P7

PRACTICE 7: REPRESENTATION OF FUNCTIONS OF SEVERAL VARIABLES

▼ Proposed Exercise P- 7.1

Study the existence of the limit of this function in the point (0,0):

$$f(x,y)=\frac{x\,y^2}{x^2+y^4}$$

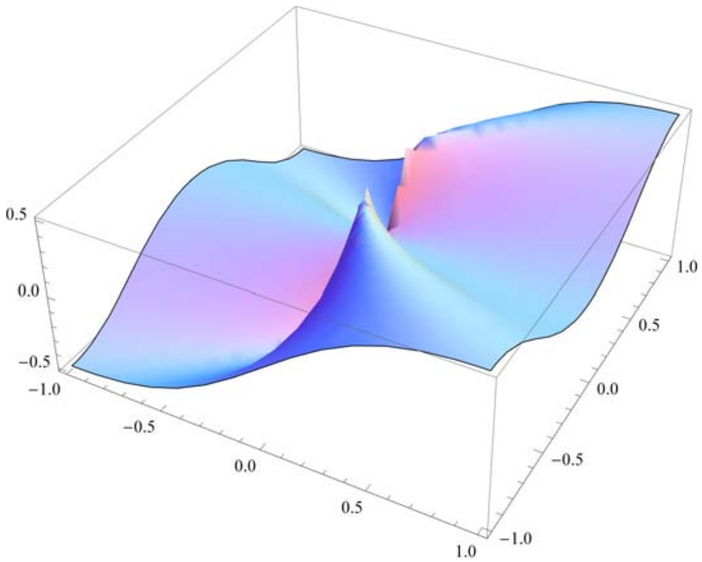
▼ Resolution P- 7.1

★ Definition and graphical representation of the function

```
Clear["Global`*"]
f[x_, y_] = (x * y^2) / (x^2 + y^4)
```

$$\frac{x\,y^2}{x^2+y^4}$$

```
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}, Mesh → False]
```



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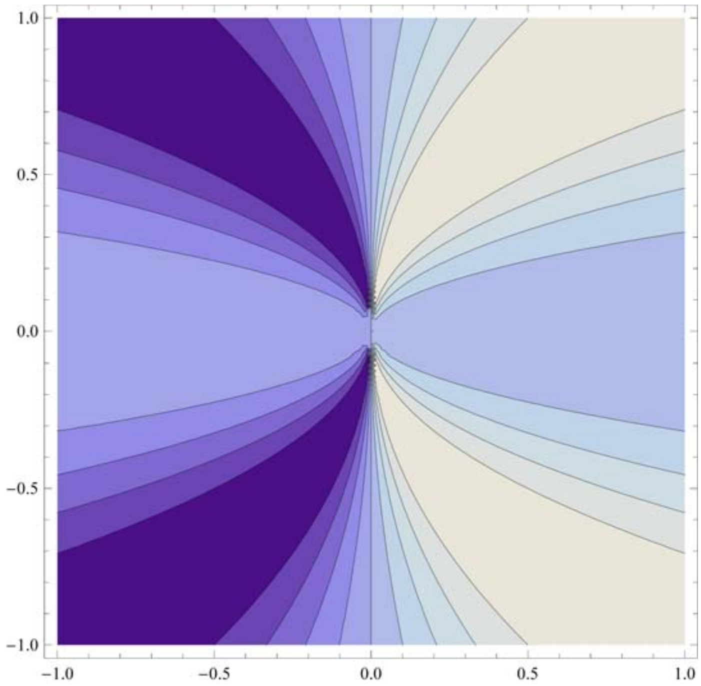
★ Plotting the level curves

```
ContourPlot[f[x, y], {x, -1, 1}, {y, -1, 1}]
```



As the level curves are parabolas, the limit will not exist

★ Limit calculations

Repeated limits

```
(* The marginal function f2[y] do not exist when x->0 *)
l1 = Limit[Limit[f[x, y], x -> 0], y -> 0]
0
l2 = Limit[Limit[f[x, y], y -> 0], x -> 0]
0
```

Directional limits

```
Limit[f[x, m * x], x -> 0]
0
(* The radial limits do not exist *)
```

