

P9

PRACTICE 9: GRAPHICAL REPRESENTATION OF SURFACES

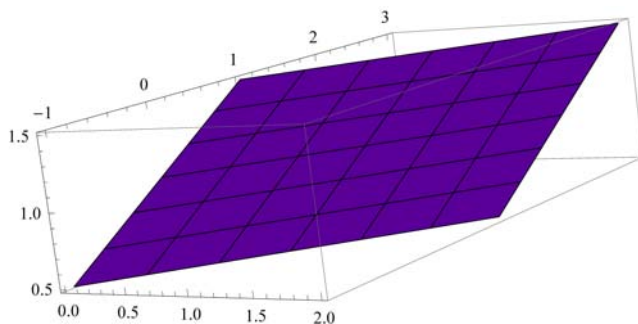
▼ Proposed Exercise P- 9.1

Plot the plane that contains the points $P_1(1,1,0)$, $P_2(0,0,-1)$ and $P_3(1,2,1/2)$ using its parametric equations.

▼ Resolution P- 9.1

★ Plane

```
Clear["Global`*"]
{w1, w2} = {{1, 1, 0}, {0, 1, 1/2}};
ParametricPlot3D[{1, 1, 1} + u w1 + v w2, {u, -1, 1},
  {v, -1, 1}, Mesh → 5, BoundaryStyle → Black, PlotStyle → Purple]
```

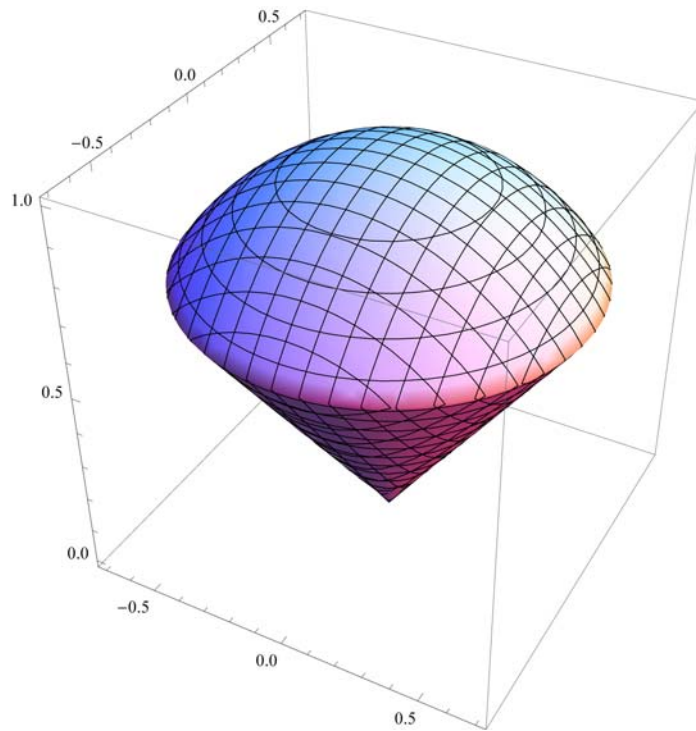


▼ Proposed Exercise P- 9.2

Given the cone $x^2 + y^2 = z^2$ and the sphere $x^2 + y^2 + z^2 = 1$, make the graphical representation of their intersection.

▼ Resolution P- 9.2

```
RegionPlot3D[x^2 + y^2 + z^2 < 1 && x^2 + y^2 < z^2,  
{x, -1, 1}, {y, -1, 1}, {z, 0, 1}, PlotPoints -> 35, PlotRange -> All]
```

