9

GRAPHICAL REPRESENTATION OF SURFACES

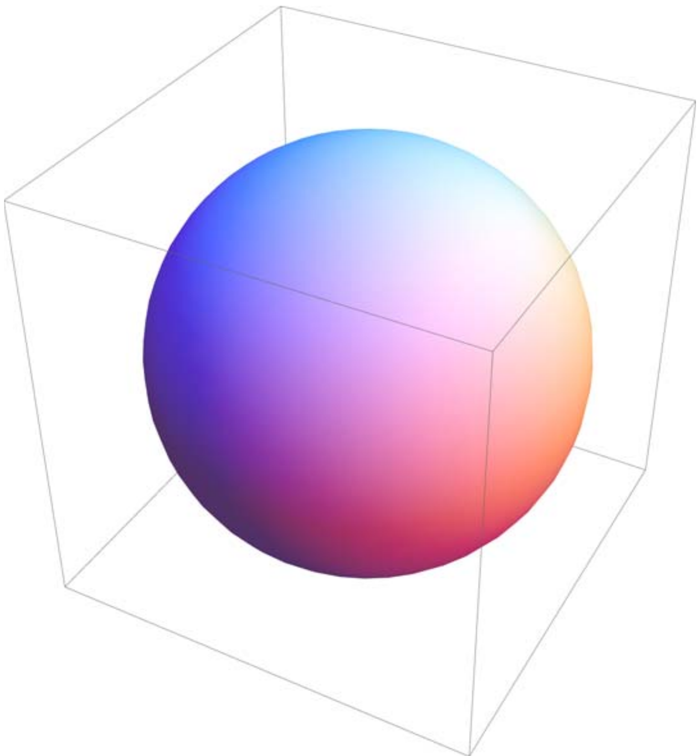
9.1. Figures defined in Mathematica

▼ Graphics3D[ ]

★ Spheres

Sphere of centre {1, 1, 1} and radius 2

```
Clear["Global`*"]
Graphics3D[Sphere[{0, 0, 0}, 2]]
```



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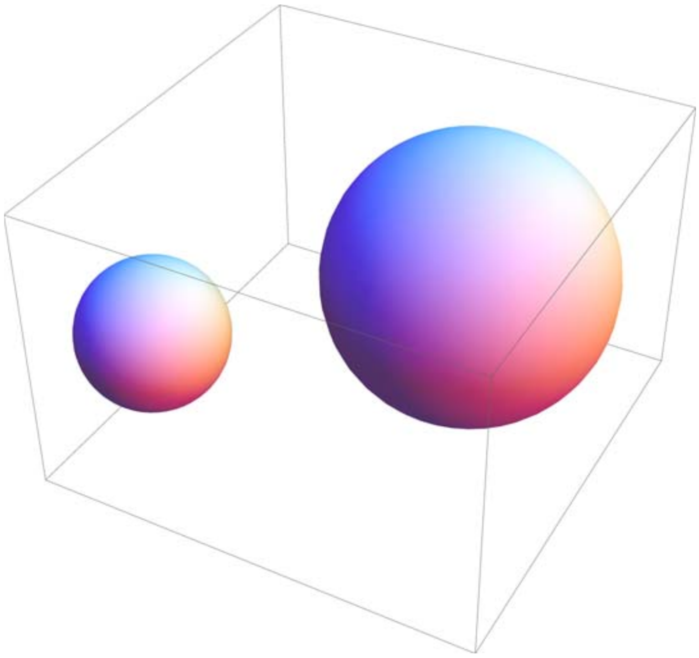
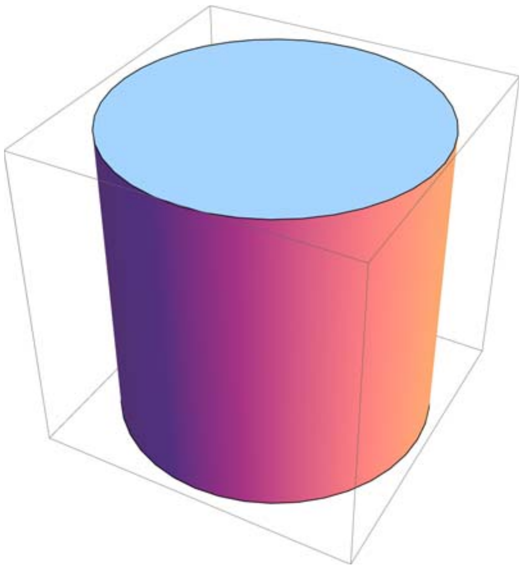
Various spheres in the same graph

```
Graphics3D[{Sphere[{0, 0, 0}, 1], Sphere[{3, 3, 0}, 2]}]
```



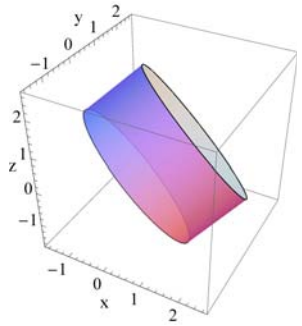
★ **Cylinders**

```
Graphics3D[Cylinder[]]
```



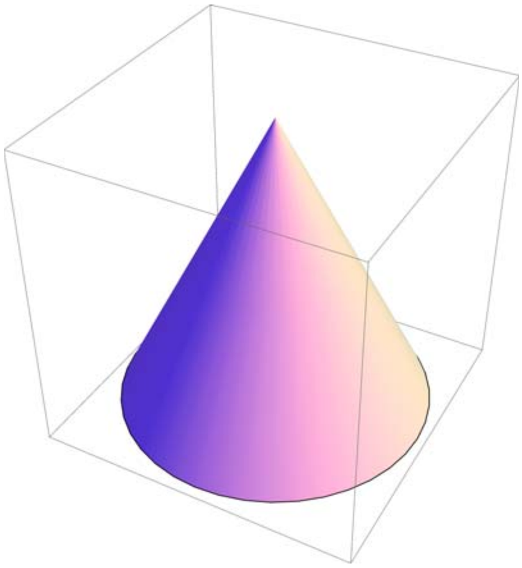
Cylinder of radius r=2 that goes from the origin to the point {1, 1, 1}

```
Graphics3D[{Opacity[0.8], Cylinder[{{0, 0, 0}, {1, 1, 1}}, 2]},
 Axes → True, AxesLabel → {"x", "y", "z"}]
```

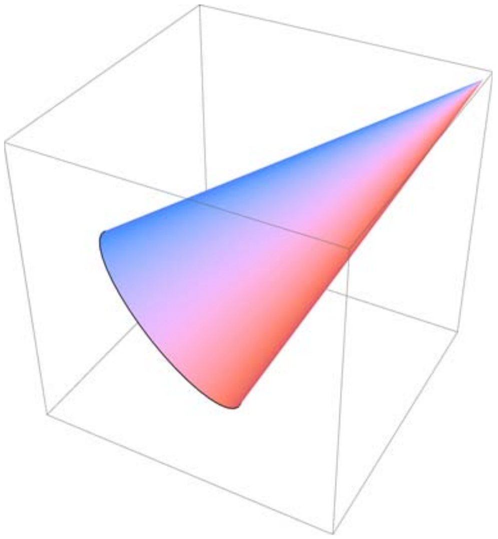


★ Cones

```
Graphics3D[Cone[]]
```



```
Graphics3D[Cone[{{0, 0, 0}, {1, 1, 1}}, 1 / 2]]
```