Directional limits along the parabolas

```
Limit[f[m * y^2, y], y -> 0]
```

$$\frac{m}{1+m^2}$$

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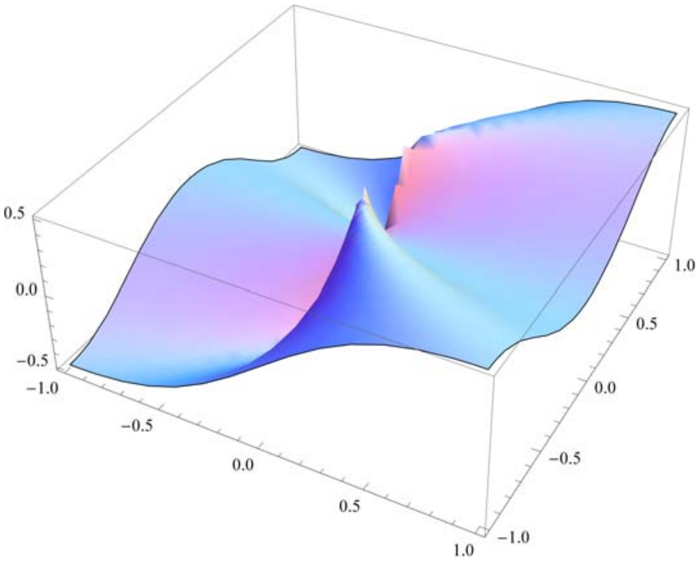
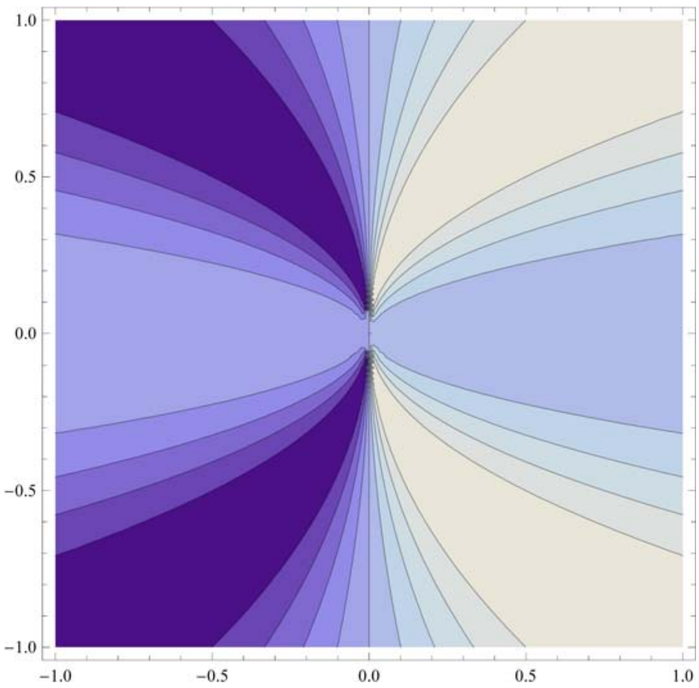
```
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}, Mesh → False]
```




```
ContourPlot[f[x, y], {x, -1, 1}, {y, -1, 1}]
```



▼ Proposed Exercise P- 7.2

Make the graphical representation of the hyperboloid $x^2 + y^2 - z^2 = 0.1$ Plot the sections of the parallel planes with respect to the axes.

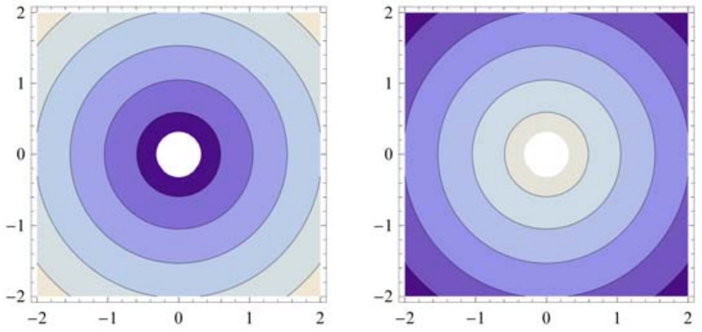
▼ Resolution P- 7.2

★ Definition of the two functions that define the hyperboloid and their graphical representation

