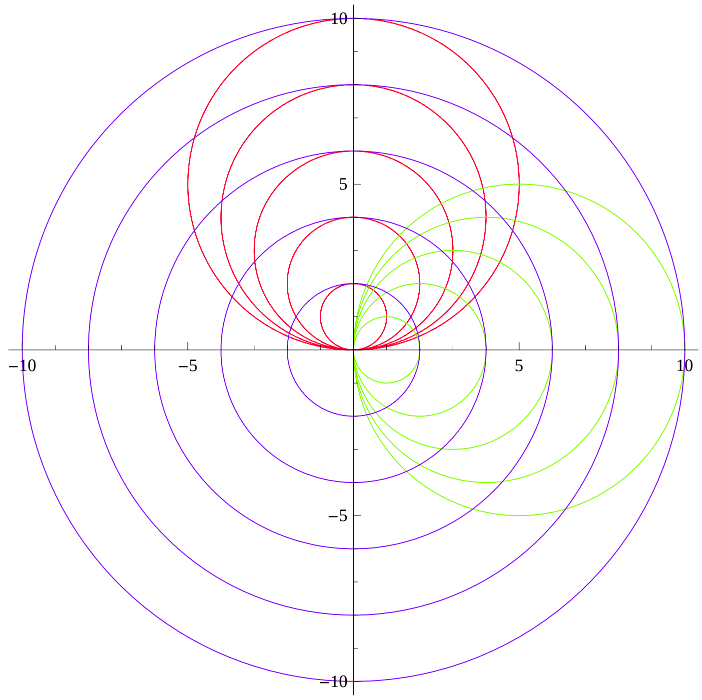
P5

PRACTICE 5: REPRESENTATION OF CURVES IN POLAR COORDINATES

▼ Proposed Exercise P-5.1

Plot the following family of circle curves:



▼ Resolution P-5.1

The equation of a circumference of centre (a,b) and radious c

```
Clear["Global`*"]
eq = (x - a) ^2 + (y - b) ^2 == c^2
```

$(-a+x)^2+(-b+y)^2 == c^2$

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1. Circle: Being its centre in the OY axis, and being (a,b)=(0,b), a=0, c=b

```
eq1 = eq /. {a → 0, c → b}
```

$x^2 + (-b + y)^2 == b^2$

```
polar1 = eq1 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$r[t]^2 == 2\, b\, r[t]\, Sin[t]$

```
r[t]^2 == 2 b r[t] Sin[t]
```

$r[t]^2 == 2\, b\, r[t]\, Sin[t]$

```
Solve[polar1, r[t]]
```

{{r[t] → 0}, {r[t] → 2 b Sin[t]}}

```
circ1[t_, b_] = 2 * b Sin[t];
```

2. Circle: Being its centre in the OY axis, and being (a,b)=(a,0), b=0, c=a

```
eq2 = eq /. {b → 0, c → a}
```

$(-a + x)^2 + y^2 == a^2$

```
polar2 = eq2 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$2\, a\, Cos[t]\, r[t] == r[t]^2$

```
Solve[polar2, r[t]]
```

{{r[t] → 0}, {r[t] → 2 a Cos[t]}}

```
circ2[t_, a_] = 2 * a Cos[t];
```

3. Circle: Being ist center in the origin, and being (a,b)=(0,0), a=0 and b=0

```
eq3 = eq /. {a → 0, b → 0}
```

$x^2 + y^2 == c^2$

```
polar3 = eq3 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$c^2 == r[t]^2$

```
Solve[polar3, r[t]]
```

{{r[t] → -c}, {r[t] → c}}

```
circ3[t_, a_] = a;
```


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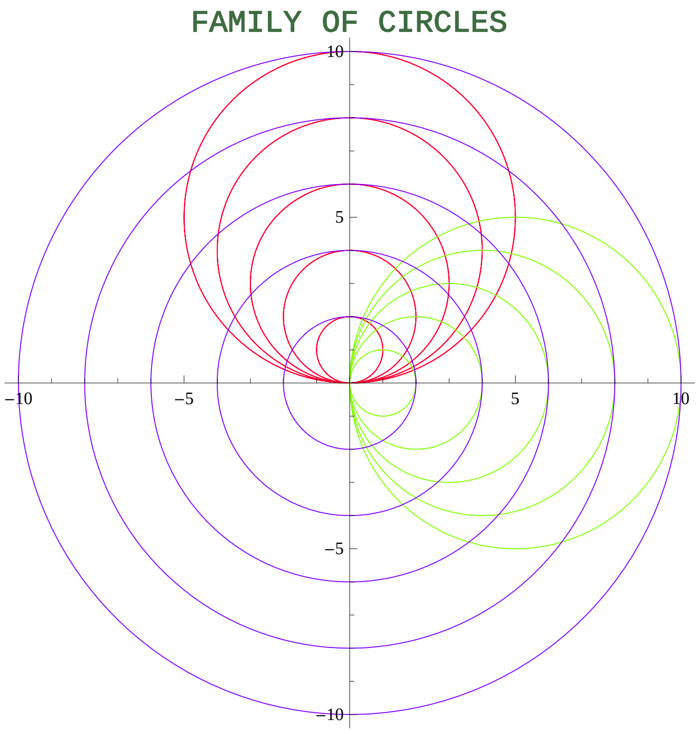
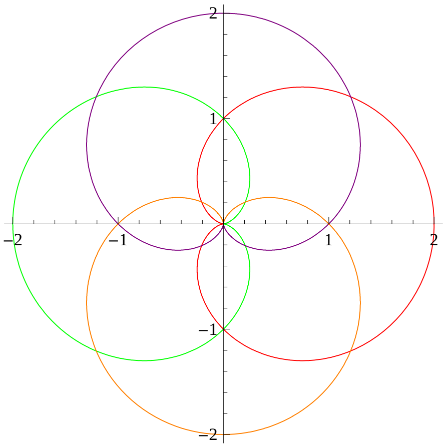
Family of circles