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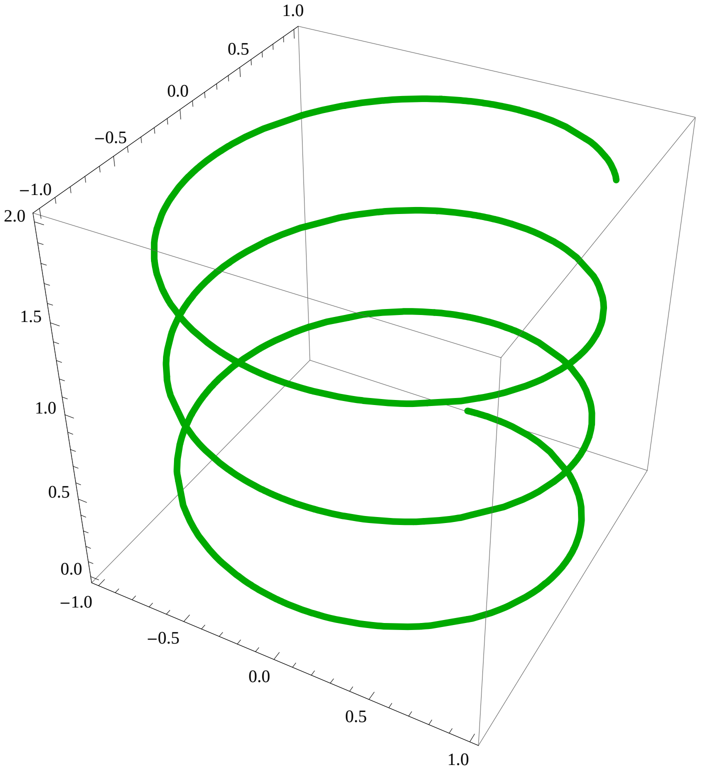
9.2. Parametrized curves and surfaces

▼ ParametricPlot3D[ ]

★ ParametricPlot3D[{functionx ,functiony, functionz},{u, umin, umax}]

This command represents a parametrized curve in the space, where the variable "u" takes values between umin and umax

```
c = ParametricPlot3D[{Sin[u], Cos[u], u / 10},
  {u, 0, 20}, PlotStyle → Directive[Darker[Green], Thickness[0.01]]]
```



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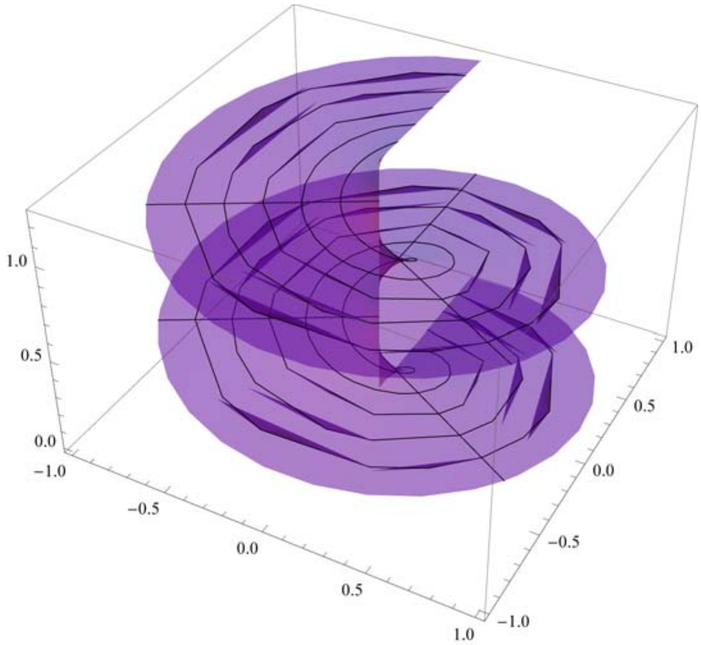
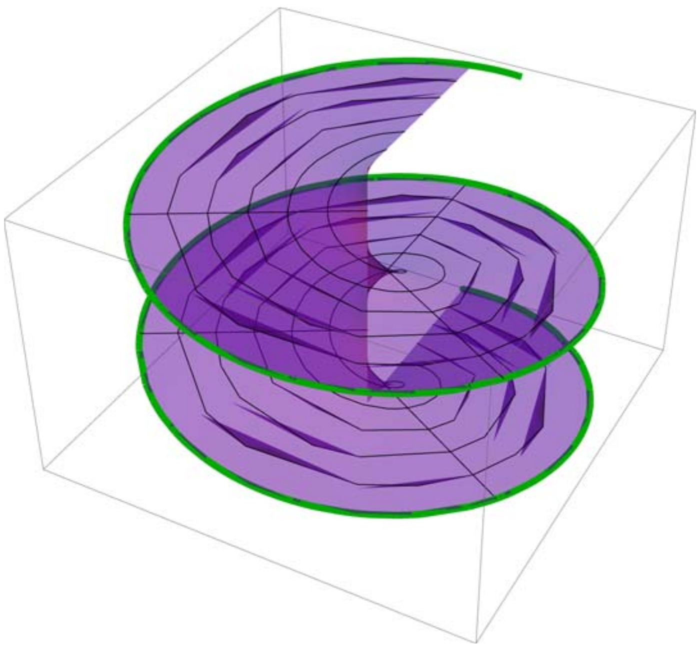
★ **ParametricPlot3D[{functionx ,functiony, functionz},{u, umin, umax},{v, vmin, vmax}]**

This command represents a parametrized surface in the space, where the variables “u” and “v” take values between umin and umax, and vmin and vmax respectively

```
s = ParametricPlot3D[{v * Sin[u], v * Cos[u], u / 10}, {u, 0, 4 Pi}, {v, 0, 1},
  PlotStyle → Directive[Purple, Opacity[0.5]], BoxRatios → Automatic, Mesh → 5]
```




```
Show[{s, c}, Axes → False]
```