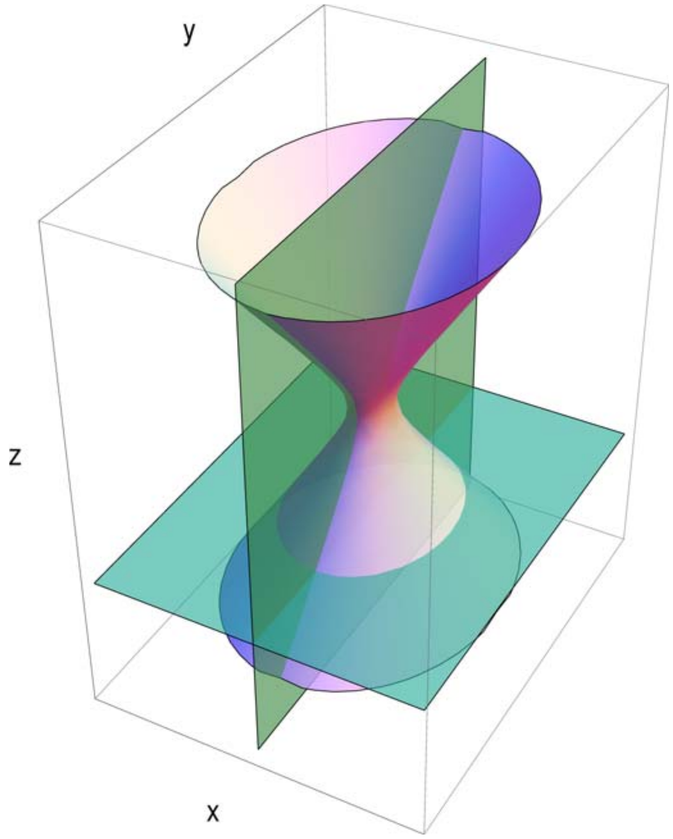
```
hyp1 = ContourPlot[Sqrt[(x^2 + y^2 - 0.1)], {x, -2, 2}, {y, -2, 2}];

hyp2 = ContourPlot[-Sqrt[(x^2 + y^2 - 0.1)], {x, -2, 2}, {y, -2, 2}];
```

```
Show[GraphicsGrid[{{hyp1, hyp2}}, Spacings → Scaled[0.2]]]
```




```
g2 = ContourPlot3D[{z == -1.2}, {x, -2.5, 2.5}, {y, -2.5, 2.5}, {z, -2.5, 2.5},
    ContourStyle → Directive[RGBColor[0.2, 0.8, 0.5], Opacity[0.6]], Mesh → False];
g3 = ContourPlot3D[{x == 0}, {x, -2.5, 2.5}, {y, -2.5, 2.5}, {z, -2.5, 2.5},
    ContourStyle → Directive[RGBColor[0.2, 0.8, 0.5], Opacity[0.6]], Mesh → False];
g4 = ContourPlot3D[x^2 + y^2 - 0.1 == z^2,
    {x, -2, 2}, {y, -2, 2}, {z, -2, 2}, Mesh → None, Ticks → None,
    ContourStyle → Directive[Opacity[0.8]], AxesLabel → {"X", "Z", "y"}];
g6 = Show[g2, g3, g4, Ticks → None, AxesLabel → {"X", "y", "Z"},
   BoxRatios → {1, 1.3, 1.5}, PlotRange → {-2.5, 2.5}]
```



▼ Proposed Exercise P- 7.3

Study the maximum and minimum values of this function graphically:

$$f(x,y)=x^3 + 3 * x * y^2 - 15 * x - 12 * y$$

▼ Resolution P- 7.3

★ Stationary points

Definition of the function

```
f[x_, y_] = x^3 + 3 * x * y^2 - 15 * x - 12 * y
```

$-15\,x + x^3 - 12\,y + 3\,x\,y^2$

Definition of the first order partial derivatives

```
dfx[x_, y_] = ∂x f[x, y]
```

$-15 + 3\,x^2 + 3\,y^2$

```
dfy[x_, y_] = ∂y f[x, y]
```

$-12 + 6\,x\,y$

```
gradf[x_, y_] = {dfx[x, y], dfy[x, y]}
```

$\{-15 + 3\,x^2 + 3\,y^2,\ -12 + 6\,x\,y\}$

Resolution of the system of equations

$$\text{gradf} = 0 \Longleftrightarrow \begin{cases} f_x(x, y) = 0 \\ f_y(x, y) = 0 \end{cases}$$

```
s = Solve[gradf[x, y] == {0, 0}]
```