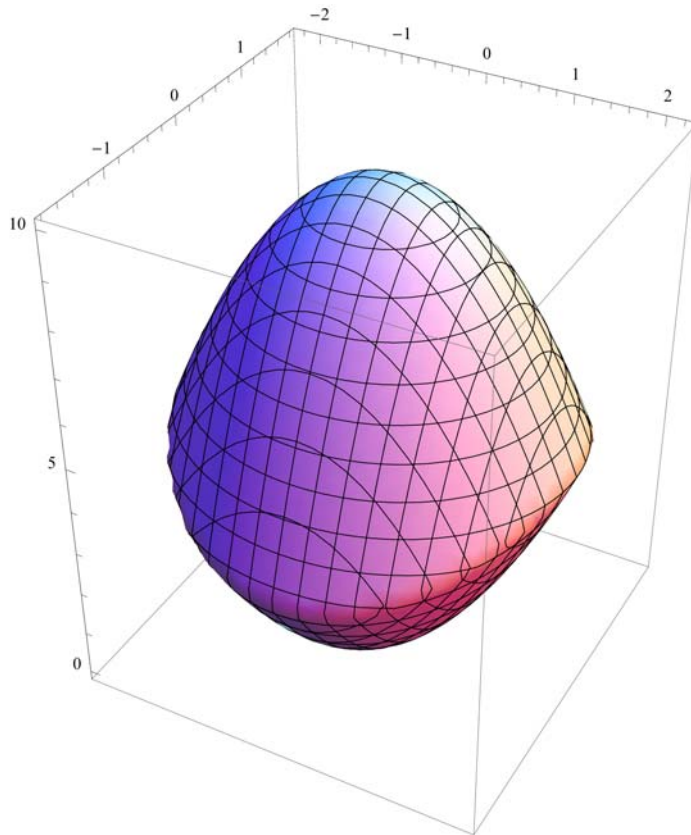
**▼ Proposed Exercise P- 9.3**

Make the graphical representation of the interior part of the following paraboloids:

$$z = x^2 + y^2 \text{ and } z = 10 - x^2 - 2y^2$$

▼ Resolution P- 9.3

```
RegionPlot3D[x^2 + y^2 < z && 10 - x^2 - 2 y^2 > z, {x, -3, 3}, {y, -2, 2},
{z, 0, 10}, PlotPoints -> 35, BoxRatios -> {2, 2, 2.5}, PlotRange -> All]
```



▼ Proposed Exercise P- 9.4

Make the graphical representation of the domain that is exterior to the cylinder $y = x^2 + y^2$, interior to the sphere $x^2 + y^2 + z^2 = 1$ and delimited by the plane $z = 0$ in the second quadrant.

▼ Resolution P- 9.4

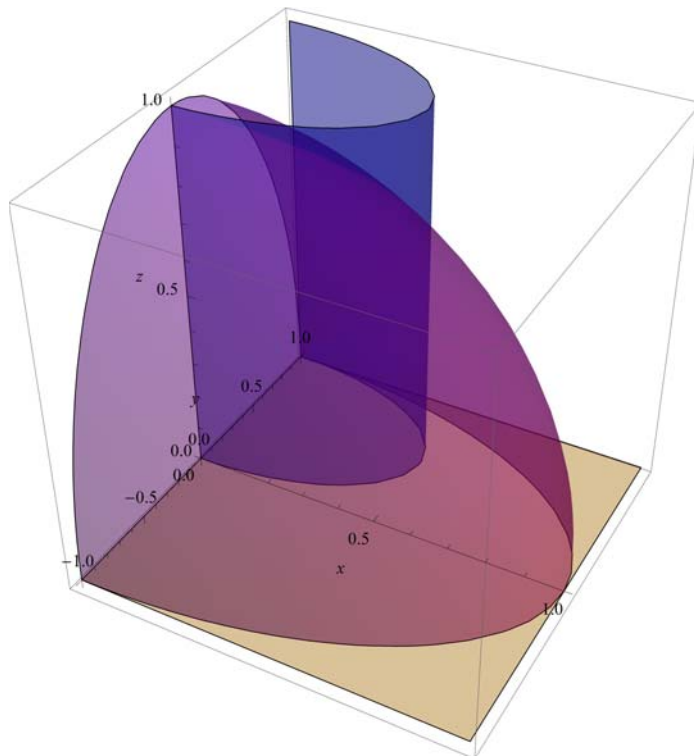
$$\text{cil}[\mathbf{x}_-, \mathbf{y}_-, \mathbf{z}_-] = -y + x^2 + y^2$$

$$x^2 - y + y^2$$

$$\text{sf}[\mathbf{x}_-, \mathbf{y}_-, \mathbf{z}_-] = x^2 + y^2 + z^2 - 1$$

$$-1 + x^2 + y^2 + z^2$$

```
ContourPlot3D[{z == 0, -y + x^2 + y^2, x^2 + y^2 + z^2 - 1}, {x, 0, 1}, {y, -1, 1},
  {z, 0, 1}, BoxRatios -> {2, 2, 2}, Mesh -> False, AxesLabel -> {x, y, z},
  AxesOrigin -> {0, 0, 0}, ContourStyle -> {Directive[Orange, Opacity[0.4]],
  Directive[Blue, Opacity[0.5]], Directive[Purple, Opacity[0.5]]}]
```