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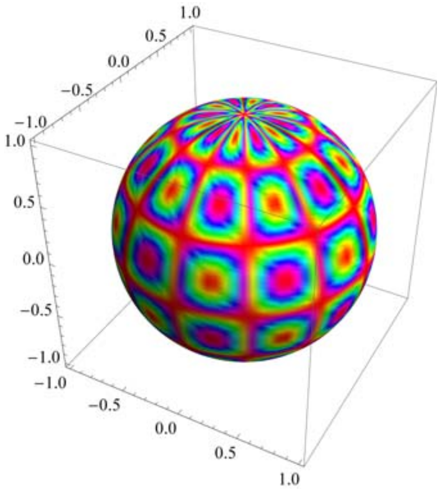
▼ Options of the command ParametricPlot

```
ParametricPlot3D[ {Cos[ ϕ] Sin[θ], Sin[ϕ] Sin[θ], Cos[θ]},
 {ϕ, 0, 2 π}, {θ, 0, π}, PlotPoints → 100, Mesh → None,
 ColorFunction → Function[{x, y, z, ϕ, θ}, Hue[Sin[6 ϕ] Sin[6 θ]]],
 ColorFunctionScaling → False]
```



▼ Spherical coordinates

The spherical coordinates are given by:

x(u,v)= r Sin[u] Sin[v]

y(u,v)= r Cos[u] Sin[v]

z(u,v)= r Cos[v]

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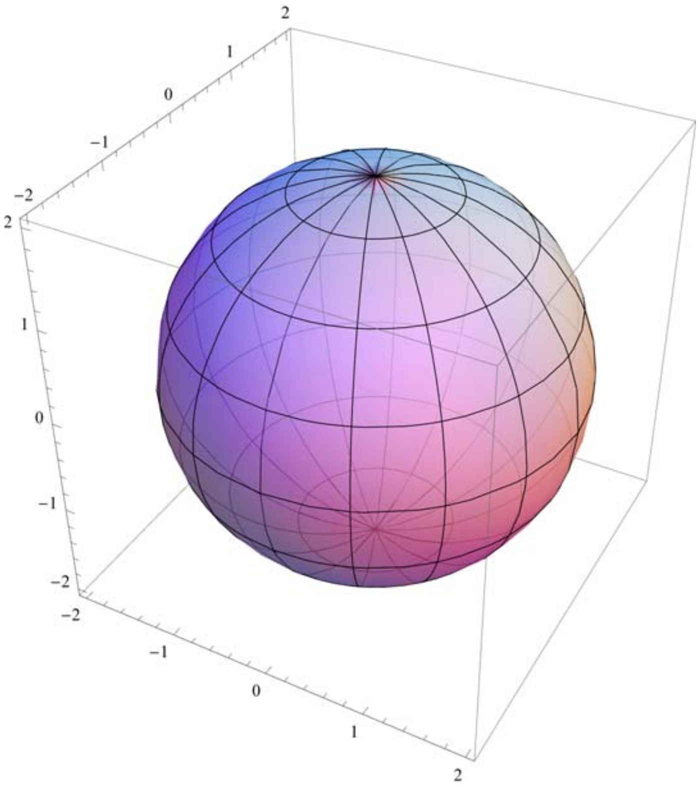
⋆ Sphere

```
ParametricPlot3D[ {2 Sin[u] Sin[v], 2 Cos[u] Sin[v], 2 Cos[v]},
 {u, -π, π}, {v, -π, π}, PlotStyle -> Opacity[0.5]]
```



▼ Cylindrical coordinates

The cylindrical coordinates are given by:

x(u,v)= r Cos[u]

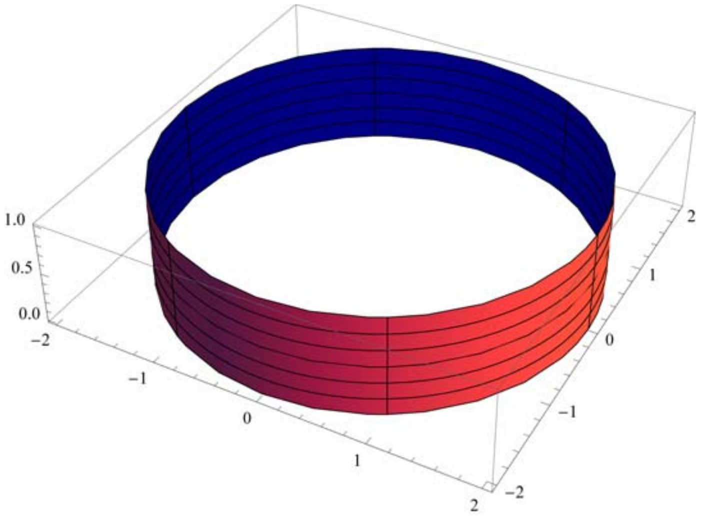
y(u,v)= r Sin[u]

z(u,v)= v

⋆ Cylinder

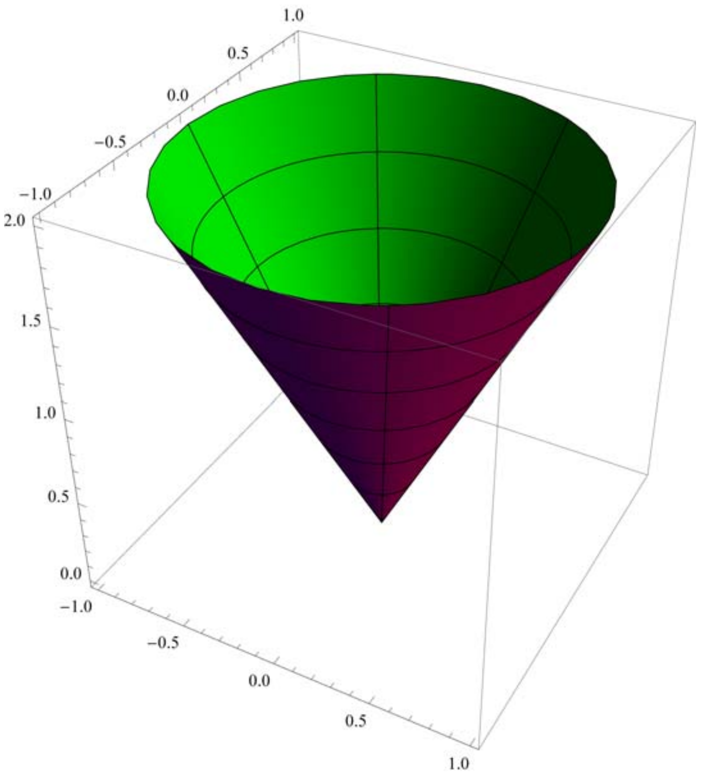
Cylinder of radius r=2, and delimited by the planes z=0 and z=1

```
ParametricPlot3D[{2 Cos[u], 2 Sin[u], v}, {u, 0, 2 Pi}, {v, 0, 1},
 Mesh → 5, BoundaryStyle → Black, PlotStyle → FaceForm[Pink, Blue]]
```