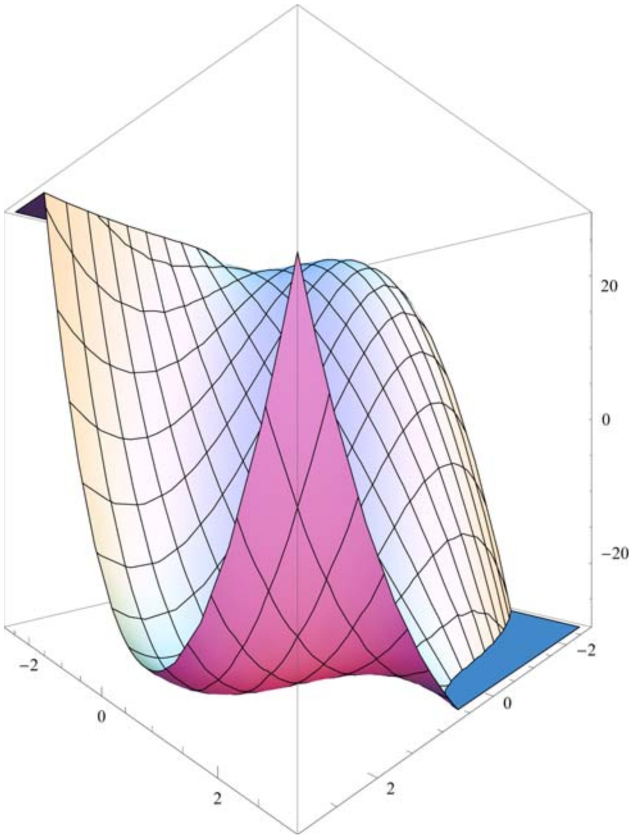
$\{\{x \to -2,\ y \to -1\},\ \{x \to -1,\ y \to -2\},\ \{x \to 1,\ y \to 2\},\ \{x \to 2,\ y \to 1\}\}$

Definition of the points that are the solution of the system of equations

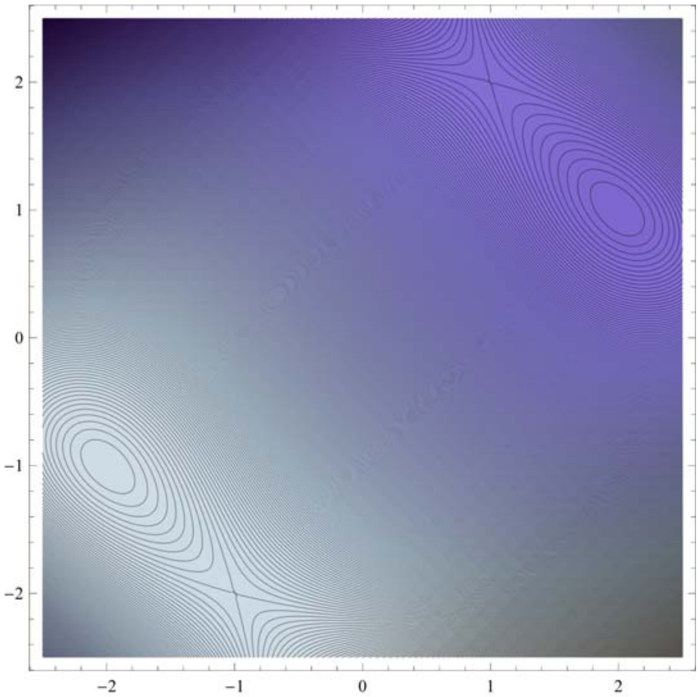
```
pc[n_] := {x, y} /. s[[n]];
```

⇒Graphical representation of the function and the level curves in a range that contain the point