▼ Proposed Exercise P- 9.5

Using parametric coordinates, make the graphical representation of the domain that is interior to the cylinder $1/2=x^2+y^2$ and which is delimited by the sphere $x^2+y^2+z^2=1$.

▼ Resolution P- 9.5

★ Graphical representation of the surfaces that delimitate each figure

```
Clear["Global`*"]
```

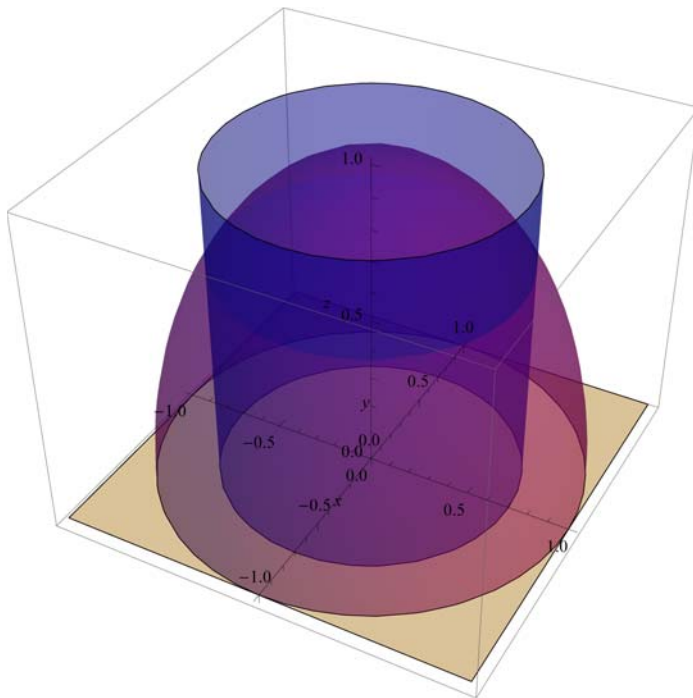
```
cir[x_, y_, z_] = x^2 + y^2 - 1 / 2
```

```
- 1 / 2 + x^2 + y^2
```

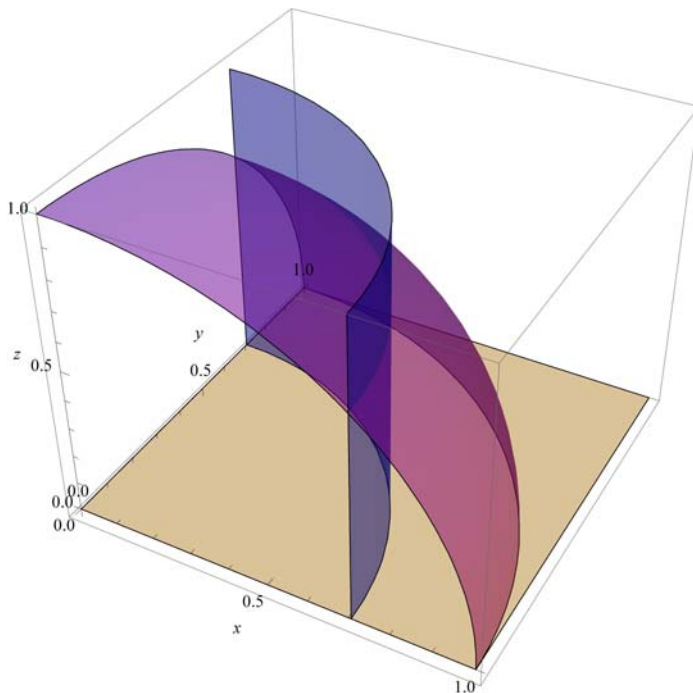
```
sph[x_, y_, z_] = x^2 + y^2 + z^2 - 1
```

```
- 1 + x^2 + y^2 + z^2
```

```
ContourPlot3D[{z == 0, x^2 + y^2 - 1/2 == 0, x^2 + y^2 + z^2 - 1 == 0}, {x, -1, 1}, {y, -1, 1},
{z, 0, 1}, BoxRatios -> {1, 1, 0.8}, Mesh -> False, AxesLabel -> {x, y, z},
AxesOrigin -> {0, 0, 0}, ContourStyle -> {Directive[Orange, Opacity[0.4]],
Directive[Blue, Opacity[0.5]], Directive[Purple, Opacity[0.5]]}]
```



```
ContourPlot3D[{z == 0, x^2 + y^2 - 1/2 == 0, x^2 + y^2 + z^2 - 1 == 0}, {x, 0, 1}, {y, 0, 1},
{z, 0, 1}, BoxRatios -> {1, 1, 0.8}, Mesh -> False, AxesLabel -> {x, y, z},
AxesOrigin -> {0, 0, 0}, ContourStyle -> {Directive[Orange, Opacity[0.4]],
Directive[Blue, Opacity[0.5]], Directive[Purple, Opacity[0.5]]}]
```