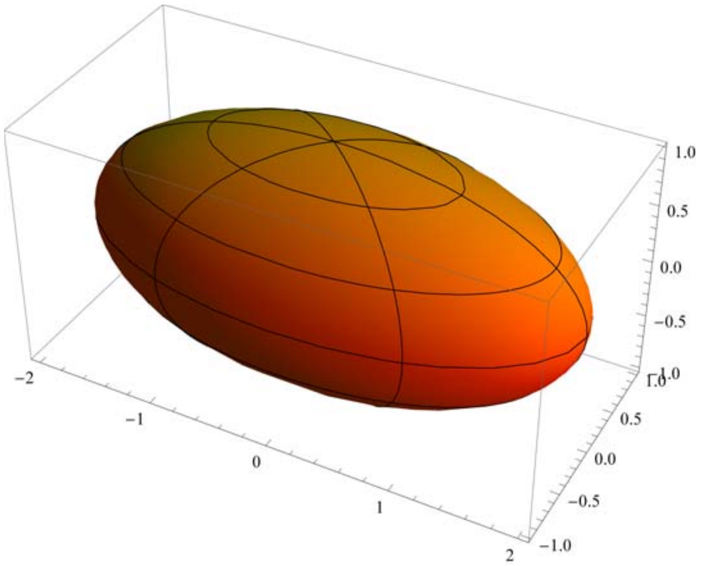
★ **Cone**

```
ParametricPlot3D[{v Cos[u], v Sin[u], 2 v}, {u, 0, 2 Pi}, {v, 0, 1},
 Mesh → 5, BoundaryStyle → Black, PlotStyle → FaceForm[Purple, Green]]
```



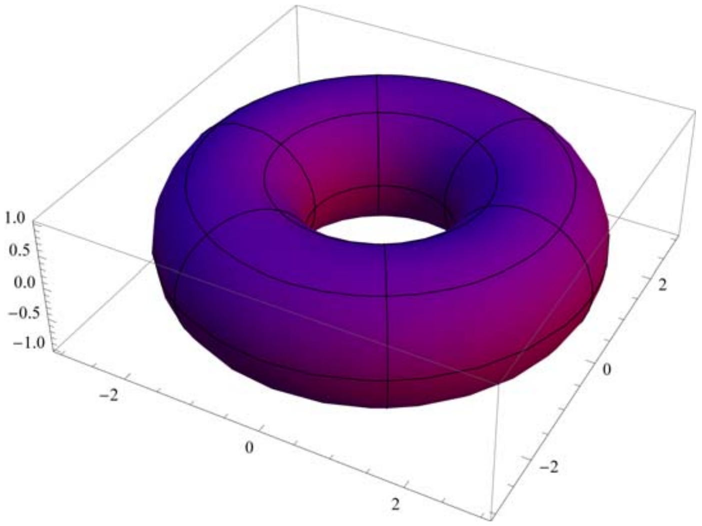
★ **Ellipsoid**

```
ParametricPlot3D[{2 Cos[u] Sin[v], Sin[u] Sin[v], Cos[v]}, {v, 0, Pi}, {u, 0, 2 Pi},
 Mesh → 5, BoundaryStyle → Black, PlotStyle → FaceForm[Orange, Yellow]]
```



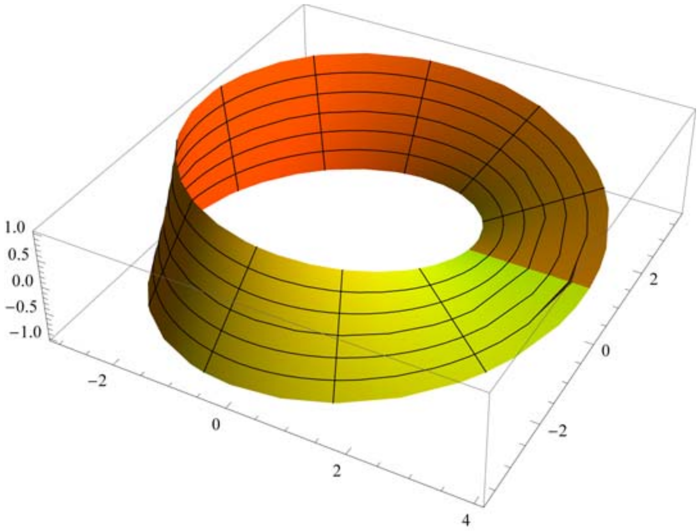
★ **Torus**

```
ParametricPlot3D[{(2 + Cos[v]) Cos[u], (2 + Cos[v]) Sin[u], Sin[v]}, {u, 0, 2 Pi},
 {v, 0, 2 Pi}, Mesh → 5, BoundaryStyle → Black, PlotStyle → FaceForm[Purple, Green]]
```



★ **Möbius band**

```
ParametricPlot3D[{Cos[t] (3 + r Cos[t / 2]), Sin[t] (3 + r Cos[t / 2]), r Sin[t / 2]},
 {r, -1, 1}, {t, 0, 2 Pi}, Mesh → {5, 10}, PlotStyle → FaceForm[Orange, Yellow]]
```