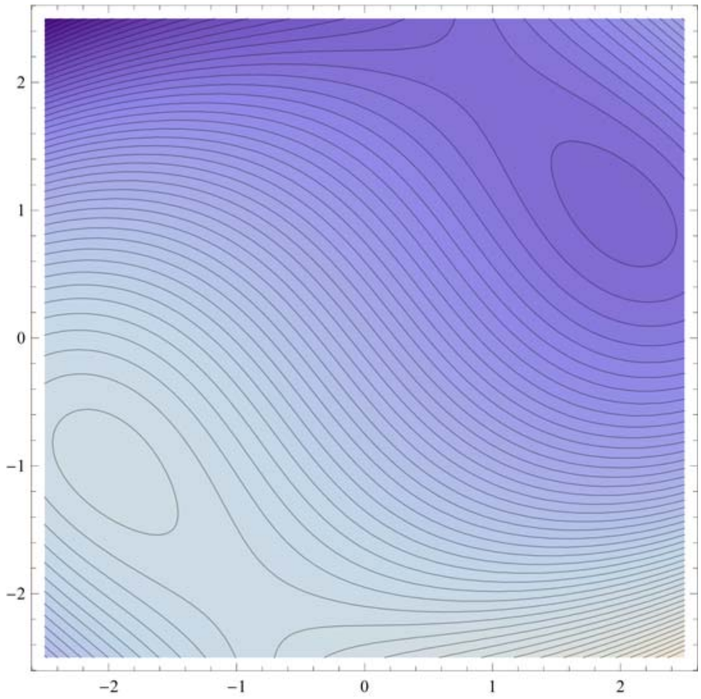
```
Plot3D[f[x, y], {x, -2.8, 2.8}, {y, -2.8, 2.8},
 PlotRange → {-29, 29}, BoxRatios → {1, 1, 1}, ViewPoint → {1, 1, 0}]
```




```
ContourPlot[f[x, y], {x, -2.5, 2.5}, {y, -2.5, 2.5},
 Contours → Function[{min, max}, Range[min, max, 0.2]]]
```

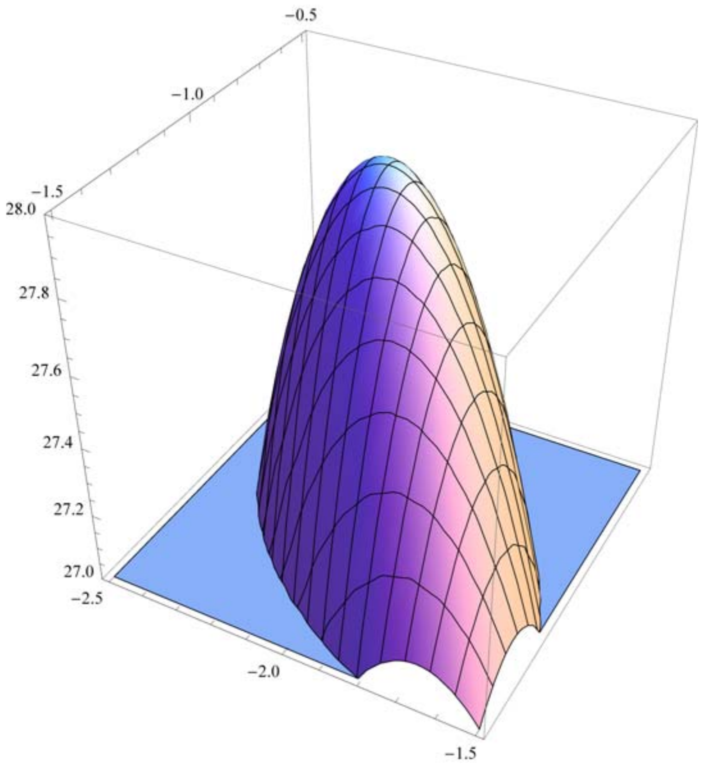


```
ContourPlot[f[x, y], {x, -2.5, 2.5}, {y, -2.5, 2.5}, Contours → 60]
```

