collected in these two devices (that is, the mass emitted to the atmosphere). For example, for PM with a size between 0 and 10 μm is:

$$\frac{2000\ \text{kg PM}_{0-10}}{\text{day}} - \frac{1800\ \text{kg PM}_{0-10}\ \text{collected}}{\text{day}} = \frac{200\ \text{kg PM}_{0-10}\ \text{not collected}}{\text{day}}$$

Results are summarized in the following two tables:

Size particles (μm)	***0-10***	***10-20***	***20-44***	***> 44***
Mass (kg·day⁻¹)	2000	3500	3000	1500
Mass collected by the EF (kg·day⁻¹)	1800	3395	2985	1500
Mass not collected by the EF (kg·day⁻¹)	200	105	15	0

Size particles (μm)	***0-10***	***10-20***	***20-44***	***> 44***
Mass (kg·day⁻¹)	2000	3500	3000	1500
Mass collected by the cyclone (kg·day⁻¹)	1100	2730	2700	1485
Mass not collected by the cyclone (kg·day⁻¹)	900	770	300	15

The total mass of PM collected by the EF and the cyclone are:

$$1800 + 3395 + 2985 + 1500 = \frac{9680\ \text{kg PM collected EF}}{\text{day}}$$

$$1100 + 2730 + 2700 + 1485 = \frac{8015\ \text{kg PM collected EF}}{\text{day}}$$

The global efficiency of these two collectors is:

$$\eta_{\text{EF}} = \frac{9680\ \text{kg PM collected EF}}{10000\ \text{kg PM}} \cdot 100 = \mathbf{96.8}\%$$

$$\eta_{\text{cyclone}} = \frac{8015\ \text{kg PM collected EF}}{10000\ \text{kg PM}} \cdot 100 = \mathbf{80.15}\%$$

To estimate the mass of PM_{10} released to the atmosphere, we subtract the mass of PM with a diameter < 10 micrometers collected to the mass input of this size range:

$$\frac{2000\ \text{kg PM}_{0-10}}{\text{day}} - \frac{1800\ \text{kg PM}_{0-10}\ \text{collected EF}}{\text{day}} = \frac{200\ \text{kg PM}_{0-10}\ \text{not collected EF}}{\text{day}}$$

$$\frac{2000\ \text{kg PM}_{0-10}}{\text{day}} - \frac{1100\ \text{kg PM}_{0-10}\ \text{collected cyclone}}{\text{day}} = \frac{900\ \text{kg PM}_{0-10}\ \text{not collected cyclone}}{\text{day}}$$

The mass of PM_{10} emitted to the atmosphere would be more than 4 times higher if a cyclone were installed.

To compute the number of PM_{10} particles that would be released daily to the atmosphere, we first calculate the mass of a particle with a 5 μm diameter (average of the size range 0 – 10 μm):

$$\rho_{\text{particle}} \cdot \frac{\pi\, d^3_{\text{particle}}}{6} = \frac{2300\text{ kg}}{\text{m}^3} \cdot \frac{\pi\,(5 \cdot 10^{-6}\text{ m})^3}{6} = \frac{1.5\ 10^{-13}\ \text{kg}}{\text{particle}}$$

The number of particles emitted into the atmosphere would be:

$$\frac{200\text{ kg PM}_{0-10}\text{ not collected EF}}{\text{day}} \cdot \frac{\text{particle}}{1.5\ 10^{-13}\ \text{kg}} = \mathbf{\frac{13 \cdot 10^{14}\ particles\ PM_{0-10}}{day}}$$

$$\frac{900\text{ kg PM}_{0-10}\text{ not collected cyclone}}{\text{day}} \cdot \frac{\text{particle}}{1.5\ 10^{-13}\ \text{kg}} = \mathbf{\frac{60 \cdot 10^{14}\ particles\ PM_{0-10}}{day}}$$

So, we can conclude that the cyclone is less efficient removing small particles than the EF.

Problem 10.-

To compute the global collection efficiency we multiply the fraction of each size range (m_i, 2nd column) by the collection efficiency for that size (n_i, 3rd column) in percentage and we sum the individual values (4th column). The result is 73.5%.

range diameter (μm)	m_i (%)	n_i (%)	$m_i \cdot (ni/100)$ (%)	$m_i \cdot [(100\text{-}ni)/100]$ (%)
< 5	10	20	2	8
5 – 10	15	40	6	9
10 – 20	35	80	28	7
20 – 40	20	90	18	2
40 – 80	10	95	9.5	0.5
> 80	10	100	10	0

The percentage of the particles emitted to the atmosphere can be obtained multiplying the fraction of each size range by the result of subtracting the fractional efficiency to 100, or, alternatively, subtracting the global efficiency to 100. The result is 26.5%.

To obtain the mass of dust of PM_{20} in the emission, we sum the percentage of dust emitted corresponding to size ranges below this diameter (20 μm):

$$8 + 9 + 7 = 24$$

Finally, we estimate the mass of dust with a diameter < 20 μm from the ratio between this result and the percentage of total particles emitted to the atmosphere:

$$\frac{24}{26.5} \cdot 100 = \mathbf{90.56}\%$$