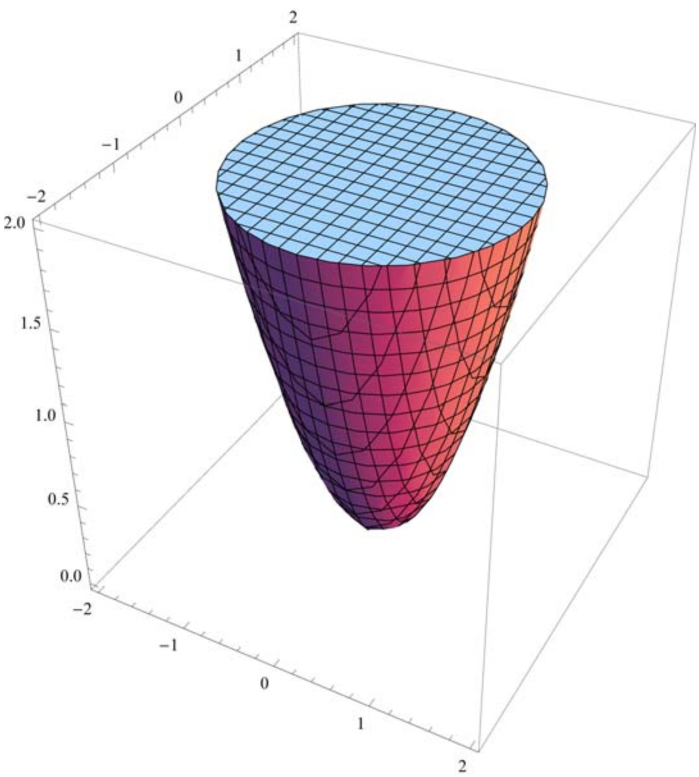


9.3. Regions in the space

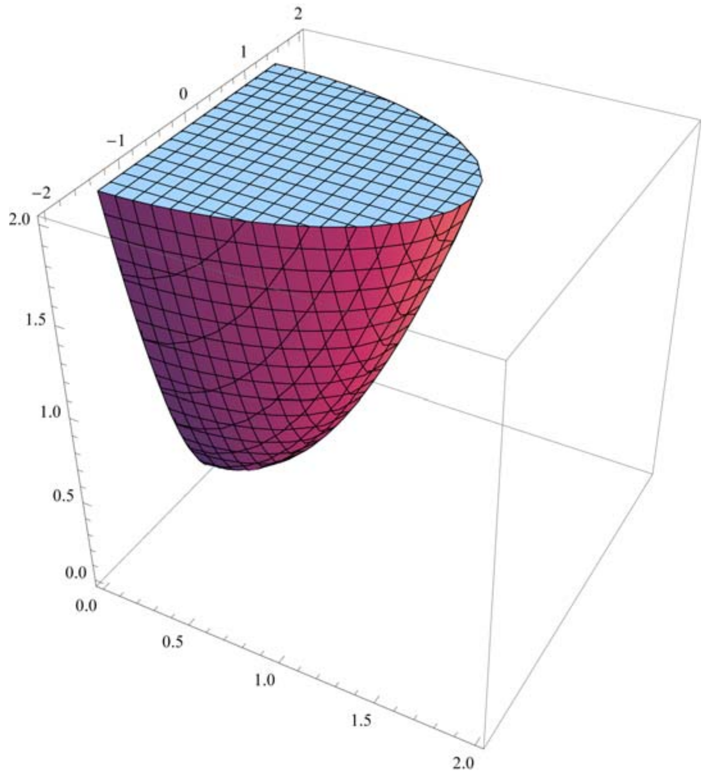
▼ RegionPlot3D[ ]

It is to plot the interior part of the specified curve. For example, the interior part of the following paraboloid can be plotted as:

```
RegionPlot3D[x^2 + y^2 < z, {x, -2, 2}, {y, -2, 2}, {z, 0, 2}]
```



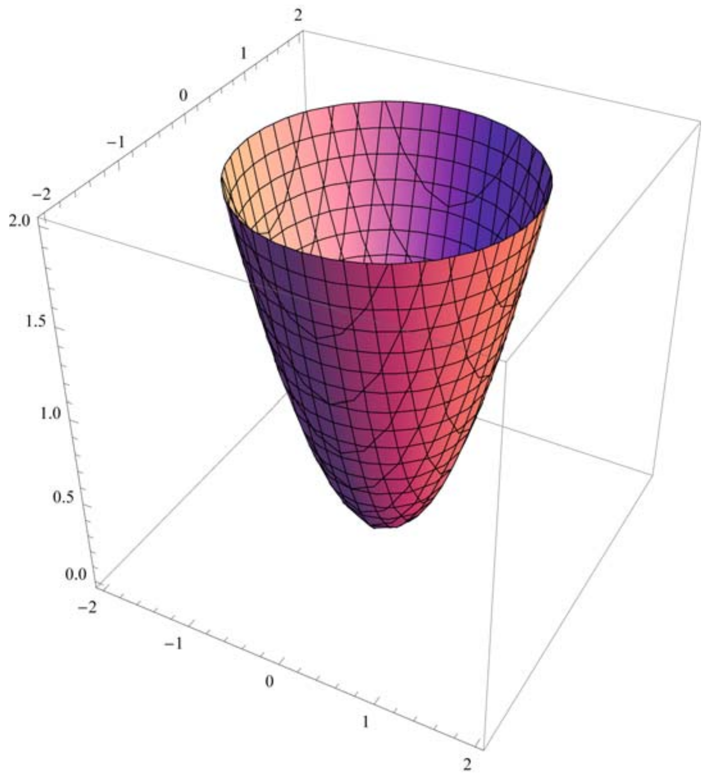
```
RegionPlot3D[x^2 + y^2 < z, {x, 0, 2}, {y, -2, 2}, {z, 0, 2}]
```



▼ ContourPlot3D[ ]

It is to plot the surface of the specified curve. For example, the surface of the following paraboloid can be plotted as:

```
ContourPlot3D[x^2 + y^2 == z, {x, -2, 2}, {y, -2, 2}, {z, 0, 2}]
```



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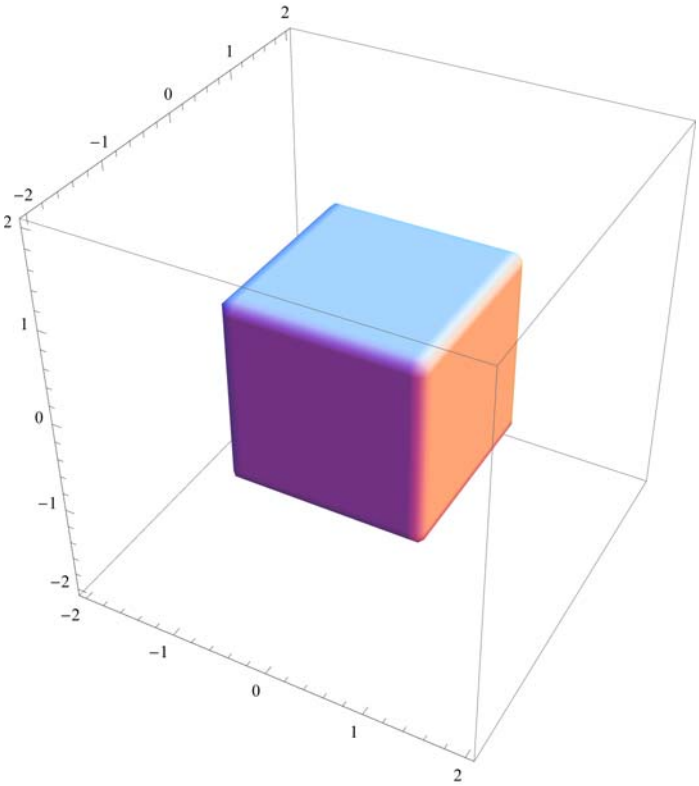
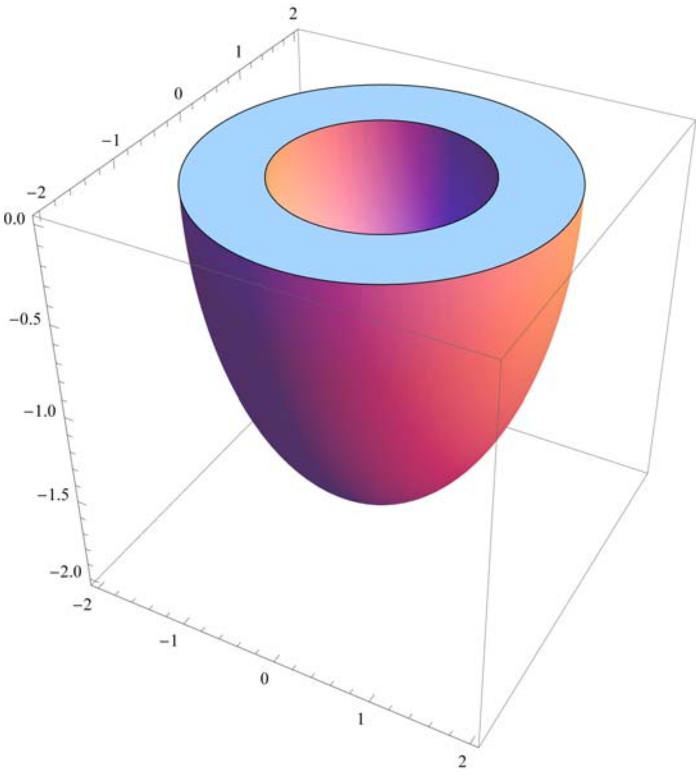
★ **Cube**

```
RegionPlot3D[-1 ≤ x ≤ 1 && -1 ≤ y ≤ 1 && -1 ≤ z ≤ 1,
 {x, -2, 2}, {y, -2, 2}, {z, -2, 2}, Mesh → None, PlotPoints → 50]
```