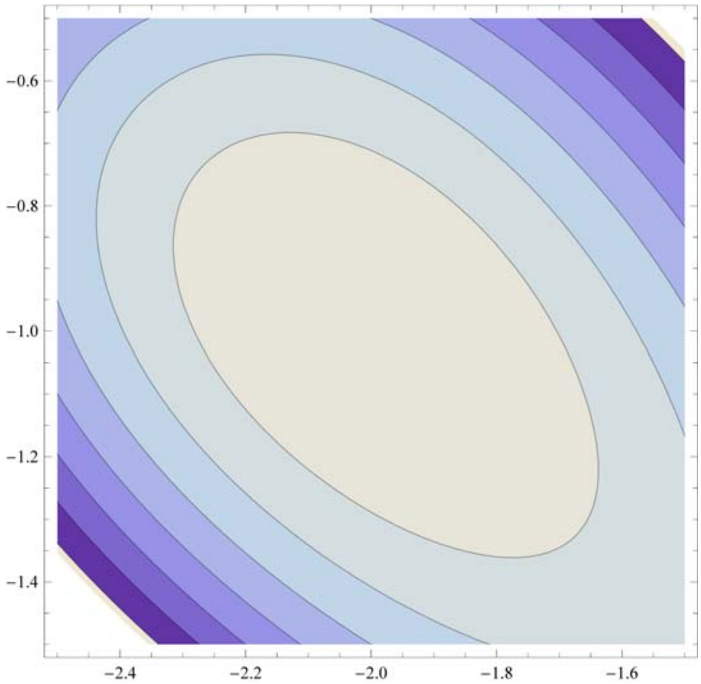


The point P[1] is a maximum

```
Plot3D[f[x, y], {x, -2.5, -1.5}, {y, -1.5, -0.5},
 PlotRange → {27, 28}, BoxRatios → {1, 1, 1}]
```

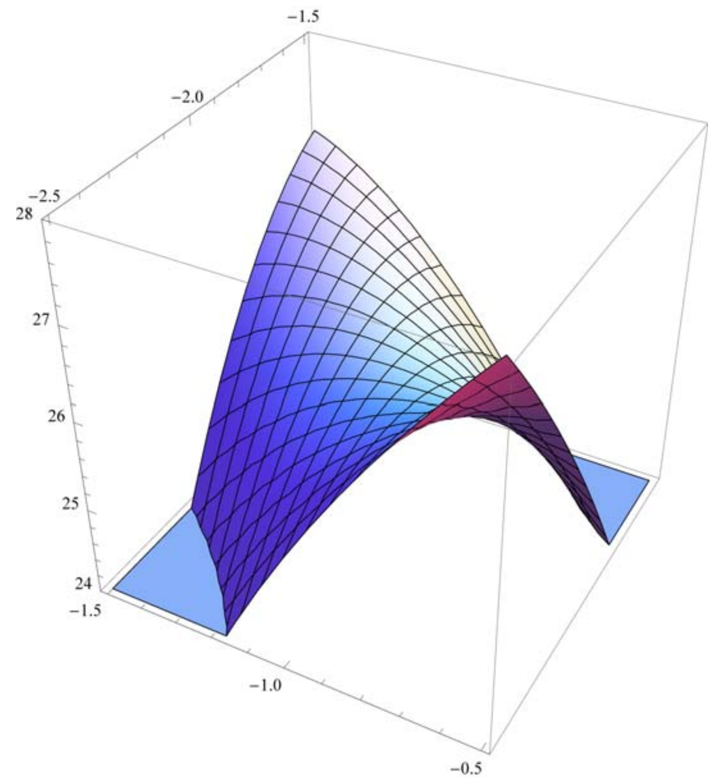


```
ContourPlot[f[x, y], {x, -2.5, -1.5}, {y, -1.5, -0.5}]
```