```
c1 = PolarPlot[Evaluate[Table[circ1[t, b], {b, 1, 5}]],
    {t, 0, 2 * π}, PlotStyle → RGBColor[1, 0, 0.2], PlotLabel → r == 2 b Sin[t]];
c2 = PolarPlot[Evaluate[Table[circ2[t, a], {a, 1, 5}]], {t, 0, π},
    PlotStyle → RGBColor[0.5, 1, 0], PlotLabel → r == 2 a Cos[t]];
c3 = PolarPlot[Evaluate[Table[circ3[t, c], {c, 2, 10, 2}]], {t, 0, 2 * π},
    PlotStyle → RGBColor[0.5, 0, 1], PlotLabel → r == c];
Show[c1, c2, c3, PlotLabel → Style["FAMILY OF CIRCLES",
    Bold, 14, RGBColor[0.3, 0.2, 0.7]]]
```



▼ Proposed Exercise P-5.2

Obtain the graph of the family of cardioids:



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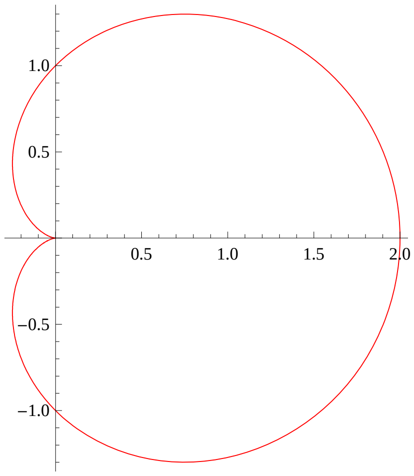
▼ Resolution P-5.2

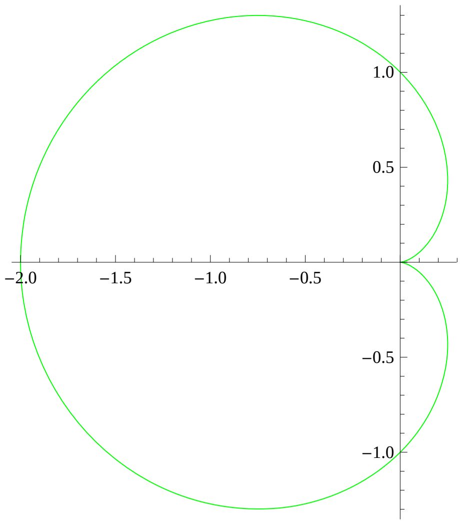
1st Cardioid

```
cardioid1[t_, a_] = a (1 + Cos[t]);
car1 = PolarPlot[cardioid1[t, 1], {t, 0, 2 π}, PlotStyle → Red]
```



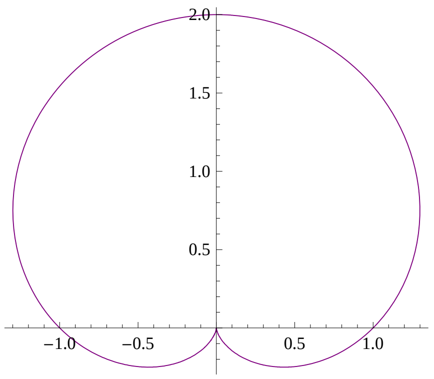
2nd Cardioid

```
cardioid2[t_, a_] = a (1 - Cos[t]);
car2 = PolarPlot[cardioid2[t, 1], {t, 0, 2 π}, PlotStyle → Green]
```



3rd Cardioid