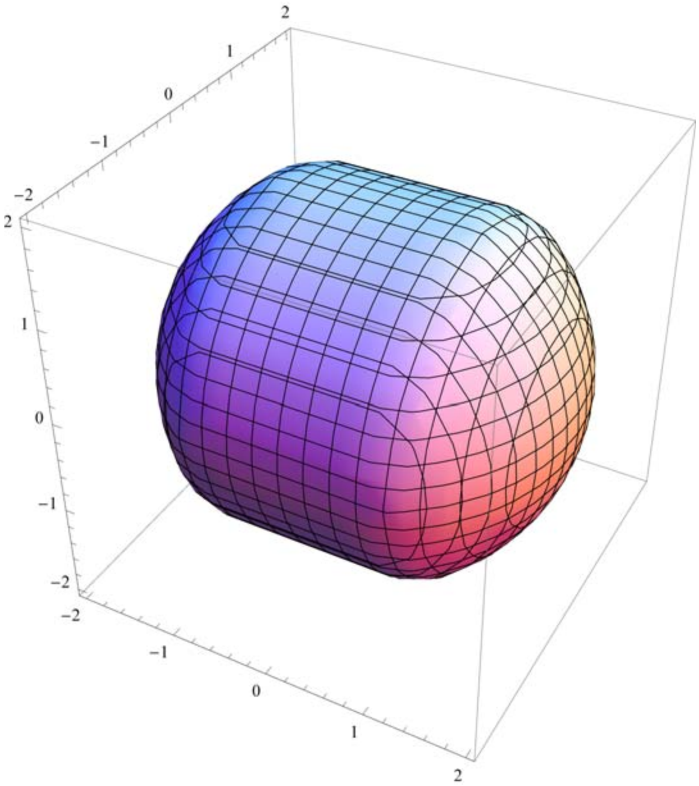
★ **Spherical surface**

```
RegionPlot3D[1 ≤ x^2 + y^2 + z^2 ≤ 3, {x, -2, 2},
 {y, -2, 2}, {z, -2, 0}, Mesh → None, PlotPoints → 50]
```

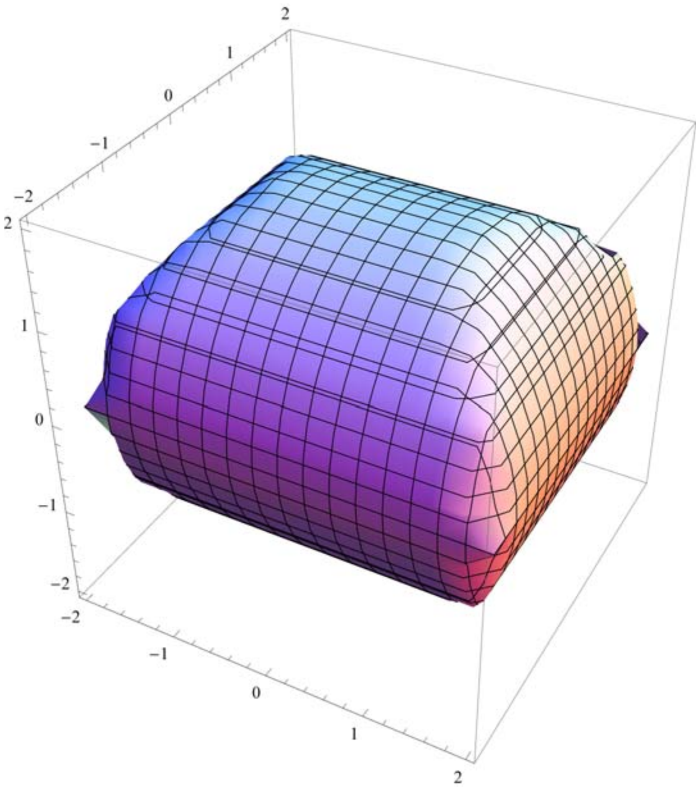


★ **Some other regions**

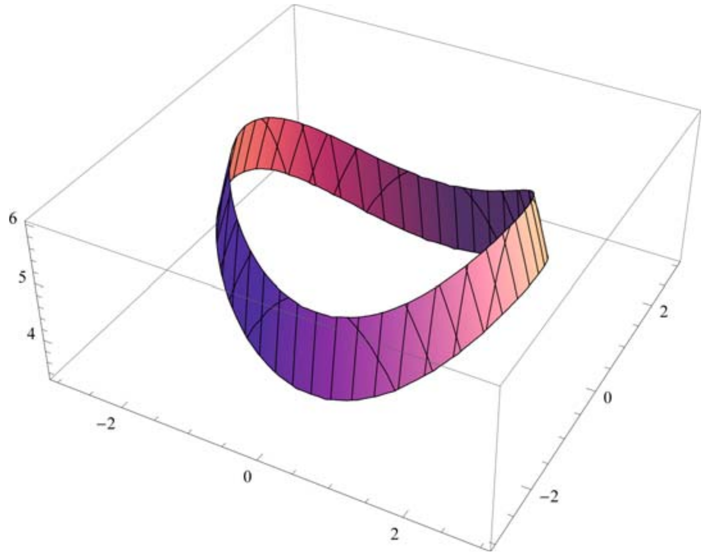
```
RegionPlot3D[x^2 + y^2 + z^2 ≤ 4 && y^2 + z^2 ≤ 3, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]
```




```
RegionPlot3D[x^2 + z^2 ≤ 4 && y^2 + z^2 ≤ 3, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]
```