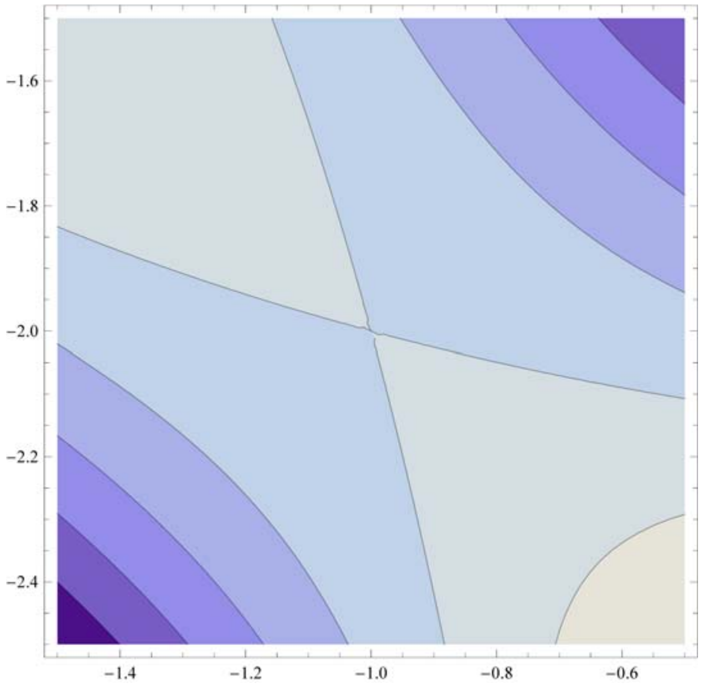


The point P[2] is a saddle point

```
Plot3D[f[x, y], {x, -1.5, -0.5}, {y, -2.5, -1.5},
 PlotRange → {24, 28}, BoxRatios → {1, 1, 1}]
```

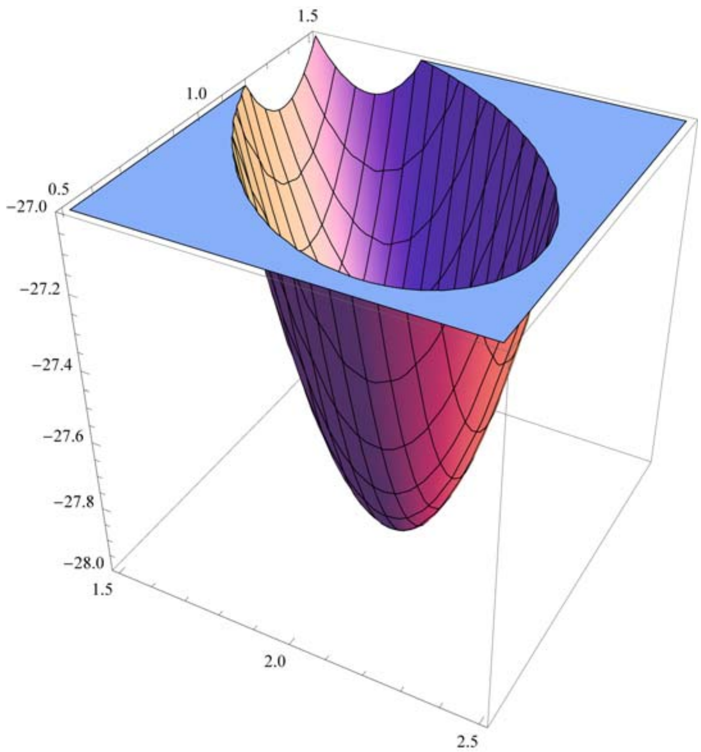


```
ContourPlot[f[x, y], {x, -1.5, -0.5}, {y, -2.5, -1.5}]
```

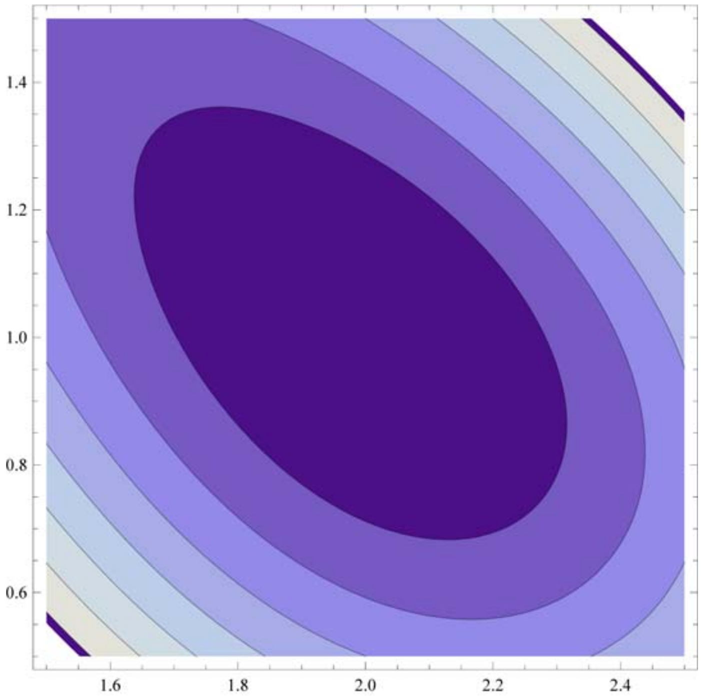


The point P[4] is a minimum

```
Plot3D[f[x, y], {x, 1.5, 2.5}, {y, 0.5, 1.5}, PlotRange → {-27, -28}, BoxRatios → {1, 1, 1}]
```



```
ContourPlot[f[x, y], {x, 1.5, 2.5}, {y, 0.5, 1.5}]
```