★ Change to cylindrical coordinates

Domain

$$\begin{aligned}x &= r \cdot \cos[t]; \\y &= r \cdot \sin[t];\end{aligned}$$

Limits of the radius "r": $r_1=0$ eta $r_2=1/\sqrt{2}$

Rango

$$\begin{aligned}ec &= \text{sph}[x, y, z] == 0 // \text{Simplify} \\r^2 + z^2 &= 1 \\ \text{Solve}[ec, z] // \text{Simplify} \\ \{ \{z \rightarrow -\sqrt{1-r^2}\}, \{z \rightarrow \sqrt{1-r^2}\} \}\end{aligned}$$

★ The domain in cylindrical coordinates

$$\{t, 0, 2 \text{ Pi}\}, \{r, 0, 1/\sqrt{2}\}, \{z, 0, \sqrt{1-r^2}\}$$

$$d = \{x, y, 0\}$$

$$\{r \cos[t], r \sin[t], 0\}$$

$$\text{reg1} = \{x, y, \sqrt{1-r^2}\}$$

$$\{r \cos[t], r \sin[t], \sqrt{1-r^2}\}$$

$$\begin{aligned}R1 &= \text{ParametricPlot3D}[\{\text{reg1}, d\}, \{t, 0, \text{Pi}\}, \{r, 0, 1/\sqrt{2}\}, \\ &\quad \text{Mesh} \rightarrow 5, \text{BoxRatios} \rightarrow \{1, 1, 1.4\}, \text{PlotRange} \rightarrow \{-1, 1\}, \{0, 1\}, \{0, 1\}\}, \\ &\quad \text{PlotStyle} \rightarrow \{\text{Directive}[\text{Purple}, \text{Opacity}[0.4]], \\ &\quad \quad \text{Directive}[\text{Orange}, \text{Opacity}[0.4]], \text{Directive}[\text{Blue}, \text{Opacity}[0.5]]\}];\end{aligned}$$

$$\text{reg2} = \{1/\sqrt{2} \cos[t], 1/\sqrt{2} \sin[t], r\};$$

$$\begin{aligned}R2 &= \text{ParametricPlot3D}[\{\text{reg2}\}, \{t, 0, \text{Pi}\}, \{r, 0, 1/\sqrt{2}\}, \\ &\quad \text{Mesh} \rightarrow 5, \text{BoxRatios} \rightarrow \{1, 1, 1.2\}, \text{PlotRange} \rightarrow \{-1, 1\}, \{0, 1\}, \{0, 1\}\}, \\ &\quad \text{PlotStyle} \rightarrow \{\text{Directive}[\text{Blue}, \text{Opacity}[0.4]], \\ &\quad \quad \text{Directive}[\text{Green}, \text{Opacity}[0.4]], \text{Directive}[\text{Blue}, \text{Opacity}[0.5]]\}];\end{aligned}$$

```
Show[{R1, R2}, AxesLabel -> {"x", "y", "z"},  
AxesOrigin -> {0, 0, 0}, PlotRange -> {{-1, 1}, {0, 1/√2}, {0, 1}}]
```