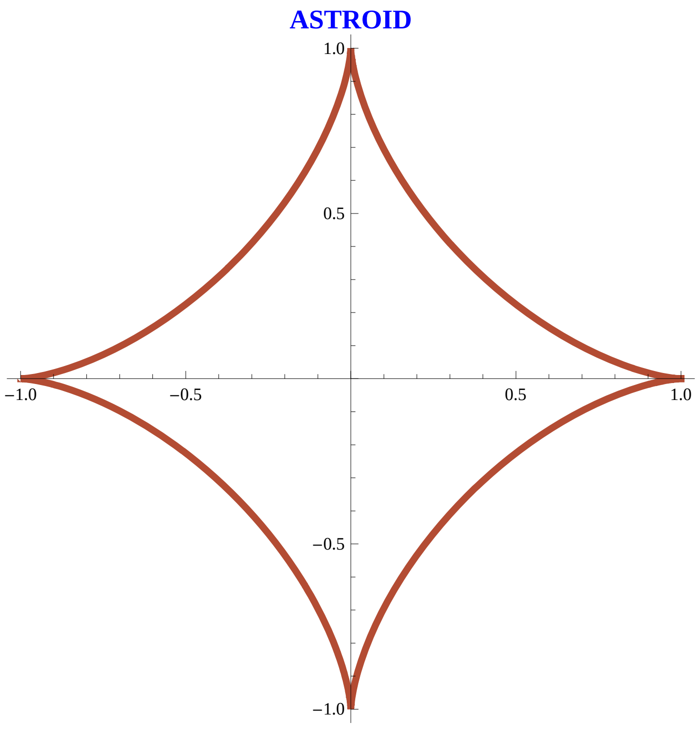
```
Plot3D[10 - x^2 - 2 y^2, {x, -3, 3}, {y, -3, 3},
 RegionFunction → Function[{x, y, z}, 8 < 2 x^2 + 3 y^2 < 10]]
```



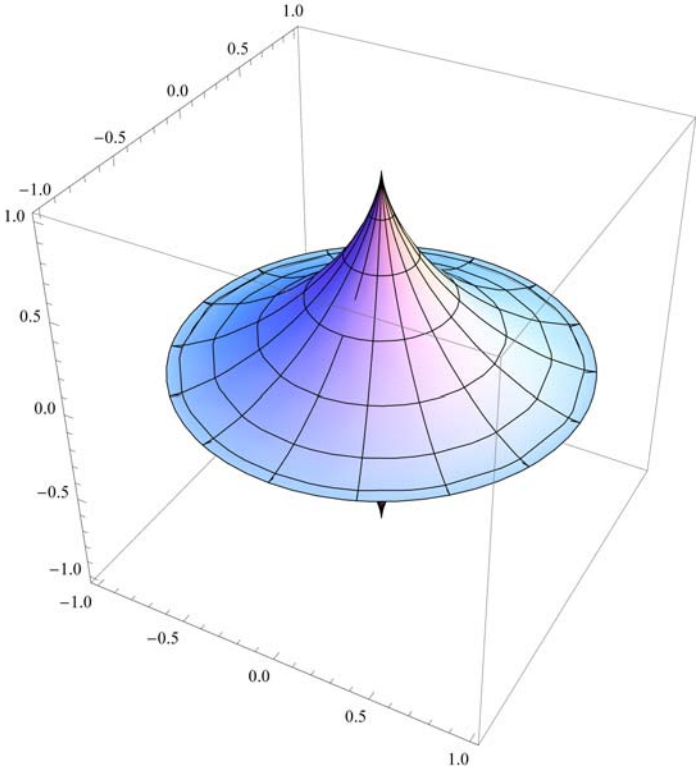
9.4. Revolution surfaces

★ **Astroid**

```
ParametricPlot[{Cos[t]^3, Sin[t]^3}, {t, 0, 2 π}, AspectRatio → Automatic,
 PlotStyle → {RGBColor[0.7, 0.3, 0.2], Thickness[0.01]},
 PlotLabel → Style["ASTROID", Bold, Blue, 14]]
```

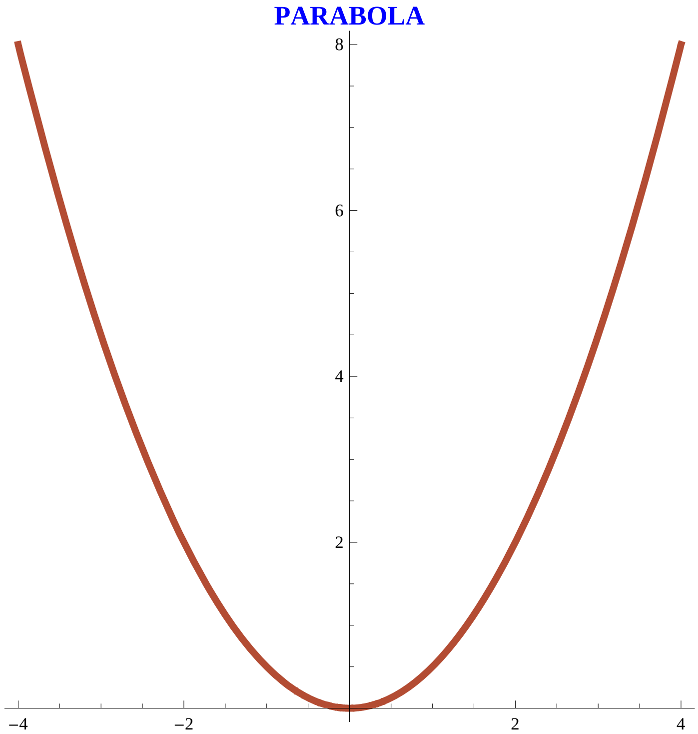


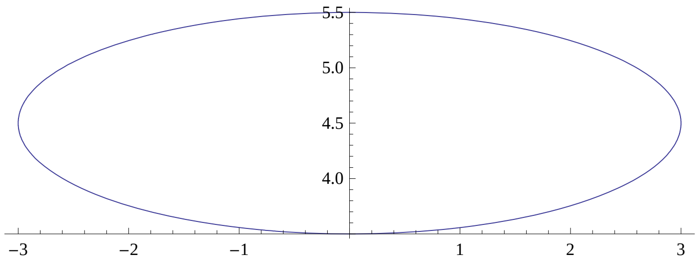
```
RevolutionPlot3D[{Cos[t]^3, Sin[t]^3}, {t, -Pi / 2, Pi / 2}, AspectRatio → Automatic]
```