The point P[3] is a saddle point

```
f[1, 2]
```

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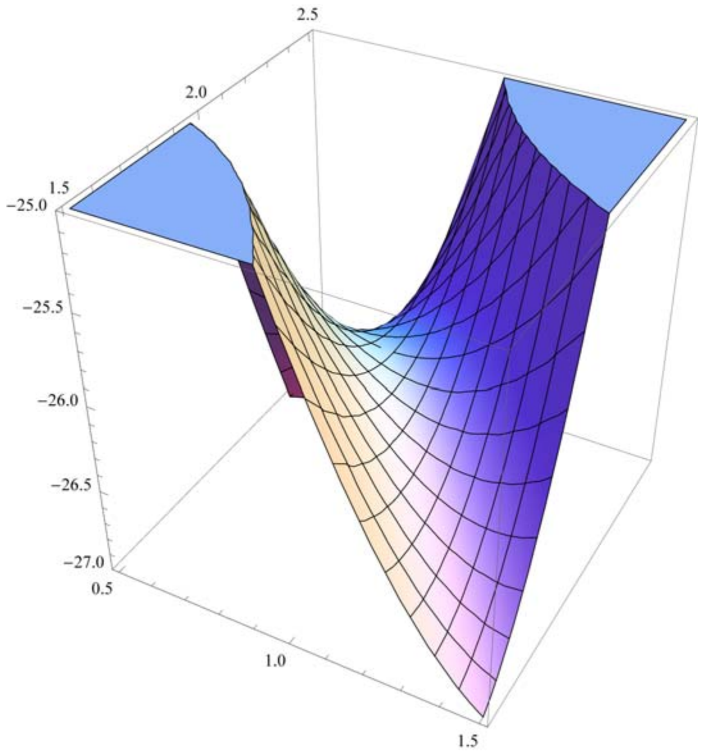
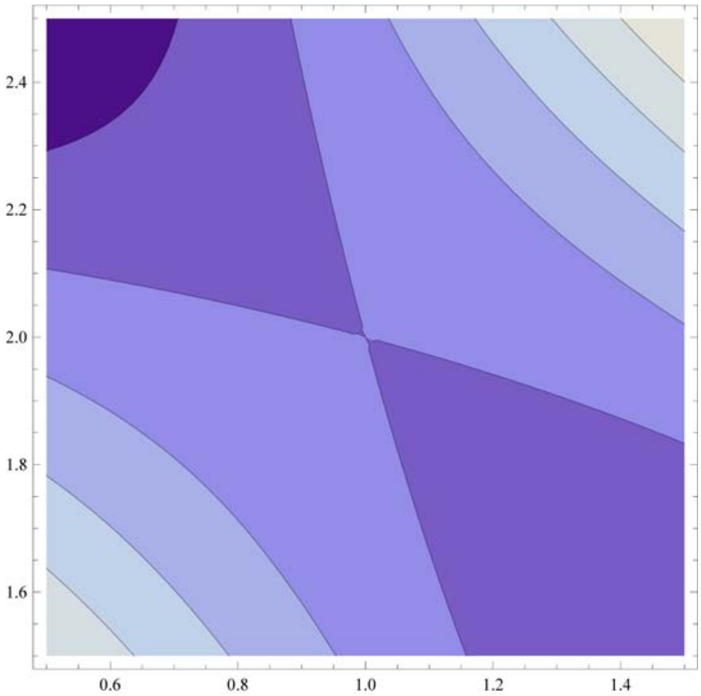
```
Plot3D[f[x, y], {x, 0.5, 1.5}, {y, 1.5, 2.5}, PlotRange → {-25, -27}, BoxRatios → {1, 1, 1}]
```



```
ContourPlot[f[x, y], {x, 0.5, 1.5}, {y, 1.5, 2.5}]
```



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