```
cardioid3[t_, a_] = a (1 + Sin[t]);
car3 = PolarPlot[cardioid3[t, 1], {t, 0, 2 π}, PlotStyle → Purple]
```



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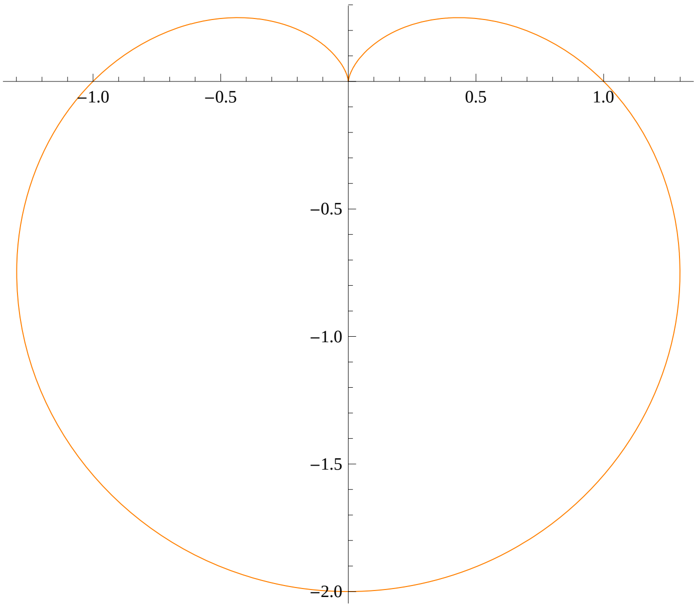
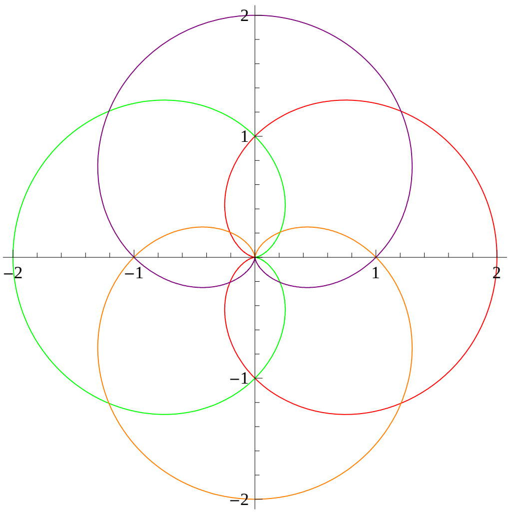
4th Cardioid

```
cardioid4[t_, a_] = a (1 - Sin[t]);
car4 = PolarPlot[cardioid4[t, 1], {t, 0, 2 π}, PlotStyle → Orange]
```



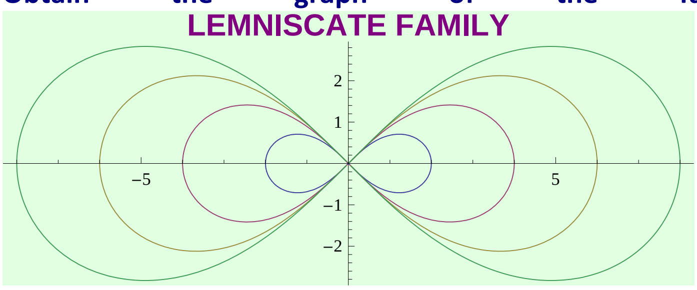
Family of cardioids

```
Show[car1, car2, car3, car4]
```



▼ Proposed Exercise P-5.3

Obtain the graph of the family of lemniscate curves:



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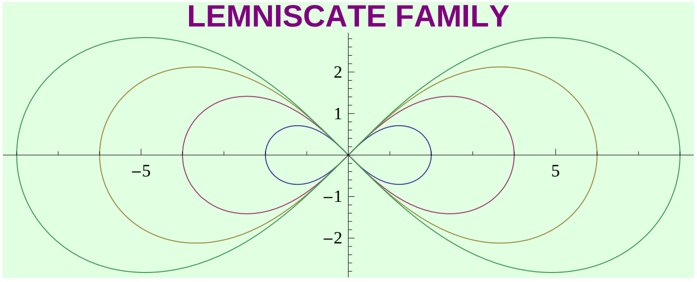
▼ Resolution P-5.3

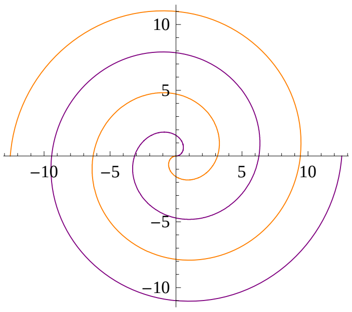
```
lemniscate[t_, a_] = a (Cos[2 * t]) ^ (1 / 2);

PolarPlot[Evaluate[Table[lemniscate[t, a], {a, 2, 8, 2}]], {t, 0, 2 Pi},
 PlotLabel → Style["LEMNISCATE FAMILY", Bold, 14], Background → LightGreen]
```



▼ Proposed Exercise P-5.4

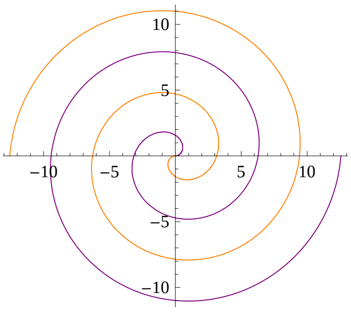
Obtain the graph of these spirals:



▼ Resolution P-5.4

```
esparq[θ_, a_, b_, x_] = a + b θ^ (1 / x);

PolarPlot[{esparq[θ, 0, 1, 1], esparq[θ, 0, -1, 1]},
 {θ, 0, 4 Pi}, PlotStyle → {Purple, Orange}]
```



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