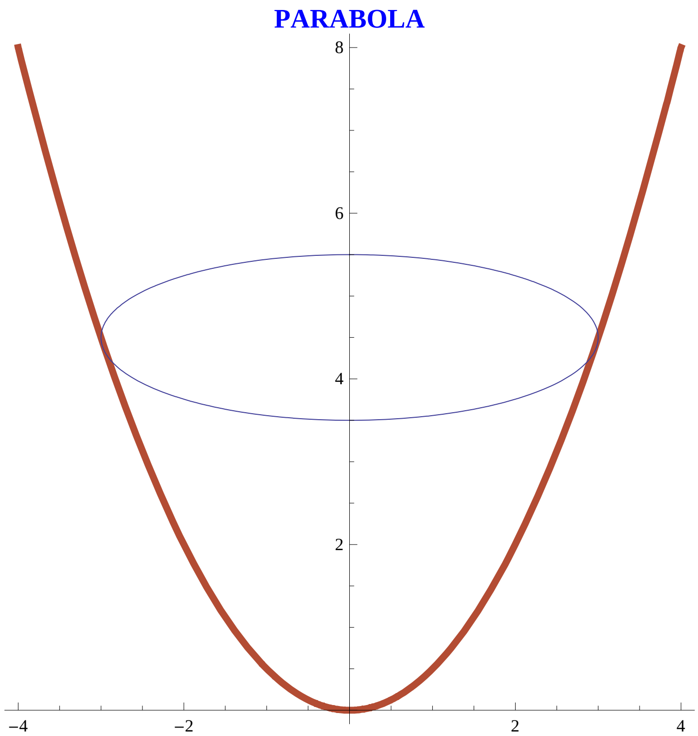


★ **Revolution paraboloid**

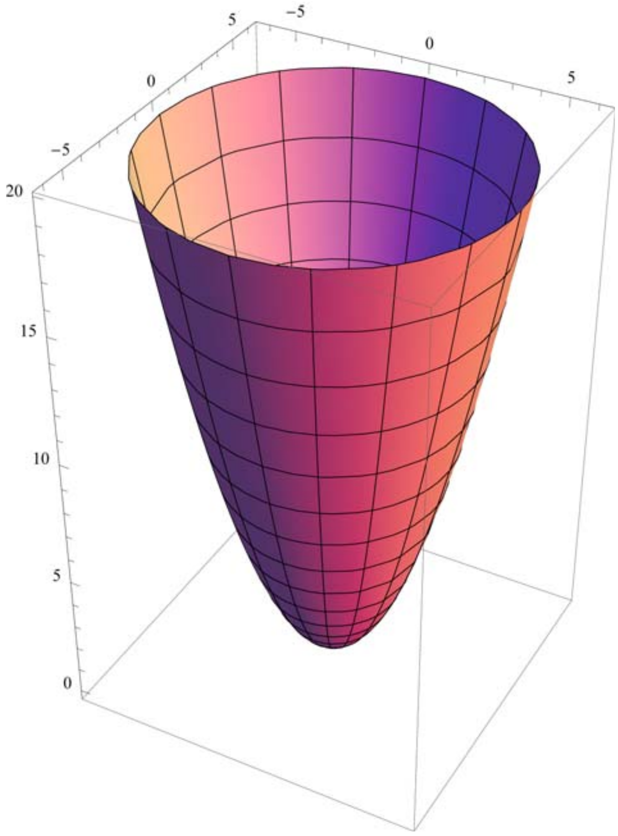
```
Clear["Global`*"]
splot1 = ParametricPlot[{t, 0.5 t^2}, {t, -4, 4}, AxesOrigin → {0, 0},
  AspectRatio → Automatic, PlotStyle → {RGBColor[0.7, 0.3, 0.2], Thickness[0.01]},
  PlotLabel → Style["PARABOLA", Bold, Blue, 14]]
ellipse[t_, a_, b_, c_, d_] = {a * Sin[t], b * Cos[t]} + {c, d};
splot2 = ParametricPlot[
  Evaluate[{ellipse[t, 3, 1, 0, 0] + {0, 4.5}}, {t, 0, 2 Pi}, AspectRatio → Automatic]]
Show[splot1, splot2]
```


PARABOLA
8
6
4
2
−4
−2
2
4


5.5
5.0
4.5
4.0
−3
−2
−1
1
2
3

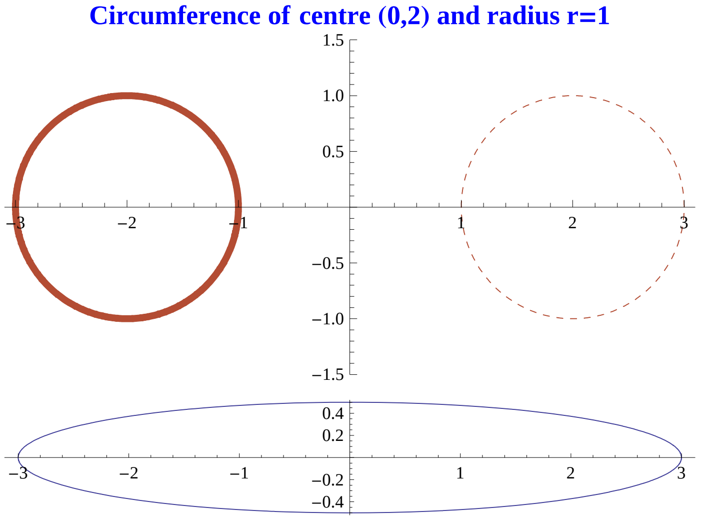

PARABOLA
8
6
4
2
−4
−2
2
4

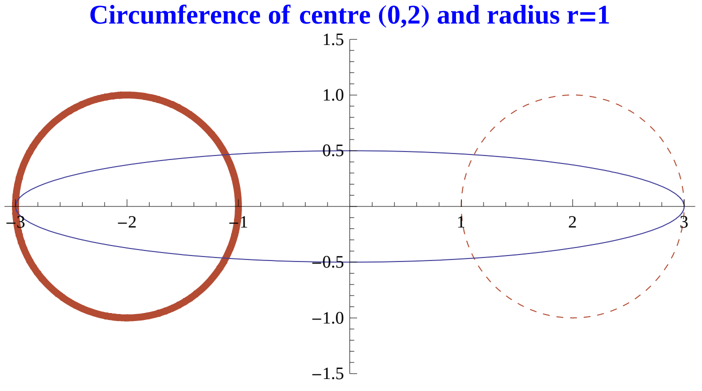
```
RevolutionPlot3D[{t, 0.5 t^2}, {t, 0, 2 Pi}]
```



★ **Torus**

```
splot3 = ParametricPlot[{{-2 - Cos[t], Sin[t]}, {2 + Cos[t], Sin[t]}},
   {t, 0, 2 π}, PlotRange -> {{-3.1, 3.1}, {-1.5, 1.5}}, AxesOrigin → {0, 0},
   AspectRatio → Automatic, PlotStyle → {{RGBColor[0.7, 0.3, 0.2], Thickness[0.01]},
     {RGBColor[0.7, 0.3, 0.2], Dashing[0.01]}},
   PlotLabel → Style["Circumference of centre (0,2) and radius r=1", Bold, Blue, 14]]
ellipse[t_, a_, b_, c_, d_] = {a * Sin[t], b * Cos[t]} + {c, d};
splot4 = ParametricPlot[
   Evaluate[{ellipse[t, 3, 0.5, 0, 0] + {0, 0}}, {t, 0, 2 Pi}, AspectRatio → Automatic]]
Show[splot3, splot4]
```





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```
ParametricPlot3D[{(2 + Cos[v]) Cos[u], (2 + Cos[v]) Sin[u], Sin[v]}, {u, 0, 2 Pi},
 {v, 0, 2 Pi}, Mesh → 5, BoundaryStyle → Black, PlotStyle → FaceForm[Purple, Green]]
```